Thesis on the Charles River Bridge.

E. Grover.

1877.

The Sudbury River Aqueduct for bringing an additional supply of water to Boston was begun in July 1875 and is now nearly completed. It is a very interesting line from its beginning at Farm Pond in Framingham to Chestnut Hill Reservoir (15.61 miles). Aside from the general varied character of the line, running as it does across the B. & A. R.R. the M. & F. R.R. and a branch road, thro' quicksand, peat-bogs, swamps and woods, through several rock-cuttings and over several brooks at four of which there are waste weirs constructed it includes four tunnels, three in rock and one in gravel, (one being 4635 ft. long), and also two handsome and substantial bridges of granite and brick. The Waban Brook Bridge consists of nine semicircular arches of about 45 ft. span each.

Charles River Bridge – General description.

The most important structure on the line, however, is the bridge which carries the conduit across the Charles River valley at Newton Upper Falls. This bridge which is the subject of this thesis is 475 ft long between the terminal chambers; At the westerly end is a semicircular arch of 37 ft span partly cut off by the slope of the hill which is quite steep; Next is a segmental arch of 130 ft span and about 45 ft rise, of 69 ft radius; then there are on the easterly side of the river four semicircular arches of 37 ft. span and lastly a flat arch having a span of 28 ft over Ellis St. a town road. At the point where the Cochituate aqueduct crosses the Charles about a mile below a siphon is employed to carry the water down the valley which then crosses the river over three small arches; and the same means of crossing had been under consideration in the new work, but it was finally decided to build the bridge. In making the survey for the line the best location for the bridge was easily determined the river here being quite narrow (a little more than 100 ft wide) and the slopes of the valley especially the westerly slope being abrupt and the whole valley, river bed and banks being of firm rock, making a good foundation for the piers and abutments.

The springing of the river arch is ~~a little or~~ considerably above the water level which latter changes but slightly during the year; accordingly there is no obstruction of the water-way. The width of the bridge is 18 ft. below the lower string course, the piers have a slight batter and the face walls of the large arch have a curved batter corresponding to the curve of pressures.

Materials and Construction.

Foundations. These consist either of the natural rock or of large blocks of granite laid in the proper place and manner and at such depths as to be sufficiently firm and immovable. The steep western slope, the river-bed and the east bank are composed of a compact rock which makes a perfectly good foundation. In preparing the western slope for the abutment the soil and loose rock was entirely removed and the solid natural rock was then dressed with the hammer to plane surfaces which were perpendicular to the thrust of the arches at their springing and level or vertical elsewhere, thus forming sets of steps: Upon these prepared surfaces were laid in mortar, large rectangular blocks of stone, with dressed beds and builds well jointed together, the bed joints being 3/8" thick and vertical joints not exceeding 2 inches. The bed of the river was prepared as follows to receive the vertical timbers which support the centring:- the mud was removed and large blocks of stone laid on the natural rock and levels taken at each point of support. The tops of these blocks were made as nearly level as possible and timbers then laid horizontally, the lower surface being cut to fit neatly to the rock and so as to bring the upper surfaces of the timbers level.

On the easterly slope the ground was less rocky and the earth was excavated a few feet to receive the large rectangular blocks of granite, laid as above mentioned each course below the finished ground line being wider and longer than the one above thus distributing the pressure over a larger area.

The Piers as also the facing of the outside spandrel walls and the exposed facing of the abutments and terminal structures are composed of ashlar with dressed builds beds and vertical joints, laid in mortar in regular horizontal courses. The piers are composed of very large stones well bonded together and have a batter of one in twenty.(?) The capping for the piers is dressed smooth and has a slight projection beyond the face. Voussoirs. The foundations piers and centring having been completed the arches were begun and carried up at the same height on each side so as to keep the load on the centring symmetrical. The keystone has a depth of five feet and the springing stone a depth of six feet in the large arch; the thickness of each voussoir is about two feet there being 78 in the arch. The voussoirs are hammered smooth the intrados to the curve and the ring and coursing joints to plane and true surfaces, and no cavities are left. In the large arch the faces ~~of~~ the voussoirs are bounded by neat lines which are 3 inches distant from the plane of the spandrel wall and the projection is chamfered off at angle 45°; beyond the neat lines the stone is quarry faced, the projection being bolder than that of the spandrel walls. The voussoirs of the roadway arch are also chamfered, the projection being 2 inches. The voussoirs of the smaller arches have smooth hammered joints and quarry faces with no chamfering.

Spandrel walls. The exterior spandrel walls are 20 in. thick at the top and increase to 24 in. at the springing of the small arches below which they were not required to be over 24 in thick; and have the vertical joints in the face stones dressed to 8 in. from the face, the rest of the joint not being more than 1½ in thick. The 18 to 24 inches behind the face stones are composed of split stones of fair rectangular shape, laid in mortar with joints not exceeding 1½ in. and containing some spawls. The face stones are quarry faced with no projection of 2 in. with the edges pitched off to neat and true lines. In these walls are numerous headers extending through the whole 20 or 24 inches and serving to bond the wall together. Where the spandrels rest upon the voussoirs or upon the quoins of the great arch the stones are cut to fit neatly,

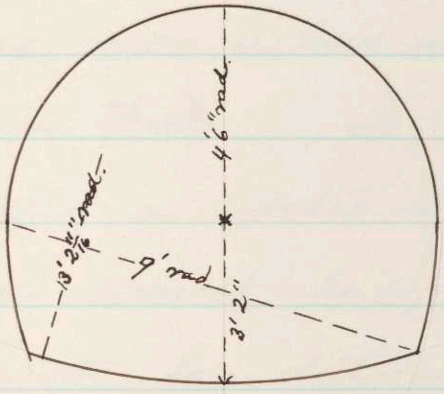
avoiding sharp angles of stone.
Between the two exterior spandrel walls of ashlar are two walls of brick laid in mortar. These are 1 ft 4 in in thickness and are bonded together and to the exterior walls by long split stone blocks. Blocks of split-stone are also laid on the top of the four walls to support the conduit. Near the top of these walls the spaces between are arched over thus forming continuous passages between the between the arches by which access may be had under the conduit. These passages are lighted and ventilated by narrow openings in the spandrel stones immediately under the lower string course. Part of the space between the walls is filled with cement concrete. Good masonry work may be rapidly destroyed by the action of frost upon any moisture which might remain in it and consequently great care has been taken to make, in the designing and construction of the bridge, the conduit as nearly as possible water tight and also to provide for the thorough drainage of any water which may

leak through. The floor of the passages in the top of the spandrel above described are cemented and then in order that any percolation may not affect the spandrel walls the water is conducted over the concrete filling which extends about two thirds the length of the arches, that is, above the point of rupture, and is covered with coal tar concrete, to gutters in or near the centre of the piers and comