Enhancements of Electric Power Systems Operation Using FACTS Devices

by

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Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degrees of

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Abstract

In this thesis, the enhancement of electric power systems operation using FACTS devices is studied.

The study consists of two parts: a study of transmission capacity and steady state analysis of FACTS devices.

In a study of transmission capacity (TC) a set of possible definitions of TC suitable for power networks of arbitrary topology is given first in order to correctly describe the operational status of any system. For given set of definitions a novel method for accurately evaluating TC is developed, through which the problem of calculating TC in an electric power network of arbitrary topology and size is posed as a problem of solution existence to a non-linear DC resistor network.

In steady state analysis of FACTS devices we explore one of the benefits of FACTS controllers: an increase in security region resulting in maximum network loadability. Through simulations the optimum increase in maximum network loadability is found associated with employing a FACTS device in a power network.

Thesis Supervisor: Marija D. Ilić
Title: Senior Research Scientist

To my family and friend:

Mom and Dad, Peter,

In-sook, Jane, Han,

and Ji-Young

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Chapter 1

Introduction

As a result of the current deregulation of the power industry, the role of transmission system has a greater importance in the operation and planning of electric power systems than it has in the past. In the traditional regulated industry environment, where global system planning and strong design of generation and transmission systems were possible the demand for power was met with little difficulty. However, global system planning has become virtually impossible under the deregulated industry environment, especially with the presence of non-utility generators (NUG). Moreover, recent environmental issues and cost problems have made addition of new transmission lines extremely arduous [1]. Demand, on the other hand, has grown continuously. Consequently, it has become increasingly necessary to operate power systems closer to their transmission capacity limits.

1.1 Network Limits

In a transmission network capacity limits exist because the amount of power that can be transferred safely through each transmission line is bounded by thermal and stability limitations imposed on the line.

Thermal limitations arise because physical properties of transmission lines are functions of temperature. For instance, the temperature of a transmission line exceeds its maximum specification, the transmission line could sag between supporting towers potentially resulting in a violation of the minimum ground clearance restrictions. Although various factors affect the temperature of a line, the loss dissipated through heat due to the conducting current on the line, I^2R, is the most significant. Thus, the thermal limitation can be expressed as a conducting current limitation. Since lines are designed to be used close to certain nominal voltage level, the conducting current limitation can easily be converted to transmission capacity limits (either given in terms of the maximum apparent power (MVA) or the maximum real power (MW)).

Stability limitations exist in order to ensure that a system can recover from a fault. There are two types of stability limitations: steady state limits and transient limits. For a lossless transmission line as shown in Figure 1-1 the real power from bus i to bus j is given by

$$\begin{aligned} P_{ij} &= \frac{V_i V_j}{X_{ij}} \sin(\delta_i - \delta_j) \\ &= \frac{V_i V_j}{X_{ij}} \sin \delta_{ij} \end{aligned} \tag{1.1}$$

where $\sin \delta_{ij} = \sin(\delta_i - \delta_j)$.

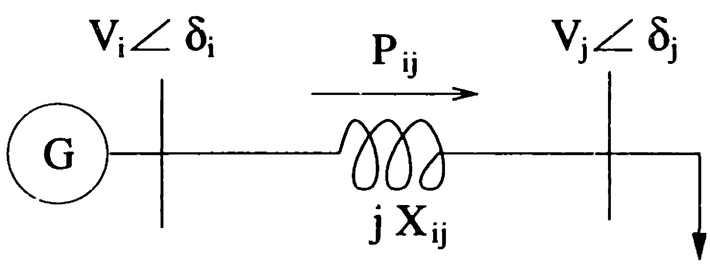

Figure 1-1: One line diagram of lossless line

The steady state limit of a line is set by the maximum power, P_{max}, that can be transferred through the line. Figure 1-2 shows that

$$P_{max} = \frac{V_i V_j}{X_{ij}}. \tag{1.2}$$

Transient limits are set to ensure that a system recovers from a fault. A typical

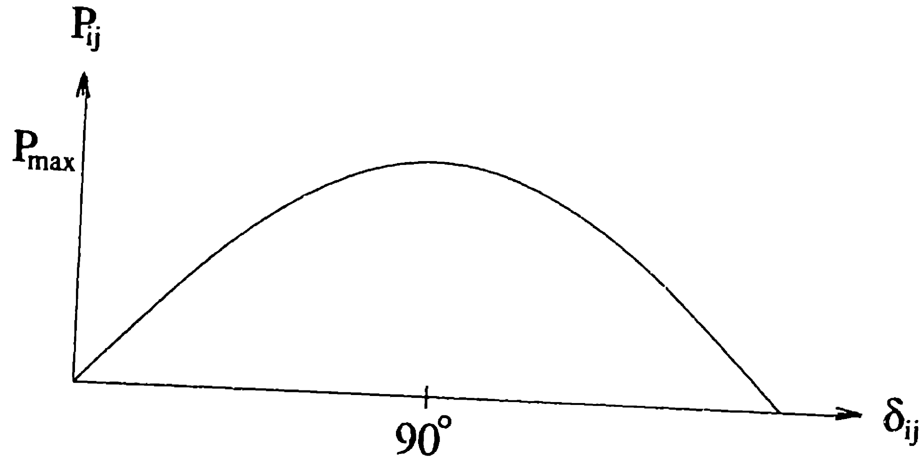

Figure 1-2: Real power through line ij as a function of δ_{ij}

transient limit is usually given by

$$|\delta_{ij}|_{max} = 30^\circ \tag{1.3}$$

It is interesting to note that the maximum possible transfer of power specified by a typical transient limit is exactly the half of P_{max} given in Eq. (1.2). The transmission line capacity limits are the most restrictive of above limits.

In addition to transmission limits, a power system is subject to other limits imposed on the buses of a system. Specifically, these are voltage magnitude at generators and loads, V_g's and V_l's respectively, real power generation outputs, P_g's, and reactive power generation outputs, Q_g's.

1.2 Network Security

In the industry, the term *network security* refers to the operation of a network within its capacity limits. Network security simply means that the operational status of the system is able to meet the required present and future demands safely, even in the event of a possible large disturbance [2]. When network security is threatened, various corrective actions must be applied to the system so that network security can be restored. Network security is an instantaneous state which changes with time because it depends on the network operating conditions which are also time dependent. Network security is frequently quantified by what is often referred to as the $N-1$

contingency requirement. The $N-1$ contingency requirement states that a system must be operated at all times with a level of network security such that the system can withstand the occurrence of any single contingency without losing system integrity - none of the stability limits are violated. Possible contingencies are network component losses such as transmission line outage, generator outage, etc. In the presence of automated protection devices this requirement is particularly important because a single contingency could result in cascading outages throughout a system. Cascading outages occur when a loss of one component results in a violation of stability limits and activates an automatic protection device which shuts off components under stress, the protective action could lead to further violations of other stability limits thus activating more automatic protection devices and shutting off more components in the system.

As more automatic protection devices are being employed, the accurate evaluation of network security, or security analysis is of greater importance to the operation of the power system. Therefore, security analysis should be done regularly and frequently to ensure safe operation of the system. Such an analysis requires three basic steps: monitoring, assessment, and control.

1.2.1 Network Monitoring

Through network monitoring the current operational status of the system is measured. First, a system operator measures system variables such as voltage magnitudes, power injections, the status of circuit breakers and switches at the buses, and current and power flows on lines wherever possible. Then, using a mathematical procedure called state estimation, the operator determines values for the remaining system variables [3]. This yields a complete picture of the current operational status of the network. Once the network monitoring is complete, the system assessment starts.

1.2.2 System Assessment

Through system assessment the current operational status of the system is evaluated with respect to the set of limits imposed on the system. Based on this evaluation the operator determines whether the state of operation is normal, alert, emergency, or restorative [4]. Figure 1-3 shows the relationship among the four states of operation.

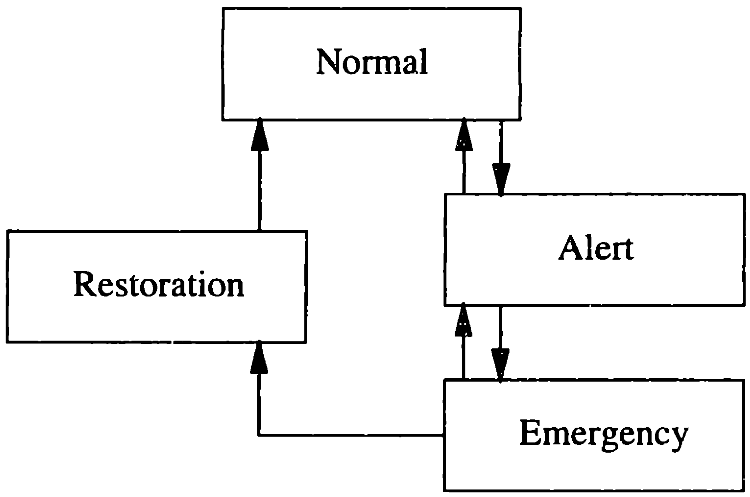

Figure 1-3: Dy Liacco diagram of system operation states

In the normal state the system is operated within all limits while also satisfying the $N-1$ contingency requirement.

In the alert state the system is operated very close to, but still within, the limits. Unfortunately, the $N-1$ contingency requirement can no longer be satisfied. This means that the occurrence of certain next contingencies could cause some system limits to be violated.

In the emergency state the system is operated while violating a few system limits. Obviously, the system cannot be operated in this state for very long without damaging some equipment in the system.

In the restorative state the operating condition of the system is being altered to restore the stability of the system. The system is again operated within the stability limits.

Once the system assessment is finished, appropriate security control begins.

1.2.3 Security Control

Security control drives the system to the desired state of operation using various control actions available on the system. Control actions can be characterized as preventive, corrective, or restorative depending on the way they are applied.

Suppose the system is being operated in the alert state. Then preventive actions are applied to avoid a possible violation of stability limits in the future in case of the occurrence of certain contingencies. After applying the appropriate actions, the system should be ideally operated in the normal state satisfying the $N-1$ contingency requirement.

Suppose the system is being operated in the emergency state. Then, corrective controls are applied to the system so that network security is restored. By implementing these actions the system can operate without fear of damaging the equipment as long as there are no other contingencies in the system.

Suppose the system is forced to a collapse despite all control actions. Then, restorative control actions are applied to bring the system back to its normal or alert state of operation.

The possible corrective actions on a system depend on the availability of control devices on the system. A few examples of corrective actions are:

- redispatching of generation
- adjustment of interchanges with neighboring power networks
- switching of reactive sources
- adjustment of transformer taps
- load shedding [5]

1.2.4 Security Optimization

In the security control stage a system operator must be aware that two different control actions may lead to the same desired goal but at very different expenses concerning operation costs, system losses, and the complexity of operation. Therefore, the system operator must choose control actions with a clear understanding of the consequences of various control actions. Finding the most suitable control actions thus becomes the optimization problem of operation costs, system losses, system load that can be served and the number of control actions required.

1.3 Security Region

For a given network the security region prescribes the range of system variables, $\mathbf{x}$ such that $\mathbf{x}$ satisfies both the equality and inequality constraints, $\mathbf{f}(\mathbf{x})$, imposed on the network. Mathematically, the security region, $\mathbf{S}$, of the given network can be expressed by

$$\mathbf{S} = \{\mathbf{x}|\mathbf{f}(\mathbf{x}) \leq 0\}. \tag{1.4}$$

The system variable, $\mathbf{x}$, will be defined more concretely in later chapters. It is important at this juncture, however, to point out that $\mathbf{x}$ consists of two different types of variables: controllable and uncontrollable. Examples of controllable variables, $\mathbf{u}$, are voltage magnitude and the real power output of generator buses. Examples of uncontrollable variables, $\mathbf{s}$, are voltage angle and the reactive power output of generator buses. Because not all system variables are controllable (i.e. set to be any arbitrary values), the security region does not include the whole space. Thus, the most optimal operating condition cannot always be found. This leads to the study of power system devices to increase the size of the security region because a larger security region allows more flexible and robust system operation and thus provides a better chance of finding the optimal operating condition.

1.4 Potential Benefit of FACTS devices

The control devices on today's power systems are mostly mechanical since switching of the devices is done manually. So switching of such devices cannot be done frequently due to the fear of wearing out mechanical devices. This translates into somewhat inefficient operation since it means that network security is met mainly through generation scheduling (overdesign as mentioned earlier) and only slow switching of control devices, such as the changing of power transformer tap settings and the switching of shunt capacitors.

New technological developments in power electronics have provided an alternative solution to the above mentioned inefficiency. The new devices typically referred to as Flexible AC Transmission Systems (FACTS) controllers, make it possible to directly control power flow through certain lines using fast electronic switching. FACTS devices allow certain parameters of transmission lines to be changed thus enabling the re-routing of certain power transfers without resorting to generation scheduling.

According to a study done by Electric Power Research Institute (EPRI), the direct control of power flows through fast and frequent switching of FACTS devices presents the following advantages over old, mechanically switched control devices [6]:

1. Better response to stability problems which results in a reduction of the required transmission stability margin
2. Greater control of power so that it flows on the prescribed transmission routes
3. Reduction in the cost of operation by avoiding expensive generation scheduling

However, the devices being passive have inherent limits in how much they affect the overall situation on the system [].

1.5 Problem statement

The study consists of two parts: a study of transmission capacity and steady state analysis of FACTS.

The study of transmission capacity on a power network begins with re-visiting and understanding of the term *transmission capacity.* While this notion is uniquely defined for the simplest two bus power network, different definitions are possible for an arbitrary size system. A set of possible definitions suitable for power networks of arbitrary topology is given first in order to correctly describe the operational status of any system. Once a set of definitions is chosen, methods for accurately evaluating transmission capacity are developed. This enables bottlenecks or the weak points of power flow in a system to be located for any given operating points, which gives good insight into how the power flow on lines within a system relate to individual component variables of the system, such as power injections and voltage magnitudes on buses.

Once some insights are gained about a system, a steady state analysis of the effect of FACTS controllers is performed to find ways to optimize the use of the FACTS controllers for the system. A set of different models that closely describe currently available FACTS devices is developed. Based on the models, the impact of FACTS at various locations on an arbitrary network is examined using sensitivity studies. The study is then generalized to formulate rules that suggest ways to employ FACTS controllers so that their benefits are maximized. Since this analysis involves large signal changes, use of nonlinear methods is essential. These methods are compared to linear approximation methods.

The theoretical development is to be done through simulations on simple power systems.

Chapter 2

Modeling

This chapter reviews the theoretical background needed for the work in this thesis. First, modeling of a power system and its components is introduced. Using the system variables and system matrices introduced in the modeling the load flow problem is formulated. Then, the notion of power flow control is presented along with a brief description of power flow control devices either currently available or under development. Finally, the modeling of an ideal FACTS device used throughout the thesis is explained.

2.1 Power System Modeling

Figure 2-1 shows a typical general representation of a simple power system. The simplest system consists of three basic individual components: loads, generators and transmission lines. Four system variables, namely voltage magnitude, V_i, phase angle, δ_i, real power injection, P_i and reactive power injection, Q_i are used to characterize each component i; usually for each component two system variables are controllable and the other two are uncontrollable. In addition, for steady state analysis one of the generator us designated to have a reference phase angle unchanged. This is to deal with the fact that not all power inputs can be specified prior to the analysis. Losses are not known ahead of time. Mathematically, a slack generator compensates for system-wide losses and eliminates the singularity problem created by not being

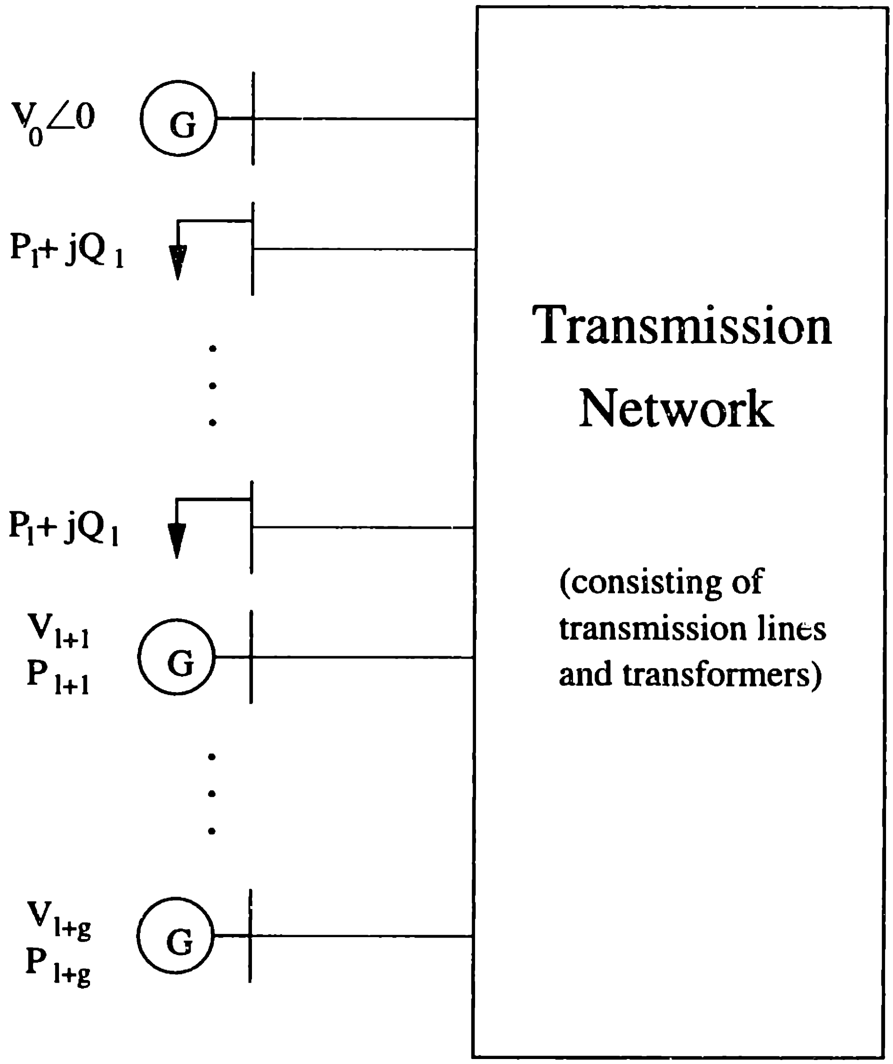

Figure 2-1: General representation of a simple power system

able to pre-specify its power into the system. In the actual operation this function is achieved by means of AGC.

A slack generator, generators and loads are distinguished by the different set of controllable system variables.

slack generator: a power source of infinite power output capability.
controllable variables are voltage magnitude, $V_0 = 1$ $(p.u.)$, and phase angle $\delta_0 = 0$ $(°)$. ideal voltage source.

loads: power sinks.
controllable variables are real and reactive power injection, P_i, and Q_i respectively where $i = 1, ..., l$.[1]

generators: power sources.
controllable variables are real power injection, P_i, and voltage magnitude, V_i where $i = l + 1, ..., l + g$.

For the network operating in steady state or in quasi-sinusoidal steady state, all system variables are assumed to be constant for a given operating point or vary slowly with respect to steady state frequency.

The π-equivalent model shown in Figure 2-2 is often used to describe transmission line ij [7].

Two system matrices are used to characterize the topological properties of the transmission network, namely reduced incidence matrix **A** and admittance matrix **Y**.

A is useful for describing purely topological properties of the system; how buses are connected by transmission lines in the system. Suppose b is the number of transmission lines in the system. Then **A** is the matrix whose dimensions are $b \times (l + g)$ and whose entries, a_{ij}, are given by:

$$a_{ij} = \begin{cases} 1 & \text{if there is a transmission line connecting } i \text{ and } j, \text{ and } i < j \\ -1 & \text{if there is a transmission line connecting } i \text{ and } j, \text{ and } i > j \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

Y combines the topological properties of the system with the characteristic properties of transmission lines. The dimensions of **Y** are given by $(l + g) \times (l + g)$, and

[1]This modeling assumption is not always met, and sometimes different load models are needed.

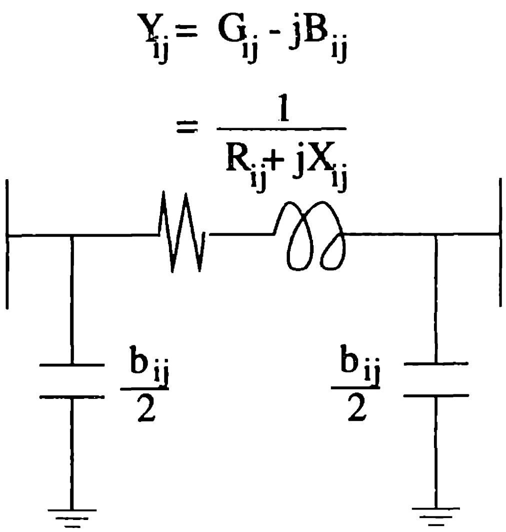

Figure 2-2: π-equivalent model of transmission line

the entries of $\mathbf{Y}$, y_{ij}, are given by:

$$\begin{aligned} y_{ii} &= \sum_{j \neq i} Y_{ij} \\ y_{ij} &= -Y_{ij} \end{aligned} \tag{2.2}$$

where Y_{ij} is the admittance of transmission line ij as shown in Figure 2-1. $\mathbf{Y}$ provides a convenient notation of relating various system variables. For example, the network bus voltages, $\mathbf{V}$, and currents, $\mathbf{I}$, are related simply by:

$$\mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_{l+g} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1(l+g)} \\ y_{21} & y_{22} & \cdots & y_{2(l+g)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_{(l+g)1} & y_{(l+g)2} & \cdots & y_{(l+g)(l+g)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_{(l+g)} \end{bmatrix} = \mathbf{YV}. \tag{2.3}$$

2.1.1 Load Flow Problem

From the definition of complex power at bus i, $\hat{S}_i$,

$$\begin{aligned} \hat{S}_i &= \hat{V}_i \hat{I}_i^* \\ &= \hat{V}_i \sum_{j=0}^{l+g} \hat{Y}_{ij}^* \hat{V}_j^* \qquad i = 0,\ 1,\ \dots\ ,\ \text{l+g}. \end{aligned} \tag{2.4}$$

Let

$$\begin{aligned} \hat{V}_i &\equiv V_i e^{j\delta_i} \\ \delta_{ij} &\equiv \delta_i - \delta_j \\ \hat{Y}_{ij} &\equiv G_{ij} + jB_{ij} \end{aligned}$$

where G_{ij} and B_{ij} are the conductance and susceptance of transmission line ij respectively. Then Eq. (2.4) becomes

$$\begin{aligned} \hat{S}_i &= \sum_{j=0}^{l+g} V_i V_j e^{j\delta_{ij}} (G_{ij} - jB_{ij}) \\ &= \sum_{j=0}^{l+g} V_i V_j (\cos\delta_{ij} + j\sin\delta_{ij})(G_{ij} - jB_{ij}) \qquad i = 0,\ 1,\ \dots\ ,\ \text{l+g}. \end{aligned} \tag{2.5}$$

Separating the real and imaginary part of $\hat{S}_i$ into P_i and Q_i

$$\begin{aligned} P &= \sum_{j=0}^{l+g} V_i V_j (G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) \\ Q &= \sum_{j=0}^{l+g} V_i V_j (G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \end{aligned} \tag{2.6}$$

Thus, the load flow equations are of the form:

$$\begin{aligned} \mathbf{P} &= \mathbf{P}(\mathbf{V}, \delta) \\ \mathbf{Q} &= \mathbf{Q}(\mathbf{V}, \delta). \end{aligned} \tag{2.7}$$

Given the controllable system variables, the objective (often referred to as the *full blown load flow problem*) is to solve Eq. (2.7) for the uncontrollable system variables. This is a nonlinear problem and the solution cannot be found analytically in most case. The Gauss iterative method and Newton-Raphson method are often used to find numerical solutions to the problem.

Simplifications to The Load Flow Problem

Some simplifications to Eq. (2.6) can be made to reduce the number of iteration steps that must be taken. One such simplification comes from considering only the real power part of the load flow problem. It turns out that the real power at any bus i is not affected much by voltage magnitude deviations at buses during the iteration steps [7]. For this reason, the voltage magnitude at bus i can be assumed to be fixed during the iteration steps when solving only the real power part of the load flow problem. The reduced problem is given by:

$$P = \sum_{j=0}^{l+g} V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \tag{2.8}$$

where both V_i and V_j are fixed. This simplified load flow problem is called *decoupled* $(P - \delta)$ *load flow.*

Further simplification can be made to the decoupled $(P-\delta)$ load flow by assuming:

- the voltage magnitude at bus i is fixed at 1 ($p.u.$); $V_i = 1$ $p.u.$, or some other value corresponding to nominal operation

- typically the resistance R_{ij}, of transmission line ij is much smaller than the reactance X_{ij} of the same line; $X_{ij} \gg R_{ij}$.

- the network is made up of short lines; $|\delta_i - \delta_j| \ll 1$ so that $\sin \delta_{ij}$ can approximated by $\delta_i - \delta_j$

The linearized form of Eq. (2.8) under the above mentioned assumptions is then

$$P = \sum_{j=1}^{l+g} \frac{1}{X_{ik}} \delta_{ij}. \tag{2.9}$$

This most simplified form of the load flow problem is called *DC load flow.*

For the rest of this thesis only decoupled $(P - \delta)$ load flow and DC load flow will be considered.

2.2 Power Flow Control

Transmission systems have been designed with fixed or mechanically switched series and shunt reactive compensations together with voltage regulating and phase shifting transformer tap changers to optimize line impedance, minimize voltage variation, and control power flow under steady-state or quasi-sinusoidal steady state conditions. The designs also have included enough stability margins to recover from a single contingency such as equipment failure faults and transmission line or generator outages.

During the last two decades major, if not revolutionary, advances have been made in high power semiconductor devices and sophisticated electronic control technologies.

Power electronics technology, using ordinary thyristors, provides a means for the rapid control of power flows on a transmission network. Modern implementation of traditional methods (such as switched reactive shunt compensation, series capacitive compensation and phase-shifting with a tap changing transformer) using ordinary thyristor switches has led to static VAR compensators for transmission voltage control, controllable series compensators for line impedance variation, and phase-shifters for transmission angle adjustment. This ordinary thyristor-controlled equipment can provide rapid power control but requires a facility of considerable size with significant labor installation [8].

The use of recently developed gate turn-off thyristors, however, allows an advanced, all solid state, uniform implementation of static VAR compensators, controllable series compensators, and phase-shifters with drastic reduction in equipment size and installation labor. Significant improvements in operating performance and flexibility and a progressive decrease in capital cost have been achieved through the gate turn-off thyristors.

The following subsections present the basic notion, descriptions and modeling of various FACTS devices which take full advantage of the fast and efficient power flow control, conversion and conditioning of bulk electric power, using solid state switching devices.

2.2.1 Notion of Power Flow Control

As can be seen from Eq. (1.1) the real power flow through lossless transmission line ij is a function of the transmission line reactance X_{ij}, the voltage magnitudes V_i at bus i and V_j, at bus j, and the phase angle differences between buses i and j, δ_{ij}. By controlling one or a combination of the four parameters mentioned above the flow through transmission line ij can be controlled.

Figure 2-3 shows two lossless transmission lines with impedances X_1 and X_2 connecting buses i and j. For convenience transmission lines are labelled as transmission

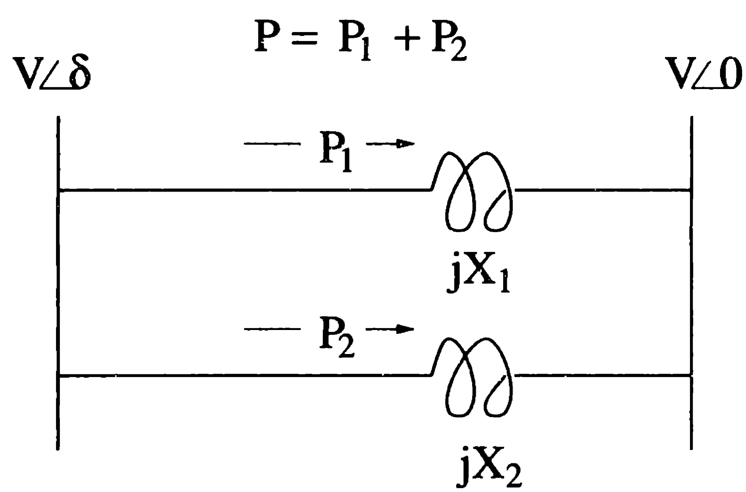

Figure 2-3: Two transmission lines connecting buses i and j

♯1 and ♯2 as shown in the figure. Suppose the phase angles at buses i and j are computed as δ and 0 respectively by solving $(P - \delta)$ load flow where the voltage magnitudes at buses i and j are fixed at V. Using Eq. (1.1) real power flows through transmission lines ♯1 and ♯2 are given by:

$$\begin{aligned} P_1 &= \frac{V^2}{X_1}\sin\delta \\ P_2 &= \frac{V^2}{X_2}\sin\delta \end{aligned} \quad (2.10)$$

Assuming that the power from bus i to bus j is fixed at P,

$$\begin{aligned} P_1 &= \frac{X_2}{X_1+X_2}P \\ P_2 &= \frac{X_1}{X_1+X_2}P \end{aligned} \tag{2.11}$$

since the sum of P_1 and P_2 must equal P. As Eq. (2.11) shows, the uncontrolled power flow obeys the simple ratio of line reactances.

In subsequent paragraphs an observation is made on how the power flows on lines ♯1 and ♯2 change as

1. the line reactance of transmission line ♯1 is controlled
2. the phase angle at the end of transmission line ♯1 is controlled
3. the voltage magnitude at the end of transmission line ♯1 is controlled.

The concept of power flow control in more complex topology can be generalized from this example of simple two transmission line network but very arduous [].

Power Flow Control through Series Compensation

Suppose a series compensation of value X_c is added to the transmission line ♯1 as shown in Figure 2-4. The series compensation can be achieved by either mechanically switched or thyristor switched series capacitors.

The altered power flows are given by:

$$\begin{aligned} P_1 &= \frac{V^2}{X_1-X_c}\sin\delta \\ P_2 &= \frac{V^2}{X_2}\sin\delta \end{aligned} \tag{2.12}$$

Comparing Eqs. (2.10) and (2.12) it is obvious that the flow as well as transmission capacity limit on transmission line ♯1 has been changed, but it is not so obvious that the flow on transmission line ♯2 has also been affected.[2] The change in flow on transmission line ♯2 comes from the assumption that power from bus i to bus j is

[2] The better notation would have been δ' since δ's in Eqs. (2.10) and (2.12) are not equal to each other.

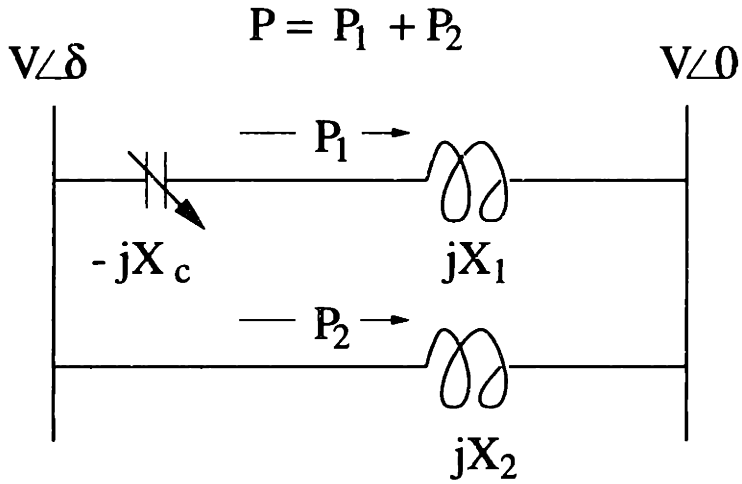

Figure 2-4: Power flow control using series compensation

fixed at P. The equation equivalent to Eq. (2.11) is given by

$$\begin{aligned} P_1 &= \frac{X_2}{X_1+X_2-X_c} P \\ P_2 &= \frac{X_1-X_c}{X_1+X_2-X_c} P \end{aligned} \tag{2.13}$$

which clearly demonstrates the effect of controlling the power flow of one line on the power flow on the other line.

Power Flow Control through Phase Shifting Transformer

Suppose a phase shifting transformer with phase shift ϕ is added to the transmission line ♯1 as shown in Figure 2-5.

The new power flows on lines are given by

$$\begin{aligned} P_1 &= \frac{V^2}{X_1} \sin(\delta - \phi) \\ P_2 &= \frac{V^2}{X_2} \sin\delta \end{aligned} \tag{2.14}$$

Since the sum of P_1 and P_2 must equal P by assumption, the change in power flow on transmission line ♯1 implies a change in power flow on transmission line ♯2. It is interesting to note that although the power flows on both lines are affected, the

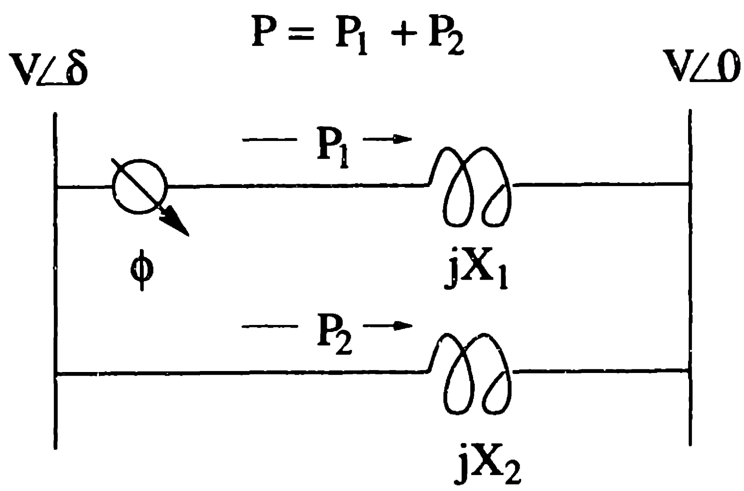

Figure 2-5: Power flow control using phase shifting transformer

transmission capacity limits are not affected by the phase shifting transformer as can be shown from Eq. (2.14); both the steady state limit and the transient stability limit remain the same since the maximum power transfer is unchanged.

Power Flow Control through Static VAR Compensator (SVC)

Suppose a static VAR compensator consisting of capacitor/ inductor shunts is placed in the middle of the transmission line ♯1 as shown in Figure 2-6.

The resulting power flows on the lines as the static VAR compensator sets the voltage at the middle of transmission line ♯1 to be constant and equal to $V\angle\delta^*$ is given by:

$$\begin{aligned} P_1 &= \tfrac{2V^2}{X_1}\sin(\delta - \delta^*) \\ P_2 &= \tfrac{V^2}{X_2}\sin\delta \end{aligned} \tag{2.15}$$

Unlike phase shifting transformers, static VAR compensators increase both the steady state limit and transient stability limit by increasing the maximum power transfer.

Power Flow Control through Power Transformer

Suppose a power transformer with a turns ratio of $1 : T$ is added to the transmission line ♯1 as shown in Figure 2-7.

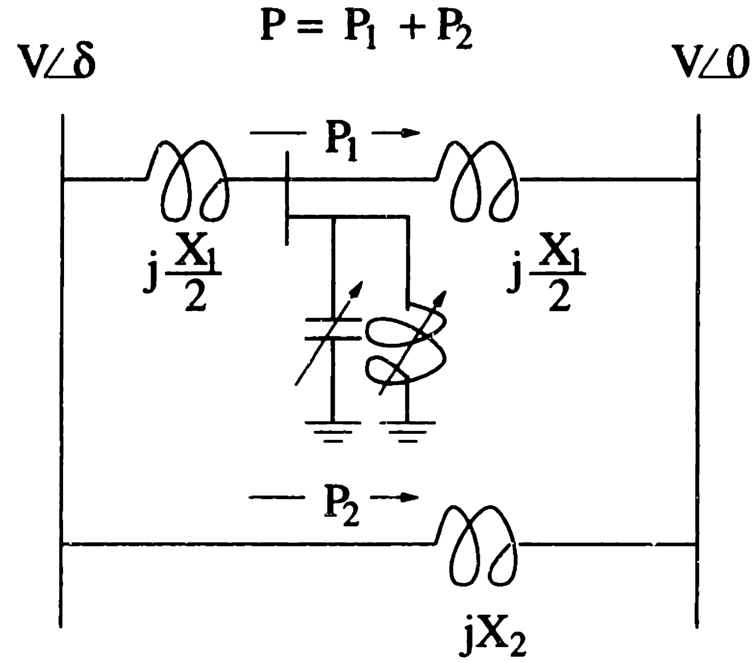

Figure 2-6: Power flow control using static VAR compensator

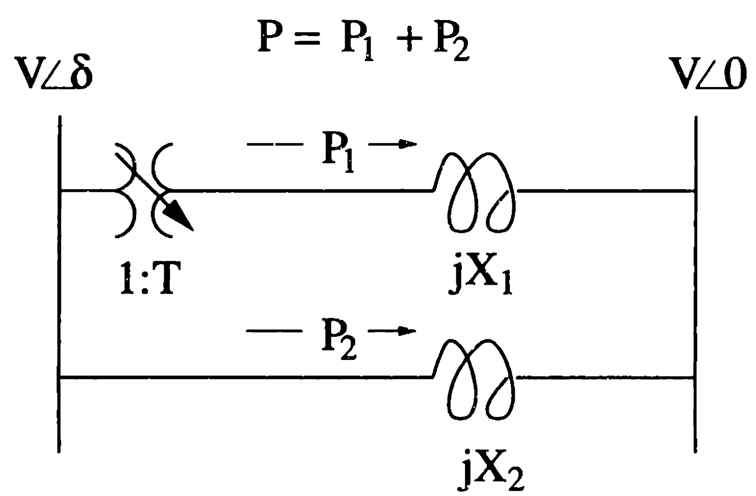

Figure 2-7: Power flow control using power transformer

The altered power flows are given by:

$$\begin{aligned} P_1 &= \tfrac{TV^2}{X_1}\sin\delta \\ P_2 &= \tfrac{V^2}{X_2}\sin\delta \end{aligned} \qquad (2.16)$$

The effect on the direct control of power flow by the power transformer can either increase or decrease the steady state limit and transient stability limit by having $T \geq 1$ or $T \leq 1$ respectively. Typically, $0.95 \leq T \leq 1.05$.

2.2.2 Various FACTS Devices

High Voltage Direct Current (HVDC)

High Voltage Direct Current (HVDC) transmission basically consists of a DC transmission line connecting two AC systems [9]. A rectifier at the sending end of the transmission line converts the power from AC to DC while at the receiving end of the line an inverter converts the power from DC back to AC. Power flow on the line is controlled by controlling the firing angle of the rectifier and inverter. The advances in high power semi-conductor devices and control technologies have made control of the firing angle of the rectifier and inverter more precise, thus making HVDC transmission more reliable. A few advantages of using HVDC transmission technology are

- feasibility of long distance transmission
- ability to link between two different frequency regions

There are, however, also downsides of using this technology. One of these is the high cost of high quality rectifiers and inverters. HVDC transmission lines are usually only used when its advantages can be fully exploited.

Series Compensators

Series compensators employ switches to add or remove reactance to or from a transmission line [10]. When conventional compensators are used, the device is basically a static flow controller by means of mechanical switching. When used properly, the device can increase the power transfer capability of a line, improve solid state stability, and help in solving voltage profile problems. Typically the series compensation levels are 50 – 60 % of the existing reactance of the line. Even though it is possible to have compensation levels greater than 50 – 60 %, above this compensation level the total effective impedance of the line would be so low (reversing the inductive character of the line to capacitive) that problems such as subsynchronous resonance could arise.

Phase Shifting Transformer

The phase shifting transformer allows for control of the phase angle between the buses connected by a transmission line, which permits direct control of the power flow through the line [10]. The phase shift can be introduced in both directions, and a typical range for phase shift is $\pm 30°$. This technology is mainly used to improve utilization of bulk power transfer facilities. Note that phase shifting transformers, unlike other FACTS devices, do not change the maximum power flow through the line.

Power Transformer

Through the transformer's turns ratio the voltage magnitudes of buses connected through a transmission line are controlled which results in direct control over the power flow through the line [10]. The typical turns ratio used is between 0.95 and 1.05. This technology is used to increase the power transfer capability and to improve the stability limit of the line.

Static VAR Compensator (SVC)

The static VAR compensator (SVC) consists of shunt compensation placed in the middle of a transmission line between the sending and receiving end buses [10]. By keeping the voltage magnitude at the place of shunt compensation equal to that of the sending end bus, the device can regulate the reactive power flow of a line. Use of a static VAR compensator allows an increase in the power transfer capability and the stability limit of the line. For instance, one SVC placed at the midpoint of a line can effectively double the line carrying capacity.

Interphase Power Controller (IPC)

The interphase power controller (IPC) has recently been developed by CITEQ (Centre d'Innocation sur le Transport d'Energie du Quebec) [11]. The device allows considerable flexibility in controlling real and reactive power flow between two network or subnetworks, but falls into the category of slow time scale control devices. A typical IPC consists of series connected passive elements such as transformers, inductors, capacitors and conventional circuit breakers.

Unified Power Flow Controller (UPFC)

The unified power flow controller (UPFC) developed by Laslo Gyugyi is considered to be the most advanced of all FACTS devices available [1]. Though passive (i.e. no power supplying capability) the UPFC provides complete control of both the active and reactive power flows on transmission lines by desired AC voltage injection in series with phase voltage. This is achieved through gate turn-off thyristor based inverters. The injected voltage allows a very flexible phase angle and voltage magnitude relationship thus resulting in flexible P and Q injection into the line.

2.2.3 Modeling of An Ideal FACTS Device

One common thread of various FACTS devices is that they are all passive devices. These devices do not change the power injections into a power system. Instead, they simply redistribute power flow through different lines of the network. A controller has the ability to both force more power through a line or restrict the amount of power through a line. One thing to keep in mind is that since the devices are passive in nature, when extra power is forced through a particular line power is taken away from other lines. Similarly, when power is restricted, extra power which is prevented from flowing through a particular line must be routed to other transmission lines in the network.

An ideal FACTS controller allows complete and independent control over four parameters which determine the complex power flow through a line, regardless of the state of the network [12]. These four parameters are the real and complex power injections at both the sending and receiving ends of the line as shown in Figure 2-8. In a lossless system this implies that the power from bus i and the power from bus

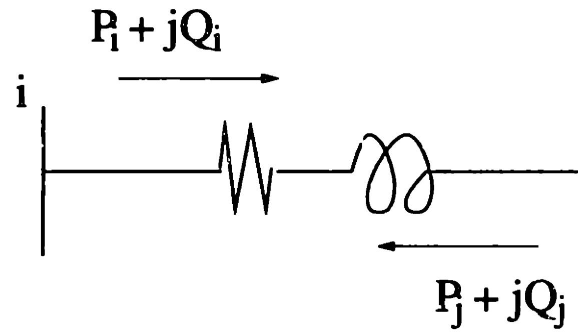

Figure 2-8: Four parameters determining the flow through transmission line ij

j must be equal in magnitude and opposite in sign (i.e., $P_i = -P_j$) regardless of the presence of an ideal FACTS controller, since the controller is assumed to be passive. Thus, based on the above mentioned assumptions and taking P_i as the reference value the model of an ideal FACTS controller should allow the power flowing through the line to remain at P_i regardless of any changes to the state of the network as shown in Figure 2-9. This can be best modeled by effectively removing the line controlled

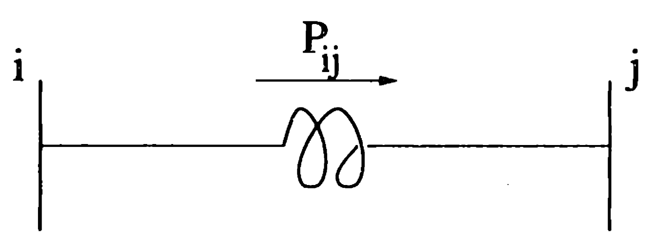

Figure 2-9: Passive ideal FACTS device on transmission line ij in a lossless system

with ideal FACTS device from the network, and replacing it with a constant power source at the sending bus of the line and a constant power sink at the receiving end bus of the line. Figure 2-10 shows the diagram of this ideal FACTS model.

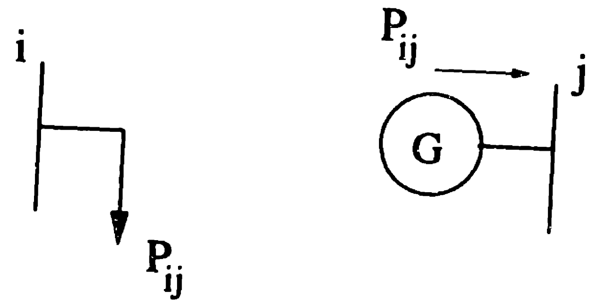

Figure 2-10: Passive ideal FACTS device model on transmission line ij in a lossless system

Removal of a line containing the ideal power flow control device introduces changes in the topology of a network (i.e. changes in system matrices: incidence matrix and admittance matrix). For a network with an ideal FACTS device located on line ij, the new admittance matrix is given by the following equation:

$$\mathbf{Y}_{new} = \mathbf{Y} - Y_{ij}[\mathbf{e}_i - \mathbf{e}_j][\mathbf{e}_i - \mathbf{e}_j]^T \tag{2.17}$$

where $\mathbf{Y}$ is the original admittance matrix of a network defined earlier, Y_{ik} is the admittance of the line between buses i and j, and $\mathbf{e}_i$ and $\mathbf{e}_j$ are vectors of dimensions $(l+g)\times 1$ whose entries are 1 at ith and jth locations respectively and 0's everywhere else.

Power injections to buses i and j from independent power sources and power sinks in Figure 2-10 also modify the system variables. The real power injection vector is changed through the following equation:

$$\mathbf{P}_{new} = \mathbf{P} + \mathbf{P}_{FACTS} \tag{2.18}$$

where $\mathbf{P}$ is the original real power injection vector and $\mathbf{P}_{FACTS}$ is the vector of power injection changes due to the presence of a FACTS device whose entries are P_i and P_j

at the ith and jth locations and 0 elsewhere.

Throughout this thesis study, this ideal FACTS device model is used extensively to illustrate the theoretical findings.

Chapter 3

Transmission Capacity in Electrical Power Networks

Transmission capacity provides a way of quantifying the degree of utilization of a power network. This chapter presents several definitions of transmission capacity suitable for the study of this thesis and an accurate way of calculating them without resorting to solving the load flow problem. A newly developed method is compared to the calculation of transmission capacity using a DC load flow which is the method often used in industry.

3.1 Transmission Capacity

In order to make an accurate assessment of the degree of power network utilization using transmission capacity a precise definition of the term is required. In this section a few possible definitions suitable for power networks of arbitrary topologies are given. The definitions are chosen so that they are sufficiently general to be applicable for quantifying power transfer capability independently of underlying study circumstances.

There are a few observations to be made before presenting the definitions of transfer capability. First, transmission capacity in a given power network is a function of many system variables. It cannot be expressed with only a single or few variables in a system, but rather requires the full knowledge of system variables and system matrices. This includes controllable as well as uncontrollable system variables. The calculation of transmission line power flows also plays an important role because transmission capacity is closely related to the security region, or more specifically security margin for a given operating point. Since the security margins ultimately determine how much power can be transferred to the load and where the limitations arise, in order to determine transmission capacity it is necessary to determine whether the system operates within acceptable limits and whether it operates without compromising system integrity following possible contingencies. The definitions presently used in industry reflect this interpretation of transmission capacity, which refers to "the amount of power, incremental above normal base power transfers" subject to a number of operational constraints and post-contingency behavior. This definition is almost identical to that of the security margin of a power system.

The following are the most commonly used definitions of transmission capacity by the industry:

1. 1st contingency incremental transmission capacity: This quantity yields the amount of incremental power from the given operating point that can be transferred over the transmission network without violating system limits even with *one* component in the system out of service due to a contingency and the rest of the components are in service and operating within the limits. It is assumed that the power system had been operated satisfying the N-1 contingency requirement before the occurrence of the contingency so that the system remains stable and is operated without violating system limits after the single contingency, such as a loss of any single generating unit, transmission line or transformer. That is to say, the system is operated in alert state after the contingency without any control actions.

2. 2nd contingency[1] incremental transmission capacity: This quantity yields the amount of incremental power from the given operating point that can be trans-

[1] the term second contingency is used to specifically exclude simultaneous outages

ferred over the transmission network without violating system limits even with *two* components in the system out of service due to contingencies and the rest of the components are in service and operating within the limits. It is assumed that the power system had been operated satisfying the N-1 contingency requirement before the occurrence of both the first and the second contingencies so that the system remains stable and is operated without violating system limits after the contingencies such as a loss of any generating units, transmission lines or transformers. That is to say, the system is operated in alert state after the contingencies without any control actions.

3. Installed incremental transmission capacity: This quantity yields the amount of incremental power from the given operating point that can be transferred over the transmission network without any contingencies.

The installed incremental transmission capacity is qualitatively different from either 1st contingency incremental transmission capacity or 2nd contingency incremental transmission capacity in the sense that the system is assumed to be operating strictly in its normal state.

The second observation is that the transmission capacity changes as the operating point of a system evolves with time because certain changes in the power system alter the security margin thus changing transmission capacity. For example, an increase in a certain load in a system at a given operating point may severely limit additional deliveries of power to a particular set of buses by pushing the use of a group of transmission lines to their capacity limits. On the other hand, the same load increase at different operating points may only marginally change the security margin.

Third, the amount of power above a normal level depends heavily on which origination and termination points to be considered. Certain sets of power exchange are easier than some other sets of transfers for a given operating point. It is, therefore, important to consider how different sets of power transfers interact with existing power transfers as defined by the operating point. It is not possible to transfer power among buses without affecting the transfer capacity among other buses.

Finally, due to the various dependencies of transmission capacity, it becomes a rather uncertain quantity when considered under the planning mode because of the uncertainty in the availability of equipment as well as the load. Thus, although the definitions in subsequent sections are suitable under operation mode, it may be necessary to alter the definitions to account for uncertainty when considered under the planning mode.

3.1.1 System Transmission Capacity (STC)

The system transmission capacity (STC) is the maximum total incremental load above a normal point of operation that can be securely transmitted through a given network from the generation to the load buses.

A mathematical definition of STC is given by

$$\begin{aligned} STC &= \max_{S} \Delta P_d \\ \Delta P_d &= \sum_{i \in Loads} \Delta P_{d,i} \end{aligned} \tag{3.1}$$

where S is the security region, $\Delta P_{d,i}$ is the incremental load at bus i, and ΔP_d is the total system incremental load. Note that the loads as well as generators at buses are free to vary as long as the resulting combination of loads and generators resides within the security region.

This definition can also be extended to the STC of generation but it has little use.

3.1.2 Load Bus Transmission Capacity (LBTC)

The load bus transmission capacity (LBTC) is the maximum incremental real power above the load that can be received at a given load bus at a given operating point without violating any of the system limits imposed on the system.

The LBTC defines the maximum possible incremental load that can be supplied by the network to a particular bus i. Thus, the incremental loads at other load buses are strictly held at 0. A mathematical definition of LBTC is given by

$$\begin{aligned} LBTC &= \max_S \Delta P_{d,i} \\ \Delta P_{d,j} &= 0, \quad j \neq i. \end{aligned} \tag{3.2}$$

Note that once again outputs at generation buses are free to vary as long as the system remains in normal state.

3.1.3 Generation Bus transmission Capacity (GBTC)

The generation bus transmission capacity (GBTC) is a dual of LBTC for generators. It defines the the maximum power that can be sent from a particular generator to the remaining system buses. A mathematical definition of GBTC is given by

$$\begin{aligned} GBTC &= \max_S \Delta P_{g,i} \\ \Delta P_{g,j} &= 0, \quad j \neq i. \end{aligned} \tag{3.3}$$

The LBTC and GBTC provide a measure of transmission capacity unique to each bus and could also provide a useful estimation of how much a given bus can participate in power interchanges.

3.1.4 Bilateral Transmission Capacity (BITC)

The bilateral transmission capacity (BITC) yields the maximum incremental real power above a given operating point that can be securely transferred from bus j to bus i. This quantity is especially useful under the deregulated industry because it sets an upper bound for potential bilateral agreements. Mathematically the BITC is given by,

$$\begin{aligned} BITC &= \max_S \Delta P_{ij} \\ \Delta P_{g,k} &= 0, \quad k \neq i \\ \Delta P_{d,l} &= 0, \quad l \neq j. \end{aligned} \tag{3.4}$$

where $\triangle P_{ij}$ is the incremental generation and load at buses i and j respectively. In calculating for the BITC, the load flow permits real power changes from the operation point at only a particular generation bus, $\triangle P_{g,i}$, and a particular load bus $\triangle P_{d,j}$. It is interesting to point out that the difference between $\triangle P_{g,i}$ and $\triangle P_{d,j}$ is the incremental system loss, $\triangle P_{ij}^{loss}$, due to the transaction between buses i and j (i.e., $\triangle P_{ij}^{loss} = \triangle P_{g,i} - \triangle P_{d,j}$).

For this thesis study only BITC is used to define transmission capacity.

3.2 Calculation of BITC using DC Load Flow

Assuming that a system is lossless and being operated strictly in normal state, the DC load flow provides the simplest way of computing BITC for general power network. In this section the calculation of BITC using the DC load flow is presented.

3.2.1 Problem Formulation

The problem is concerned with the bounds on real power input changes $\triangle P_i, i = 1, ..., (l+g)$ around a nominal steady state, for which an operationally acceptable solution exists and is characterized by

$$\delta_{ij}^{min} \leq \delta_i - \delta_j \leq \delta_{ij}^{max} \tag{3.5}$$

for all i and j. The defining equation for the DC load flow problem is given in Eq. (2.9) where the unknowns are phase angles δ_{ij} under the assumptions made for the problem. Using the system matrices and the vector form of the system variables the

DC load flow takes the general form:

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_{l+g} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \cdot\;\cdot\;\cdot & b_{1(l+g)} \\ b_{21} & b_{22} & \cdot\;\cdot\;\cdot & b_{2(l+g)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{(l+g)1} & b_{(l+g)2} & \cdot\;\cdot\;\cdot & b_{(l+g)(l+g)} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \cdot \\ \cdot \\ \cdot \\ \delta_{(l+g)} \end{bmatrix} = \mathbf{B}\delta \qquad (3.6)$$

where b_{ij} of the susceptance matrix, $\mathbf{B}$, is given by

$$\begin{aligned} b_{ii} &= \sum_{j \neq i} B_{ij} \\ b_{ij} &= -B_{ij} \end{aligned} \qquad (3.7)$$

Since for a lossless system $Y_{ij} = B_{ij}$, Eq. (3.7) becomes

$$\mathbf{P} = \mathbf{Y}\delta. \qquad (3.8)$$

The solution δ of Eq. (3.8) is $\mathbf{Y}^{-1}\mathbf{P}$ under the assumptions of

- (A1) the network is connected
- (A2) the admittance matrix, J_p is symmetric and nonsingular.

3.2.2 Computing BITC for A General Power Network Using DC Load Flow

The computation of BITC on a power network can be considered as the question of bounds on real load power change, ΔP_α at node α, such that a unique solution exists which does not violate the constraints given in (3.5). In this formulation slack bus 0 is the origination and the load bus α is the termination points of power transfer.

In order to be able to answer this question, consider a solution δ_{ij} of Eq. (3.8) with the addition of a new load power increment at bus α, ΔP_α, but with all other power inputs unchanged. Mathematically this formulation is given by,

$$\Delta \mathbf{P} = \mathbf{Y} \Delta \delta \tag{3.9}$$

where $\Delta \mathbf{P}$ is the vector of dimensions $(l+g) \times 1$ whose entries are ΔP_α at αth location and 0's everywhere else.

Suppose the solution $\bar{\Delta\delta}$ of Eq. (3.9) is computed with the new load power increment at bus α, $\Delta \bar{P}_\alpha$ equal to 1. The solution for any arbitrary ΔP_α is then $\Delta\delta_{ij} = \Delta P_\alpha \Delta\bar{\delta}_{ij}$. This allows the bounds for ΔP_α to be determined. Accordingly, we define

$$\Delta P_{\alpha,ij}^{max} = \begin{cases} \frac{\delta_{ij}^{max} - \delta_{ij}}{\Delta\bar{\delta}_{ij}} & \text{if } \Delta\bar{\delta}_{ij} > 0 \\ +\infty & \text{if } \Delta\bar{\delta}_{ij} = 0 \\ \frac{\delta_{ij}^{min} - \delta_{ij}}{\Delta\bar{\delta}_{ij}} & \text{if } \Delta\bar{\delta}_{ij} < 0 \end{cases} \tag{3.10}$$

$$\Delta P_{\alpha,ij}^{min} = \begin{cases} \frac{\delta_{ij}^{min} - \delta_{ij}}{\Delta\bar{\delta}_{ij}} & \text{if } \Delta\bar{\delta}_{ij} > 0 \\ -\infty & \text{if } \Delta\bar{\delta}_{ij} = 0 \\ \frac{\delta_{ij}^{max} - \delta_{ij}}{\Delta\bar{\delta}_{ij}} & \text{if } \Delta\bar{\delta}_{ij} < 0 \end{cases} \tag{3.11}$$

where $\Delta P_{\alpha,ij}^{min}$ is the bound on ΔP_α according to the limits imposed on transmission line ij. Since the bound must be satisfied on each branch, we can write

$$\Delta P_\alpha^{max}(\underline{g}) = \min_{ij} \Delta P_{\alpha,ij}^{max}(\underline{g}) \tag{3.12}$$

$$\Delta P_\alpha^{min}(\underline{g}) = \max_{ij} \Delta P_{\alpha,ij}^{min}(\underline{g}) \tag{3.13}$$

Since bus α is assumed to be a load, only ΔP_α^{min} is of concern for computing BITC.

The method can be easily generalized for the calculation of BITC involving an arbitrary origination bus s rather than the slack bus 0. The only modification to be made is that $\Delta \mathbf{P}$ in Eq. (3.9) becomes the vector of dimensions $(l+g) \times 1$ whose entries are ΔP_α at αth location, ΔP_s at sth location and 0's everywhere else.

3.3 Calculation of BITC using $(P - \delta)$ Load Flow

It turns out that the problem of calculating the BITC involves analyses of large signal changes. Thus, a more sophisticated method is desired to obtain more accurate results than found with the linearization method presented in the previous section. In this section a newly developed method using the $(P - \delta)$ load flow is presented. Through the new method the problem of calculating the BITC in an electric power network of arbitrary topology and size is posed as a problem of solution existence to a non-linear DC resistor network.

3.3.1 Problem Statement on A Two-bus Topology

In the simple two bus system shown in Figure 3-1 there exist two solutions of $\delta_{12} = (\delta_1 - \delta_2)$ across the transmission line defined as

$$\delta_{12} = \sin^{-1}\left(\frac{P_2}{V_1 V_2 B_{12}}\right). \tag{3.14}$$

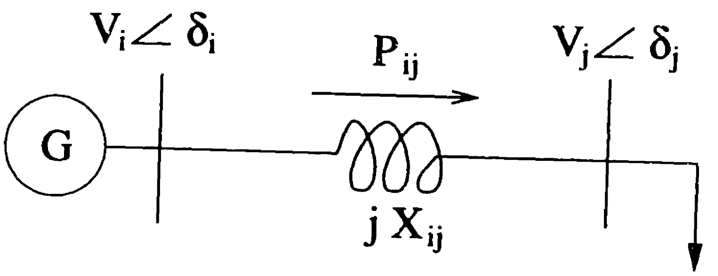

Figure 3-1: One line diagram of two bus system

At least one solution exists only as long as $\frac{P_2}{V_1 V_2 B_{12}} \leq 1$ and $P_2 \leq P_max$ given in Eq. (1.2). If two solutions exist, only one of them is operationally acceptable.

Because of being able to explicitly express the closed form solution to angle difference $(\delta_1 - \delta_2)$ and to analyze the dependence of solution existence on the required power P_2 for given voltage source and transmission system parameters, the conditions for solution existence to the decoupled $(P - \delta)$ problem in the case of a two-bus system are considered to be a fully solved problem.

3.3.2 Problem Formulation for A General Topology

For the case of general power networks the decoupled $(P - \delta)$ problem given in Eq. (2.8) is a nonlinear problem, and it generally has multiple solutions. While an explicit solution cannot be found, the calculation of BITC becomes that of finding theoretical limits on real power inputs for which a solution for phase angle differences across all transmission lines exists. Furthermore it is within the pre-specified operating limits as described in the previous section.

The real power-phase angle problem has been studied in the past primarily with the purpose of understanding a localized character of real power change propagation. As a result of these studies, one can find in the literature several early formulations of the linearized real power-phase angle $(P - \delta)$ problem as a linear resistive network problem.

These formulations are based on the recognition that the linearized decoupled $(P - \delta)$ equations of any size power system are of a general form. This form is described by the equations of a linear resistive circuit obtained from the power network by replacing each transmission line by a resistor and each generator/ load by a current source. The slack bus which is replaced by an independent (fixed) voltage source. [2]

In this equivalent resistive circuit δ_i stands for a nodal voltage measured between node i and the ground, and P_i is the current delivered by the source to the node i. The results are reviewed below.

For the tools to be truly useful for anticipating solution existence under significant input changes it is necessary to relax the linearization assumption. This is easy to recognize by recalling from the two-bus system illustration above that the static stability limit occurs in the operating regions in which a sinusoidal curve cannot be

[2] All of these early formulations require transmission lines to be modeled as lossless inductive devices.

approximated by a line.

The linearization assumption was eliminated in order to study the $(P - \delta)$ transfer problem as a non-linear circuit problem. These results are summarized in what follows.

There is an emphasis is on non-conservative measures of feasible real power transfers. Note that answers to the questions regarding bounds on system input and topology changes for which a unique solution exists cannot be provided by simply employing numerical packages for solving load flow.

3.3.3 Nonlinear Network Formulation of The Decoupled $(P - \delta)$ Problem

The formulation of the problem is similar to the linearized problem formulation given in the previous section, except that the constitutive relations defining line power flows in terms of phase angle differences are nonlinear. [3]

$$P_i = \sum_{j=0}^{l+g} V_i V_j B_{ij} \sin \delta_{ij} \tag{3.15}$$

which is constructed by removing the G_{ij} term from Eq. (2.8). Note that in the simplest case of one line being connected to one load, this constitutive relation is identical to the traditionally used real power transfer curve.

Starting from the decoupled real power equations (3.15) with the branch voltage phase angle constraint (3.5), the change in real power input at each node i can be expressed as

$$\Delta P_i = V_i \sum_{j \in K_i} V_j B_{ij} (\sin(\delta_{ij} + \Delta\delta_{ij}) - \sin \delta_{ij}), \quad i = 1, \cdots, (l + g) \tag{3.16}$$

[3]This is under the assumption that transmission lines are modeled as purely inductive devices, to be consistent with the system being lossless.

or

$$\Delta P_i = \sum_{j \in K_i} c_{ij} f_{ij}(\Delta\delta_{ij}) \tag{3.17}$$

where

$$f_{ij}(\Delta\delta_{ij}) = \sin(\delta_{ij} + \Delta\delta_{ij}) - \sin\delta_{ij}, \tag{3.18}$$

$$c_{ij} = V_i V_j B_{ij} \tag{3.19}$$

and K_i stands for the set of nodes directly connected to node i, including node 0.

Next, create a linear system of equations of the form

$$\Delta P_i = \sum_{j \in K_i} \Delta\delta_{ij} g_{ij} c_{ij} \tag{3.20}$$

where

$$g_{ij} = \frac{\sin(\delta_{ij} + \Delta\delta_{ij}) - \sin\delta_{ij}}{\Delta\delta_{ij}}. \tag{3.21}$$

If we examine solutions of (3.16) for which (3.5) holds, then it is only necessary to consider g_{ij} in (3.20) which are bounded by

$$g_{ij}^{min} \leq g_{ij} \leq g_{ij}^{max} \tag{3.22}$$

where

$$g_{ij}^{min} = \min\left(\frac{\sin\delta_{ij}^{min} - \sin\delta_{ij}}{\delta_{ij}^{min} - \delta_{ij}}, \frac{\sin\delta_{ij}^{max} - \sin\delta_{ij}}{\delta_{ij}^{max} - \delta_{ij}}\right) \tag{3.23}$$

$$g_{ij}^{max} = \max\left(\frac{\sin\delta_{ij}^{min} - \sin\delta_{ij}}{\delta_{ij}^{min} - \delta_{ij}}, \frac{\sin\delta_{ij}^{max} - \sin\delta_{ij}}{\delta_{ij}^{max} - \delta_{ij}}, \cos\delta_{ij}\right). \tag{3.24}$$

Every solution to (3.16) with the voltage phase angle constraint of (3.5) then corresponds to one solution of (3.20) with some set of coefficients bounded by (3.22). [4] Therefore, the system of non-linear equations (3.16) has a linear resistor network interpretation of the form

$$\Delta \underline{P} = H_P \Delta \underline{\delta}. \tag{3.25}$$

Given that transmission lines are modeled as lossless inductive devices, the off-

[4]This solution is unknown, though.

diagonal elements of H_P are

$$h_{ij} = c_{ij} g_{ij} \leq 0, \tag{3.26}$$

and the diagonal elements are

$$h_{ii} = - \sum_{j \in K_i, i \neq j} c_{ij} g_{ij} \geq 0 \tag{3.27}$$

The matrix H_P can be thought of as a conductance matrix of a resistive network with the same topology as the underlying power network.

3.3.4 Calculation of BITC Using $(P - \delta)$ Load Flow

Once again the calculation of BITC on an arbitrary size power network is regarded as the question of bounds on real load power change at node α, ΔP_α, such that a unique solution exists which does not violate the constraints given in (3.5).

Consider a solution δ_{ij} of (3.16) with a new load power increment at bus α, ΔP_α, but with all other power inputs unchanged. Then the increments $\Delta\delta_{ij}$, interpreted as branch voltages of a non-linear resistor network, constitute a solution of a linear resistor network. The solution is obtained from the non-linear resistive circuit by replacing the non-linear resistor connecting nodes i and j with a linear resistor whose conductance g_{ij} is given as

$$g_{ij} = \frac{\sin(\delta_{ij} + \Delta\delta_{ij}) - \sin\delta_{ij}}{\Delta\delta_{ij}}, \tag{3.28}$$

by short-circuiting the voltage source corresponding to the slack bus and by removing all current sources except the source at bus α whose current becomes ΔP_α.

If for a power deviation ΔP_α all linear resistive circuits whose conductances satisfy (3.22) have solutions which obey the branch voltage phase angle constraint (3.5), then there is exactly one solution of the non-linear resistor circuit, with power ΔP_α injected into node α and nominal inputs into the other nodes, which also satisfies the branch voltage constraint.

This can be seen as follows. By modifying the non-linear resistor characteristics,

f_{ij}, in (3.17) to $\tilde{f}_{ij}$ where

$$\tilde{f}_{ij}(\Delta\delta_{ij}) = \begin{cases} \sin\delta_{ij} + \Delta\delta_{ij}\left(\frac{\sin\delta_{ij}^{min} - \sin\delta_{ij}}{\delta_{ij}^{min} - \delta_{ij}}\right) & \text{for} \quad \delta_{ij} + \Delta\delta_{ij} < \delta_{ij}^{min} \\ \sin(\delta_{ij} + \Delta\delta_{ij}) - \sin\delta_{ij} & \text{for} \quad \delta_{ij}^{min} \leq \delta_{ij} + \Delta\delta_{ij} \leq \delta_{ij}^{max} \\ \sin\delta_{ij} + \Delta\delta_{ij}\left(\frac{\sin\delta_{ij}^{max} - \sin\delta_{ij}}{\delta_{ij}^{max} - \delta_{ij}}\right) & \text{for} \quad \delta_{ij} + \Delta\delta_{ij} > \delta_{ij}^{max} \end{cases}, \quad (3.29)$$

the modified resistor characteristics are then strictly monotonic. Therefore, the modified nonlinear resistive circuit has exactly one solution [14]. This solution is again a solution of the linear resistive circuit (3.20) whose conductances satisfy (3.22). Now, if all linear resistive circuits (3.20) with conductances that satisfy (3.22) and the voltage phase angles satisfying the constraint (3.5) as well, then this is particularly true for the solution of the linear circuit corresponding to the solution of the modified nonlinear circuit. Given that non-linear f_{ij} in (3.17) is equal to $\tilde{f}_{ij}$ for $\delta_{ij}^{min} \leq \delta_{ij} + \Delta\delta_{ij} \leq \delta_{ij}^{max}$ this proves the existence and the uniqueness of the solution of (3.20) satisfying (3.5).

It follows that we can search for the maximum and the minimum value of ΔP_α such that all linear circuits with conductances satisfying (3.22) have solutions satisfying (3.5).

For fixed conductances, $\underline{g}$, we can easily determine $\Delta P_\alpha^{max}(\underline{g})$ and $\Delta P_\alpha^{min}(\underline{g})$. Suppose we compute the solution $\Delta\bar{\delta}_{ij}(\underline{g})$ of the circuit with conductances $\underline{g}$ and the source value $\Delta\bar{P}_\alpha = 1$. The solution for arbitrary ΔP_α is then $\Delta\delta_{ij}(\underline{g}) = \Delta P_\alpha \Delta\bar{\delta}_{ij}(\underline{g})$. This allows the bounds for ΔP_α to be determined. Accordingly, we define

$$\Delta P_\alpha^{max}(\underline{g}) = \begin{cases} \frac{\delta_{ij}^{max} - \delta_{ij}}{\Delta\bar{\delta}_{ij}(\underline{g})} & \text{if} \quad \Delta\bar{\delta}_{ij}(\underline{g}) > 0 \\ +\infty & \text{if} \quad \Delta\bar{\delta}_{ij}(\underline{g}) = 0 \\ \frac{\delta_{ij}^{min} - \delta_{ij}}{\Delta\bar{\delta}_{ij}(\underline{g})} & \text{if} \quad \Delta\bar{\delta}_{ij}(\underline{g}) < 0 \end{cases} \quad (3.30)$$

$$\Delta P_\alpha^{min}(\underline{g}) = \begin{cases} \frac{\delta_{ij}^{min} - \delta_{ij}}{\Delta\bar{\delta}_{ij}(\underline{g})} & \text{if} \quad \Delta\bar{\delta}_{ij}(\underline{g}) > 0 \\ -\infty & \text{if} \quad \Delta\bar{\delta}_{ij}(\underline{g}) = 0 \\ \frac{\delta_{ij}^{max} - \delta_{ij}}{\Delta\bar{\delta}_{ij}(\underline{g})} & \text{if} \quad \Delta\bar{\delta}_{ij}(\underline{g}) < 0 \end{cases} \quad (3.31)$$

Hence, for $\Delta P_\alpha \in [\Delta P_{\alpha,ij}^{min}(\underline{g}), \Delta P_{\alpha,ij}^{max}(\underline{g})]$, the solution of the circuit satisfies

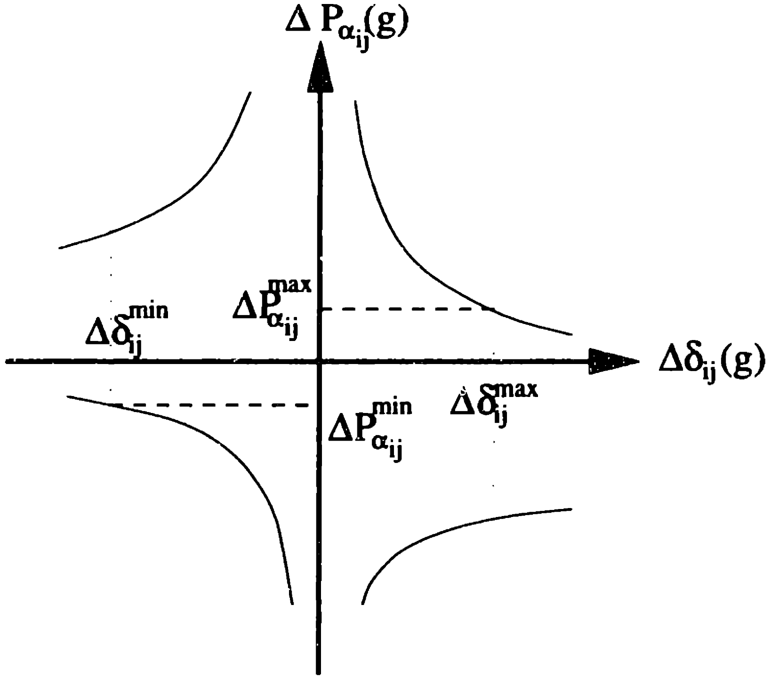

Figure 3-2: $\Delta P_{\alpha_{ij}}^{max}(g)$ as the function of $\Delta\delta_{ij}(g)$

$\delta_{ij}^{min} \leq \delta_{ij}(\underline{g}) \leq \delta_{ij}^{max}$. Since the bound must be satisfied on each branch, we can write

$$\Delta P_{\alpha}^{max}(\underline{g}) = \min_{ij} \Delta P_{\alpha,ij}^{max}(\underline{g}) \tag{3.32}$$

$$\Delta P_{\alpha}^{min}(\underline{g}) = \max_{ij} \Delta P_{\alpha,ij}^{min}(\underline{g}) \tag{3.33}$$

Now for any $\underline{g_{ij}}$ satisfying (3.22)

$$\Delta P_{\alpha}^{max} = \min_{\underline{g_{ij}}} \Delta P_{\alpha}^{max}(\underline{g_{ij}}) \tag{3.34}$$

$$\Delta P_{\alpha}^{min} = \max_{\underline{g_{ij}}} \Delta P_{\alpha}^{min}(\underline{g_{ij}}) \tag{3.35}$$

Thus, we have converted a problem involving a single non-linear resistive circuit into a problem involving an infinite number of linear resistive circuits.

Fortunately, a general circuit theory property allows the number of circuit analyses to be limited to a finite number. First, consider $\Delta P_{\alpha,ij}^{max}(\underline{g})$ and $\Delta P_{\alpha,ij}^{min}(\underline{g})$ as functions of $\Delta\bar{\delta}_{ij}(\underline{g})$ as shown in Figure 3-2.

As $\underline{g}$ varies within $[\underline{g}^{min}, \underline{g}^{max}]$ the values $\Delta\bar{\delta}_{ij}(\underline{g})$ fill out the interval $[\Delta\bar{\delta}_{ij}^{min}, \Delta\bar{\delta}_{ij}^{max}]$. Clearly, the bounds $\Delta P_{\alpha,ij}^{min}$ and $\Delta P_{\alpha,ij}^{max}$ are reached at $\Delta\bar{\delta}_{ij}^{min}$ and/or $\Delta\bar{\delta}_{ij}^{max}$. Hence, we have to determine $\Delta\bar{\delta}_{ij}^{min}$ and $\Delta\bar{\delta}_{ij}^{max}$.

The partial voltage phase angle $\Delta\delta_{ij}$ of line ij in an arbitrary resistive circuit, with given ΔP_α, is a function of g_{kl}. The function has the form

$$\Delta\delta_{ij}(\underline{g}) = \frac{Ag_{kl} + B}{Cg_{kl} + D}\Delta P_\alpha \tag{3.36}$$

where A, B, C, and D depend on all conductances except g_{kl}. Hence, for $\Delta P_\alpha = 1$,

$$\frac{\partial\Delta\bar{\delta}_{ij}(\underline{g})}{\partial g_{kl}} = \frac{AD - BC}{(Cg_{kl} + D)^2}\Delta P_\alpha. \tag{3.37}$$

As long as $Cg_{kl} + D \neq 0$, then $\Delta\bar{\delta}_{ij}(\underline{g})$ is increasing, decreasing, or constant in g_{kl}. Thus, $\bar{\Delta}\delta_{ij}(\underline{g})$ takes its extreme values, when only g_{kl} is varied, at the endpoints of the interval $[g_{kl}^{min}, g_{kl}^{max}]$.

Repeating this reasoning for all conductances, g_{ij}, we conclude that $\Delta\bar{\delta}_{ij}{}^{max}$ and $\Delta\bar{\delta}_{ij}{}^{min}$ are reached at a vertex of the hypercube defined by

$$g_{ij}^{min} \leq g_{ij} \leq g_{ij}^{max}. \tag{3.38}$$

Therefore, in order to determine ΔP_α^{min} [5], instead of infinitely many analyses of linear circuits, only 2^N have to be carried out, with N being the number of branches in the entire network.

3.3.5 Monotonic Response of A Non-linear Power Network

One class of non-linear circuits has the particularly nice property that the solution of the circuit is monotonically dependent on parameter values. This property can be applied to reducing the number of analyses in finding the bounds on real load power change at node α, ΔP_α, in the real power static stability problem discussed in the

[5] Since ΔP_α is assumed to be a load bus, only the ΔP_α^{min} is of concern.

previous section. It is shown in section 3.3.4 that 2^N is the number of linear circuit analyses to be carried out in the worst case, where N is the number of branches in the circuit. Additional consideration based on monotonicity allow to eliminate a large part of these analyses.

In [14] a criterion is given that allows a decision to be made in a nonlinear resistive circuit a branch voltage or current is, as a function of a source voltage or current elsewhere in the circuit,

- always strictly increasing
- always strictly decreasing
- always constant
- nonmonotonic – increasing, decreasing or constant depending on circuit parameters.

Here, "always" means "independently of the parameters of the circuit elements", as long as the circuit elements belong to the same class (e.g. strictly increasing resistors). For a precise formulation, cf. [14]. We shall not discuss this criterion here, but only show how to use it in our context.

We now reconsider the linear resistive circuit analyses needed to compute $\Delta P_{\alpha,ij}^{max}$ and $\Delta P_{\alpha,ij}^{min}$ for a given ij. If we can show that $\Delta\bar{\delta}_{ij}(\underline{g}) > 0$ for all $\underline{g} \in [\underline{g}^{min}, \underline{g}^{max}]$, then it can easily be seen from Figure 3-2 that

$$\Delta P_{\alpha,ij}^{max} = \frac{\delta_{ij}^{max} - \delta_{ij}}{\Delta\bar{\delta}_{ij}^{max}} \quad and \quad \Delta P_{\alpha,ij}^{min} = \frac{\delta_{ij}^{min} - \delta_{ij}}{\Delta\bar{\delta}_{ij}^{max}} \tag{3.39}$$

Thus, we only have to determine $\Delta\bar{\delta}_{ij}{}^{max}$ and need not compute $\Delta\bar{\delta}_{ij}{}^{min}$. Similarly, if $\Delta\bar{\delta}_{ij}(\underline{g}) < 0$ for all $g \in [\underline{g}^{min}, \underline{g}^{max}]$, we need only to compute $\Delta\bar{\delta}_{ij}{}^{min}$. In general, this reduces the necessary number of analyses by a factor of 2.

The property that $\Delta\bar{\delta}_{ij}(\underline{g}) > 0$ for all $\underline{g} \in [\underline{g}^{min}, \underline{g}^{max}]$ can be established in the following way, using the criterion given in [14]. Since for $\Delta P_\alpha = 0$, $\Delta\delta_{ij}(\underline{g}) = 0$ invariably, we have $\Delta\delta_{ij}(\underline{g}) > 0$ for $\Delta P_\alpha = 1$, i.e. $\Delta\bar{\delta}_{ij}(\underline{g}) > 0$ if $\Delta\delta_{ij}(\underline{g})$ is always a strictly increasing function of the source ΔP_α. The criterion [14] also allows us to detect, when $\Delta\bar{\delta}_{ij}(\underline{g}) < 0$ and when $\Delta\bar{\delta}_{ij}(\underline{g}) = 0$. This last case implies $\Delta P^{max}_{\alpha,ij} = +\infty$ and $\Delta P^{min}_{\alpha,ij} = -\infty$ and thus the branch ij imposes no constraint on ΔP_α.

Further reductions in the number of analyses for computing $\Delta P^{max}_{\alpha,ij}$ can be achieved when $\Delta\bar{\delta}_{ij}(\underline{g})$ is strictly monotonic in some conductance of $\underline{g}$. Indeed, if $\Delta\bar{\delta}_{ij}(\underline{g})$ is always strictly increasing in g_{kl} while the other conductances are held constant, then for the computation of $\Delta\bar{\delta}_{ij}{}^{max}$ we only need to set g_{kl} to g^{max}_{kl} and disregard g^{min}_{kl}. If, on the other hand, $\Delta\bar{\delta}_{ij}(\underline{g})$ is strictly decreasing in g_{kl}, while the other conductances are held constant, then the choice of extreme values is the opposite. If $\Delta\bar{\delta}_{ij}(\underline{g})$ does not depend on g_{kl}, then any value for g_{kl} can be taken in the analyses. Hence, if $\Delta\bar{\delta}_{ij}(\underline{g})$ is a strictly monotonic function or a constant function of m conductances, the number of necessary analyses computing $\Delta\bar{\delta}_{ij}{}^{max}$ and $\Delta\bar{\delta}_{ij}{}^{min}$ is reduced by 2^m.

Now we show how monotonic dependency on a conductance can be related to monotonic dependency in a source value. For this purpose consider an arbitrary linear resistive circuit with a conductance g_{kl} on branch kl. We are interested in the voltage v_{ij} on branch ij as a function of g_{kl}. Let g_{kl} and g'_{kl} be two values for this conductance and $(\underline{v}, \underline{i})$ and $(\underline{v}', \underline{i}')$ be the corresponding solutions of the circuit. Since the circuit is linear, $(\Delta\underline{v}, \Delta\underline{i}) = (\underline{v}' - \underline{v}, \underline{i}' - \underline{i})$ also satisfies all circuit equations, except that all sources are set to zero and the constitutive relation of branch kl is changed. For this branch we can write

$$\begin{aligned} \Delta i_{kl} = i'_{kl} - i_{kl} &= g'_{kl}v'_{kl} - g_{kl}v_{kl} \\ &= (g_{kl} + \Delta g_{kl})(v_{kl} + \Delta v_{kl}) - g_{kl}v_{kl} \\ &= g_{kl}\Delta v_{kl} + \Delta g_{kl}v'_{kl} \end{aligned} \tag{3.40}$$

This corresponds to the conductance, g_{kl}, and a current source connected in parallel as shown in Figure 3-3.

Hence, $(\Delta\underline{v}, \Delta\underline{i})$ is a solution of the linear resistive circuit where a current source of value $\Delta g_{kl}v'_{kl}$ has been connected in parallel to the conductance g_{kl}, and all original sources have been set to zero. Therefore, v_{ij} is a strictly increasing function of g_{kl}, i.e. v_{ij} is positive whenever Δg_{kl} is positive, if the following two conditions are satisfied:

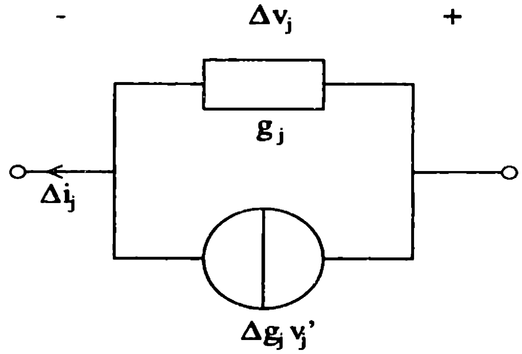

Figure 3-3: The diagram of the conductance g_j and a corresponding current source connected in parallel

- (A) v_{kl} is always positive or always negative
- (B) v_{ij} is strictly increasing (decreasing) as a function of the current of the current source connected in parallel to g_{kl} when v_{kl} is always positive (negative).

Thus indeed, condition (B) is of the required form for the application of the criteria of [14], and condition (A) has already been identified to be of the required form. In the same way, it can be established that v_{ij} is a strictly decreasing function of g_{kl} if either v_{kl} is always 0 or if v_{ij} does not depend on the source added to branch kl.

Let us now summarize which monotonic dependencies we have to check using the criterion of [14] and what are the resulting number of linear resistive circuit analyses needed for the computation of ΔP_α^{max} and ΔP_α^{min}. Equivalently, we state which among the 2^N analyses are needed to determine ΔP_α^{max} and ΔP_α^{min}.

1. Determine $\mathcal{B}_0$, the set of branches that do not depend on the source ΔP_α, $\mathcal{B}_+$, the set of branches that are strictly increasing in ΔP_α, and $\mathcal{B}_-$, the set of branches that are strictly decreasing in ΔP_α.

2. For each of the branches ij determine the sets $\mathcal{B}_0^{ij}$, $\mathcal{B}_+^{ij}$, and $\mathcal{B}_-^{ij}$ of branches kl belonging to $\mathcal{B}_+$ or $\mathcal{B}_-$ such that branch ij is a constant, a strictly increasing and a strictly decreasing function, respectively, of a current source connected in parallel to the branch kl.

3. To compute $\Delta P_{\alpha,ij}^{max}$ and $\Delta P_{\alpha,ij}^{min}$ choose the following limits of g's:

   (a) If $ij \in \mathcal{B}_0$, $\Delta P_{\alpha,ij}^{max} = +\infty$, $\Delta P_{\alpha,ij}^{max} = -\infty$, and thus, no analysis is needed.

(b) If $ij \in \mathcal{B}_+$, set g_{kl} to g_{kl}^{max} when $kl \in \mathcal{B}_+^{ij}$ or $kl \in \mathcal{B}_+^{ij}$, set g_{kl} to g_{kl}^{min} when $kl \in \mathcal{B}_-^{ij}$ and choose both g_{kl}^{max} and g_{kl}^{min} for all other branches kl. This results in $2^{N-m_{ij}}$ analyses, where $m_{ij} = |\mathcal{B}_0^{ij}| + |\mathcal{B}_+^{ij}| + |\mathcal{B}_-^{ij}|$.

(c) If $ij \in \mathcal{B}_-$ take the opposite extreme with respect to (b). This results again in $2^{N-m_{ij}}$ analyses.

(d) If $ij \notin \mathcal{B}_0\mathcal{B}_+\mathcal{B}_-$ perform all analyses of (b) and (c). This yields a total of $2^{N-m_{ij}+1}$ analyses.

4. To compute ΔP_α^{max} and ΔP_α^{min} all limits chosen under 3) for all branches ij have to be collected. The total number can be bounded by

$$\sum_{ij \in \mathcal{B}_+ \bigcup \mathcal{B}_-} 2^{N-m_{ij}} + \sum_{ij \notin \mathcal{B}_+ \bigcup \mathcal{B}_- \bigcup \mathcal{B}_-} 2^{N-m_{ij}+1} \tag{3.41}$$

The actual number might be considerably lower, because the same limits might be chosen for different branches ij.

It is expected that (3.41) is much smaller than 2^N. Of course, the price to be paid is the preliminary combinatorial analysis to check all monotonicity relations by the criterion of [14]. However, this analysis has to be performed only once. It is independent of any specific operating point, θ_{ij}, and any safety margins $[\theta_{ij}^{min}, \theta_{ij}^{max}]$. Only the topology of the power network and the bus α are considered to play a role.

Let us remark that the critical line ij can be determined when applying (3.34) and (3.35). Of course it is the critical line in the context of our bound on ΔP_α. When several lines are close to critical, the actual critical line may be different from the one indicated by our method.

Finally, the method described in this section can be generalized to the calculation of BITC involving an arbitrary origination bus s rather than the slack bus 0. If the angle differences on the transmission lines, δ_{ij}, in the power network are monotonically dependent functions on P_s and P_α, then we have a way of finding $\triangle P_s$ and $\triangle P_\alpha$ such that δ_{ij} always satisfy (3.5) for P_s and P_α within $\bar{P}_s + \triangle P_s$ and $\bar{P}_\alpha + \triangle P_\alpha$, respectively. The details of the modification to be made are given in the Appendix A.

3.3.6 Algorithms for Computing BITC in Monotonic Non-linear Networks

Based on the above analysis, one could arrive at several important features of monotonic networks for reducing the complexity of BITC.

To start with, the necessary step for a given power network is to perform a combinatorial test [15] which determines if the network has a monotonic response. This test may be time-consuming for large networks. However, it is strictly topological and needs to be done only once.

Next, provided that the power network has monotonic response one could proceed by:

- Using small signal sensitivity-based approaches described in the previous section to estimate the amount of BITC available. The most important factor here is that in circuits with a monotonic response the <u>sign</u> of large sensitivity is determined by the <u>sign</u> of the small signal sensitivity and independently of operating conditions.

- Using the method described in Section 3.3.5 for reducing the number of linear circuits that must be solved to find the corner on the hypercube defined in (3.22) such that constraints (3.5) are met. Details of this approach can be found in [16].

- Using novel approaches recently proposed in [17] which suggest a distributed compensation for eliminating transmission constraints are applicable, and can be proven effective. [6]

All of these possibilities would contribute significantly to the computational efficiency in determining BITC. In the case of distributed compensation, a systematic way for eliminating congestion by re-distributing generation/demand in a dispersed manner would be very effective.

[6] The proof for this is omitted.

Chapter 4

Maximum Loadability Problem

In this chapter we explore one of the benefits of FACTS controllers mentioned in Section 1.4, i.e. the operating cost reduction by avoiding expensive generation scheduling. This is possible by enlarging a security region created by employing FACTS controllers in an electric power network. The economic effect of FACTS devices is indirect since their presence allows for more flexibility in optimizing generation cost. One measure of the advantages associated with a larger security region is the resulting increase in maximum network loadability. The maximum network loadability is the maximum demand which can be supplied while respecting the network security constraints. Network security constraints include voltage magnitude constraints at generators and loads, V_g's and V_l's respectively, real and reactive power generation output constraints, P_g's and Q_g's, and transmission line capacity constraints. Such an optimization problem over a larger security region yields either an equivalent or improved optimal solution in terms of operational cost.

This can be illustrated with an example. Suppose a network is required to meet an unexpectedly high demand level without the required increase in transmission capacity. Due to the transmission capacity constraints some inexpensive generation capacity will remain unused. Thus, there is an obvious cost reduction associated with such generation if the maximum network loadability can be increased.

In the simulation that are performed, the maximum loadability is measured both with and without a power flow control device in the system in order to fully understand the concept described above. This provides the cptimum increase in maximum network loadability associated with employing a FACTS device in a power network.

4.1 Problem Formulation

Among the various measures of transmission capacity developed in Section 3.1, BITC - the maximum power exchange possible between two specific buses in a system - is measured when assessing the benefits of a larger security region created by employing a FACTS controller. As the power industry goes through deregulation, the exchange between power producers and consumers needs to be more flexible than at present. BITC provides a way of quantifying such flexibility of power exchanges by measuring the maximum transfer possible between a specific generator bus and a specific load bus.

In following sections detail the method and results of simulations used to determine the effect of a FACTS controller on BITC. In these simulations, the load at a specific load bus and the generation at the slack bus are both increased from their nominal operating condition until transmission capacity limits are reached, but not exceeded.[1] By choosing the slack bus to be the specific generation site, it can be assumed that an infinite amount of generation capacity is available and is used to meet the increase in demand at the load bus.

This extreme operating condition becomes a new nominal condition before a single FACTS device[2] is added to the network. Using the ideal FACTS device model developed in Section 2.2.3, the system variables and the system matrices are modified as an ideal FACTS is introduced to the system. The power flow setting of the FACTS device is then adjusted so that it re-routes the power flow from the first violation point of transmission capacity to other transmission lines in the system, thus allowing more power to be transferred from the slack bus to the specific load. The transfer between the two bus locations is increased from this new operating condition until a violation

[1] Among all constraints imposed on the system only thermal transmission capacity limits are considered.

[2] It is assumed that only one FACTS device is available.

of transmission capacity limits occurs or until the setting of the FACTS device can no longer be adjusted. The same procedure is repeated for all transmission lines that are candidates for FACTS device installation. The increased BITC's are compared for each candidate site to define the optimal installation site for the FACTS device.

4.2 Illustration of Increased BITC through FACTS

The maximum loadability problem is examined through simulations on the power system setup shown in Figure 4-1. The BITC between bus 0 and bus 2 is of the main

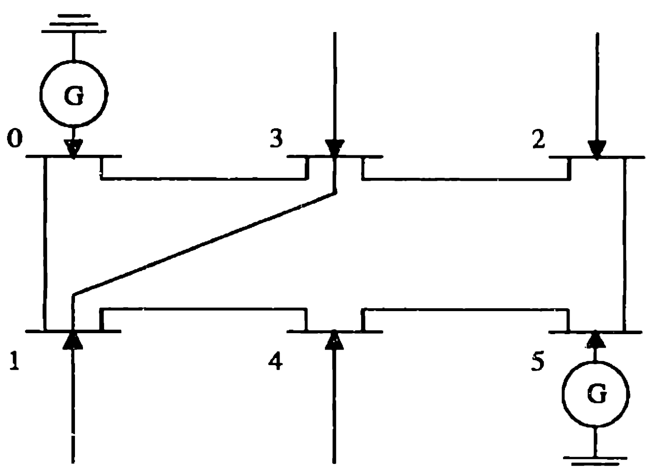

Figure 4-1: A six bus power system example

concern.

The following assumptions are made for the system:

- (A1) the system is assumed to be lossless, i.e. $Y_{ij} = iB_{ij}$ for all ij
- (A2) $V_i = 1$ $(p.u.)$ for $i = 1, ..., 6$
- (A3) the transmission capacity limits are given by $\delta_{ij}^{min} \leq \delta_{ij} \leq \delta_{ij}^{max}$ where

$$\begin{aligned} \delta_{ij}^{min} &= -0.3491\ rad = -20^{\circ} \\ \delta_{ij}^{max} &= 0.3491\ rad = 20^{\circ} \end{aligned} \tag{4.1}$$

Table 4.1 lists the susceptance values for each transmission line ij in the system.

Transmission line ij	Susceptance (p.u.)
B_{01}	1.9305
B_{03}	2.7027
B_{13}	2.4570
B_{14}	3.3333
B_{23}	7.5188
B_{25}	0.9524
B_{45}	1.5625

Table 4.1: The susceptances of transmission lines, ij in the 6 bus power system example

The following notation is used in this section:

- $\delta_{i,nom}$: phase angle at bus i at the nominal operating condition
- $P_{i,nom}$: real power injection at bus i at the nominal operating condition
- $\mathbf{A}_{nom}$: incidence matrix of the system before introducing a FACTS device to the system
- $\mathbf{Y}_{nom}$: admittance matrix of the system before introducing FACTS device to the system
- $\Delta P_{0-2,nom}$ BITC between bus 0 and bus 2 before adding a FACTS device to the system
- $\delta_{i,new}$: phase angle at bus i at the new extreme operating condition
- $P_{i,new}$: real power injection at bus i at the new extreme operating condition
- $\delta_{i,FASTS}$: phase angle at bus i after adding a FACTS device to the system
- $P_{i,FACTS}$: real power injection at bus i after adding a FACTS device to the system

- $\mathbf{A}_{FACTS}$: incidence matrix of the system after adding a FACTS device to the system

- $\mathbf{Y}_{FACTS}$: admittance matrix of the system after adding a FACTS device to the system

- $\Delta P_{0-2,FACTS}$ BITC between bus 0 and bus 2 after adding FACTS device to the system

- ΔP_{ij} directly controlled real power flow deviation on transmission line ij by a FACT device

Table 4.2 summarizes the result of solving the DC load flow of Eq. (2.9) at the nominal operating condition.

Bus no	Bus type	δ_i (rad)	P_i $(p.u.)$
0	Slack bus	0.0000	0.8750
1	PQ load	-0.2160	-0.5252
2	PQ load	-0.2206	-0.5500
3	PQ load	-0.1727	0.0000
4	PQ load	-0.2140	-0.3000
5	PV generator	-0.0157	0.5000

Table 4.2: System variable values at the nominal operating condition

The system matrices are given by:

$$\mathbf{A}_{nom} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.2}$$

$$\mathbf{Y}_{nom} = \begin{bmatrix} 7.6444 & 0 & -2.4327 & -3.3003 & 0 & -1.9114 \\ 0 & 8.3873 & -7.4444 & 0 & -0.9430 & 0 \\ -2.4327 & -7.4444 & 12.5530 & 0 & 0 & -2.6759 \\ -3.3003 & 0 & 0 & 4.8474 & -1.5470 & 0 \\ 0 & -0.9430 & 0 & -1.5470 & 2.4900 & 0 \\ -1.9114 & 0 & -2.6759 & 0 & 0 & 4.5873 \end{bmatrix} \tag{4.3}$$

Notice $\mathbf{A}^T\delta_{nom}$ yields $\delta_{\mathbf{ij,nom}}$ where

$$\delta_{\mathbf{ij,nom}} = \begin{bmatrix} \delta_{01} \\ \delta_{03} \\ \delta_{13} \\ \delta_{14} \\ \delta_{23} \\ \delta_{25} \\ \delta_{45} \end{bmatrix} = \begin{bmatrix} -0.2160 \\ -0.1727 \\ -0.0433 \\ -0.0020 \\ -0.0479 \\ -0.2049 \\ -0.1983 \end{bmatrix} \tag{4.4}$$

Applying the procedure described in Section 3.2, BITC between bus 0 and bus 2 is computed to be

$$\Delta P_{0-2,nom} = 0.6923 \ (p.u.) \tag{4.5}$$

with the critical line being transmission line 03. According to Section 3.3 the value given in Eq. (4.5) suggests a large signal change.

Table 4.3 summarizes the values of the system variables at the new extreme operating condition.

Bus no	Bus type	δ_i (rad)	P_i $(p.u.)$
0	Slack bus	0.0000	1.5673
1	PQ load	-0.3313	-0.5252
2	PQ load	-0.4803	-1.2423
3	PQ load	-0.3491	0.0000
4	PQ load	-0.3510	-0.3000
5	PV generator	-0.1992	0.5000

Table 4.3: The system variable values at the new extreme operating condition

The phase angle differences at lines δ_{ij} at the new operating condition are

$$\delta_{\mathbf{ij,nom}} = \begin{bmatrix} \delta_{01} \\ \delta_{03} \\ \delta_{13} \\ \delta_{14} \\ \delta_{23} \\ \delta_{25} \\ \delta_{45} \end{bmatrix} = \begin{bmatrix} -0.3313 \\ -0.3491 \\ 0.0177 \\ 0.0197 \\ -0.1313 \\ -0.2811 \\ -0.1518 \end{bmatrix} \tag{4.6}$$

where δ_{03} has reached the transmission capacity limit of ± 0.3491 rad (-0.3491 rad in this case).

Suppose an ideal FACTS device is added to transmission line 01 in the system. This introduces modifications to the system variables and system matrices. Table 4.4 sums up the modified values for the system variables as a result of adding a FACTS device to transmission line 01 in the system. Notice that the flow through transmission line 01 has been converted to power injections at both bus 0 and bus 1.

Bus no	Bus type	δ_i (*rad*)	P_i (*p.u.*)
0	Slack bus	0.0000	0.9338
1	PQ load	-0.3313	0.1083
2	PQ load	-0.4803	-1.2423
3	PQ load	-0.3491	0.0000
4	PQ load	-0.3510	-0.3000
5	PV generator	-0.1992	0.5000

Table 4.4: Modified system variable values due to the presence of FACTS device

The modified system matrices are

$$\mathbf{A}_{FACTS} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.7}$$

$$\mathbf{Y}_{FACTS} = \begin{bmatrix} 5.7330 & 0 & -2.4327 & -3.3003 & 0 & 0 \\ 0 & 8.3873 & -7.4444 & 0 & -0.9430 & 0 \\ -2.4327 & -7.4444 & 12.5530 & 0 & 0 & -2.6759 \\ -3.3003 & 0 & 0 & 4.8474 & -1.5470 & 0 \\ 0 & -0.9430 & 0 & -1.5470 & 2.4900 & 0 \\ 0 & 0 & -2.6759 & 0 & 0 & 2.6759 \end{bmatrix} \tag{4.8}$$

In order to increase BITC between buses 0 and 2, δ_{03} has to be fixed at -0.3491 *rad*. That is to say, the effect of ΔP_{0-2} on δ_{03} has to be offset by the effect of ΔP_{01} on δ_{03}. Mathematically,

$$\Delta P_{0-2}\frac{\partial \delta_{03}}{\partial P_{0-2}} = -\Delta P_{01}\frac{\partial \delta_{03}}{\partial P_{01}} \tag{4.9}$$

It turns out

$$\frac{\partial \delta_{03}}{\partial P_{0-2}} = -\frac{\partial \delta_{03}}{\partial P_{01}} \tag{4.10}$$

for this particular case. Thus, ΔP_{0-2} can be further increased as long as ΔP_{01} is also increased by the same amount before encountering a violation of transmission capacity constraint or until ΔP_{01} reaches its limits set by $B_{ij}\delta_{ij}^{min}$ and $B_{ij}\delta_{ij}^{max}$. The maximum BITC between buses 0 and 2 with the FACTS controller on transmission line 01 is found to be

$$\Delta P_{0-2,FACTS} = 0.7262 \ (p.u.) \tag{4.11}$$

Comparing Eq. (4.5) and Eq. (4.11) suggests that small signal analysis can be used to estimate the effect of the FACTS controller on BITC between buses 0 and 2, unlike the procedure for finding BITC without a FACTS device in the system.

Table 4.5 compares the effect of controlling different transmission lines in the system.

Candidate line ij	ΔP_{0-2} $(p.u.)$
01	0.7262
03	0.7262
13	0.7262
14	0.7205
23	0.7205
25	0.7262
45	0.7205

Table 4.5: The effect of FACTS device on BITC between buses 0 and 2 using DC load flow

The result in Table 4.5 suggests that optimal BITC can be achieved when transmission lines 01, 03, 13 and 25 are controlled.

In order to apply the newly developed method described in Section 3.3 to better estimate BITC between buses 0 and 2, a six bus power system example is converted to a nonlinear resistive circuit as shown in Figure 4-2 by replacing each transmission line by a nonlinear resistor and each generator/load by a current source, except for the slack bus which is replaced by an independent (fixed) voltage source. Some arbitrary directions shown in Figure 4-2 are chosen as reference directions. Applying the procedures described in Section 3.3 requires converting the circuit one more time

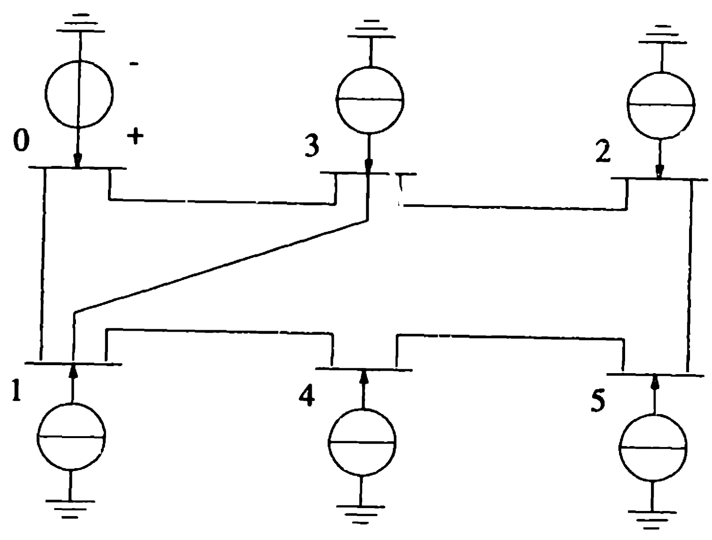

Figure 4-2: Equivalent circuit representation of 6 bus power system example

to satisfy a linear system of equations of the form given in Eq. (3.20). This is done by detaching all sources from the circuit and attaching a current source between buses of interest. Figure 4-3 shows the resulting circuit. $(P-\delta)$ load flow problem is then solved to obtain values for system variables at the nominal operating condition. Table 4.6 summarizes the result. $\mathbf{A}^T\delta_{nom}$ again yields $\delta_{\mathbf{ij,nom}}$ where

Bus no	Bus type	δ_i (rad)	P_i $(p.u.)$
0	Slack bus	0.0000	0.8750
1	PQ load	-0.2174	-0.5252
2	PQ load	-0.2218	-0.5500
3	PQ load	-0.1738	0.0000
4	PQ load	-0.2153	-0.3000
5	PV generator	-0.0156	0.5000

Table 4.6: System variables at the nominal condition solving $(P-\delta)$ load flow problem

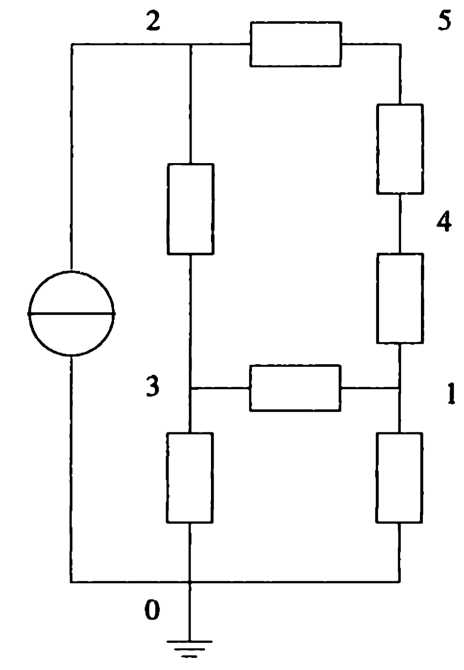

Figure 4-3: Converted circuit of 6 bus power system example

$$\delta_{\mathbf{ij,nom}} = \begin{bmatrix} -0.2174 \\ -0.1738 \\ -0.0435 \\ -0.0021 \\ -0.0480 \\ -0.2062 \\ -0.1996 \end{bmatrix} \tag{4.12}$$

The result of solving Eq. (3.24) for the minimum and maximum conductances of the resistors in the converted circuit is given in Table 4.7.

Before solving Eqs. (3.10) and (3.10) for ΔP_{0-2} the circuit is tested for monotonicity in order to reduce the number of analyses required, the worst case being $128 = 2^7$ (since there are 7 lines present in the system). Using the criterion of [14]

$$\begin{aligned} \mathcal{B}_0 &= \{\} \\ \mathcal{B}_+ &= \{01, 03, 23\} \\ \mathcal{B}_- &= \{14, 45, 25\} \end{aligned} \tag{4.13}$$

Transmission line ij	g_{ij}^{min}	g_{ij}^{max}
01	0.9595	0.9845
03	0.9648	0.9849
13	0.9770	0.9991
14	0.9797	1.0000
23	0.9767	0.9989
25	0.9609	0.9847
45	0.9617	0.9847

Table 4.7: The minimum and maximum conductances of the resistors of converted 6 bus system example circuit

where $\mathcal{B}_0$ is the set of branches that do not depend on the source, ΔP_{0-2}, $\mathcal{B}_+$ is the set of branches that are strictly increasing in ΔP_{0-2}, and $\mathcal{B}_-$ is the set of branches that are strictly decreasing in ΔP_{0-2} based on the reference direction given in Figure 4-2. Only line 13 exhibits nonmonotonic dependency.

Table 4.8 shows the result of using the same criterion given in [14] to determine the sets $\mathcal{B}_0^{ij}$, $\mathcal{B}_+^{ij}$, and $\mathcal{B}_-^{ij}$ of branches kl belonging to $\mathcal{B}_+$ or $\mathcal{B}_-$ such that branch ij is a constant, a strictly increasing and a strictly decreasing function, respectively, of a current source connected in parallel with the branch kl.

	$\mathcal{B}_+$	$\mathcal{B}_-$
$\mathcal{B}^{01}$	{}	{01, 14, 23, 25, 45}
$\mathcal{B}^{03}$	{14, 23, 25, 45}	{01, 03}
$\mathcal{B}^{13}$	{01, 14, 23, 25, 45}	{03}
$\mathcal{B}^{14}$	{03, 23, 25, 45}	{01, 14}
$\mathcal{B}^{23}$	{03, 14, 25, 45}	{01, 23}
$\mathcal{B}^{25}$	{03, 14, 23, 45}	{01, 25}
$\mathcal{B}^{45}$	{03, 14, 23, 25}	{01, 45}

Table 4.8: Monotonic dependency of lines ij to each line kl

Based on the monotonicity test and according to Eq. (3.41) the number of analyses needed to compute ΔP_{0-2} is now reduced from 128 to 16. Table 4.9 shows a summary of the number of analyses that must be performed for the computation in terms of vertices of the hypercube defined by Ineq. (3.38).

BITC	Conductance, g_{ij}	No. of Analyses
$\Delta P^{min}_{0-2,01}$	$g_{01}^{min}, g_{03}^{min}, g_{13}^{min}, g_{14}^{min}, g_{23}^{min}, g_{25}^{min}, g_{45}^{min}$ $g_{01}^{min}, g_{03}^{min}, g_{13}^{max}, g_{14}^{min}, g_{23}^{min}, g_{25}^{min}, g_{45}^{min}$	2
$\Delta P^{min}_{0-2,03}$	$g_{01}^{min}, g_{03}^{min}, g_{13}^{min}, g_{14}^{max}, g_{23}^{max}, g_{25}^{max}, g_{45}^{max}$ $g_{01}^{min}, g_{03}^{min}, g_{13}^{max}, g_{14}^{max}, g_{23}^{max}, g_{25}^{max}, g_{45}^{max}$	2
$\Delta P^{min}_{0-2,13}$	$g_{01}^{max}, g_{03}^{min}, g_{13}^{min}, g_{14}^{max}, g_{23}^{max}, g_{25}^{max}, g_{45}^{max}$ $g_{01}^{max}, g_{03}^{min}, g_{13}^{max}, g_{14}^{max}, g_{23}^{max}, g_{25}^{max}, g_{45}^{max}$ $g_{01}^{min}, g_{03}^{max}, g_{13}^{min}, g_{14}^{min}, g_{23}^{min}, g_{25}^{min}, g_{45}^{min}$ $g_{01}^{min}, g_{03}^{max}, g_{13}^{max}, g_{14}^{min}, g_{23}^{min}, g_{25}^{min}, g_{45}^{min}$	4
$\Delta P^{min}_{0-2,14}$	$g_{01}^{max}, g_{03}^{min}, g_{13}^{min}, g_{14}^{max}, g_{23}^{min}, g_{25}^{min}, g_{45}^{min}$ $g_{01}^{max}, g_{03}^{min}, g_{13}^{max}, g_{14}^{max}, g_{23}^{min}, g_{25}^{min}, g_{45}^{min}$	2
$\Delta P^{min}_{0-2,23}$	$g_{01}^{min}, g_{03}^{max}, g_{13}^{min}, g_{14}^{max}, g_{23}^{min}, g_{25}^{max}, g_{45}^{max}$ $g_{01}^{min}, g_{03}^{max}, g_{13}^{max}, g_{14}^{max}, g_{23}^{min}, g_{25}^{max}, g_{45}^{max}$	2
$\Delta P^{min}_{0-2,25}$	$g_{01}^{max}, g_{03}^{min}, g_{13}^{min}, g_{14}^{min}, g_{23}^{min}, g_{25}^{max}, g_{45}^{min}$ $g_{01}^{max}, g_{03}^{min}, g_{13}^{max}, g_{14}^{min}, g_{23}^{min}, g_{25}^{max}, g_{45}^{min}$	2
$\Delta P^{min}_{0-2,45}$	$g_{01}^{max}, g_{03}^{min}, g_{13}^{min}, g_{14}^{min}, g_{23}^{min}, g_{25}^{min}, g_{45}^{max}$ $g_{01}^{max}, g_{03}^{min}, g_{13}^{max}, g_{14}^{min}, g_{23}^{min}, g_{25}^{min}, g_{45}^{max}$	2

Table 4.9: The vertices of hypercube to be considered in order to compute ΔP_{0-2}

Completing the procedure described in Section 3.3, BITC between bus 0 and bus 2 is computed to be

$$\Delta P_{0-2,nom} = 0.6638\ (p.u.) \tag{4.14}$$

with the critical line being transmission line 03.

Since small signal analysis is permissible for determining the effect of a FACTS device in the system, the $(P - \delta)$ load flow problem is linearized at a new extreme operating condition for calculating the increased maximum loadability. Table 4.10 summarizes the values for system variables at the new extreme operating condition.

Following a similar approach to that taken for the DC load flow case, modifications are made to system variables and system matrices as a FACTS device is added to transmission line ij in the system. Once the modifications are made, ΔP_{ij} is adjusted in order to offset the effect of ΔP_{0-2} on δ_{03} by the effect of ΔP_{ij} on δ_{03} as ΔP_{0-2} is increased.

Table 4.11 compares the effect of controlling different transmission lines in the

Bus no	Bus type	δ_i (rad)	P_i $(p.u.)$
0	Slack bus	0.0000	1.5388
1	PQ load	-0.3329	-0.5252
2	PQ load	-0.4770	-1.2138
3	PQ load	-0.3487	0.0000
4	PQ load	-0.3514	-0.3000
5	PV generator	-0.1964	0.5000

Table 4.10: The system variable values at new extreme operating condition

system.

Candidate line ij	ΔP_{0-2} $(p.u.)$
01	0.6931
03	0.6931
13	0.6931
14	0.6903
23	0.6903
25	0.6931
45	0.6903

Table 4.11: The maximum BITC, ΔP_{0-2} on controlling different lines

The result in Table 4.11 confirms the result obtained from the DC analysis that optimal BITC can be achieved when transmission lines 01, 03, 13 and 25 are directly controlled.

The extensive $(P - \delta)$ calculation shows that $\Delta P_{0-2,nom} = -0.6652$ $(p.u.)$. The result indicates that the newly developed method provides a good way of estimating BITC without explicitly solving the load flow problem. Also, as proven through simulations of many cases, the linearization overestimates BITC.

Chapter 5

Conclusions and Further Research

5.1 Conclusions

This thesis examines how FACTS devices could be used to solve problems associated with transmission capacity limits in electric power networks.

Several topics are covered in this thesis. First, network limits, network security and security region are reviewed and related to transmission capacity limits for a better understanding of the terms. Then, several definitions of transmission capacity are reviewed and interpreted with regard to their use in assessing the status of a power system's operation. The notion of transmission capacity in electric power networks has recently become particularly relevant, since under the open access competitive power supply market the quantity must be posted by each utility. Among the various definitions of transmission capacity, Bilateral Transmission Capacity (BITC) is used extensively throughout this thesis.

The problem of BITC calculation is posed in a manner which relates it to the problem of solution existence of the non-linear load flow problem such that phase angle differences are within the pre-specified constraints. Two methods of computing BITC are introduced, which do not require solving extensive load flow problems: the DC load flow approximation and a newly developed $(P-\delta)$ load flow approximation. The former uses linearized steady state system model (load flow equations) to determine BITC. The latter uses the nonlinear resistor network formulation, combined with the monotonic response property. The monotonicity property is used to reduce the number of analyses need for the computation. It is shown that the method provides very reliable estimates of BITC bounds within which an acceptable solution can be found.

The model of an ideal FACTS device developed in [12] is introduced as a device which can reproduce the characteristics of any FACTS devices currently available as well as ones under development. Modeling is done in such a way that it provides an upper limit on the performance of any FACTS device. Two currently available FACTS devices which closely approximate the ideal FACTS device model are the Interphase Power Controller (IPC) and the Unified Power Flow Controller (UPFC).

The ideal FACTS device model is used to demonstrate the ability of a FACTS device to increase the size of a network's security region as defined by the transmission capacity limits. This security region increase varies depending on the location of FACTS device. The increase in network security region results in an increase in maximum loadability. It is shown that a FACTS device allows more flexibility in power exchanges by permitting increases in the maximum loadability, or specifically the maximum power transfers possible between groups of producers and consumers. Thus, FACTS devices can offer a solution which is much more cost effective than the oftentimes very expensive generation redispatching. We have only indirectly shown operating cost reduction by allowing that presence of FACTS devices at right locations enlarges the security regions.

5.2 Further Research

Based on the study done in this thesis, a number of additional studies are required for the complete analysis of the impact of FACTS devices in the secure-economical operation of power systems.

A further study needs to be performed to understand how the results presented in this thesis and how FACTS devices in general can be effectively used in network planning and deregulation. In order for the study to be useful, the economic dimensions of the problem must be analyzed. The problem should be posed as one that optimizes the system operation over the entire security region with respect to operating cost while using the FACTS setting as one of the optimization variables. By including the resistances of the transmission lines, the use of FACTS devices to reduce the losses in the system is also an interesting study topic. In this case the optimization problem is with respect to both the cost of operation and power losses.

The study of the ultimate loadability of a network should be done with an unlimited generation capacity and with unlimited number of FACTS devices in the system. The interactions among power flow control devices in a network are not fully understood at present. An investigation into the ultimate loadability might provide some insights to the matter.

A further analysis should include a study of the loadability problem with the transmission capacity also satisfying the contingency requirement as it is done in power industry. The study presented in this thesis does not fully take advantage of the fast switching capability of FACTS devices. By adding the contingency requirement to the definition of transmission capacity, the role of FACTS devices as system stability controllers must be explored.

When used as system stability controllers, FACTS devices have to be studied using transient stability analysis. It is expected that when there are multiple FACTS devices present in a system, the fast switching power electronic devices used to build FACTS controllers may have negative effect on one another. One of the problems expected is sub-synchronous resonance.

Once carried out, the above mentioned investigations should provide some guidelines to follow for choosing the locations of FACTS devices.

Appendix A

Method of Calculating BITC Involving An Arbitrary Generator

Let's denote the origination point of transfer to be bus s and the termination point of transfer to be bus r. Suppose that we set for each transmission line safety limits

$$\bar{\delta_{ij}} + r_{ij}^{min} + s_{ij}^{min} \leq \bar{\delta ij} + \triangle\delta_{ij} \leq \bar{\delta_{ij}} + r_{ij}^{max} + s_{ij}^{max} \tag{A.1}$$

with $\frac{-\pi}{2} < \bar{\delta ij} + r_{ij}^{min} + s_{ij}^{min}$ and $\bar{\delta_{ij}} + r_{ij}^{max} + s_{ij}^{max} < \frac{\pi}{2}$.

First, we determine $\triangle P_r$ using the method described [18] such that the following holds true:

$$r_{ij}^{min} \leq \triangle\delta_{ij}^{P_r} \leq r_{ij}^{max}. \tag{A.2}$$

We can then apply the same method to determine $\triangle P_s$ such that

$$s_{ij}^{min} \leq \triangle\delta_{ij}^{P_s} \leq s_{ij}^{max} \tag{A.3}$$

with $\bar{\delta_{ij}} + r_{ij}^{min}$ and $\bar{\delta_{ij}} + r_{ij}^{max}$ as new nominal operating points. Defining g_{ij} as before it is only necessary to check if g_{ij}^{min} with $\bar{\delta_{ij}} + r_{ij}^{min}$, and $\bar{\delta_{ij}} + r_{ij}^{max}$ is less than g_{ij}^{min} with the nominal operating point given by $\bar{\delta_{ij}} + \triangle\delta_{ij}^{\triangle P_r^{actual}}$.

It can be shown that g_{ij}^{min} at $\delta_{ij} = \bar{\delta_{ij}} + r_{ij}^{min}$ and $\delta_{ij} = \bar{\delta_{ij}} + r_{ij}^{max}$ is given by

$$g_{ij}^{min,\triangle P_r,\triangle P_s} = min(g_{ij,1}^{min,\triangle P_r,\triangle P_s}, g_{ij,2}^{min,\triangle P_r,\triangle P_s}, g_{ij,3}^{min,\triangle P_r,\triangle P_s}, g_{ij,4}^{min,\triangle P_r,\triangle P_s}) \quad (A.4)$$

where

$$g_{ij,1}^{min,\triangle P_r,\triangle P_s} = \frac{sin(\bar{\delta_{ij}} + r_{ij} + s_{ij}) - sin(\bar{\delta_{ij}} + r_{ij})}{s_{ij}}, \quad (A.5)$$

$$g_{ij,2}^{min,\triangle P_r,\triangle P_s} = \frac{sin(\bar{\delta_{ij}} - r_{ij} + s_{ij}) - sin(\bar{\delta_{ij}} - r_{ij})}{s_{ij}}, \quad (A.6)$$

$$g_{ij,3}^{min,\triangle P_r,\triangle P_s} = \frac{sin(\bar{\delta_{ij}} + r_{ij} - s_{ij}) - sin(\bar{\delta_{ij}} + r_{ij})}{-s_{ij}}, \text{ and} \quad (A.7)$$

$$g_{ij,4}^{min,\triangle P_r,\triangle P_s} = \frac{sin(\bar{\delta_{ij}} - r_{ij} - s_{ij}) - sin(\bar{\delta_{ij}} - r_{ij})}{-s_{ij}}. \quad (A.8)$$

This can be compared to the actual limits given by:

$$g_{ij}^{min,\triangle P_r^{actual},\triangle P_s} = min\{g_{ij,1}^{min,\triangle P_r^{actual},\triangle P_s}, g_{ij,2}^{min,\triangle P_r^{actual},\triangle P_s}\} \quad (A.9)$$

where

$$g_{ij,1}^{min,\triangle P_r^{actual},\triangle P_s} = \frac{sin(\bar{\delta_{ij}} + \triangle\delta_{ij}^{P_r} + s_{ij}) - sin(\bar{\delta_{ij}} + \triangle\delta_{ij})}{s_{ij}}, \text{ and} \quad (A.10)$$

$$g_{ij,2}^{min,\triangle P_r^{actual},\triangle P_s} = \frac{sin(\bar{\delta_{ij}} + \triangle\delta_{ij}^{P_r} - s_{ij}) - sin(\bar{\delta_{ij}} + \triangle\delta_{ij})}{s_{ij}}. \quad (A.11)$$

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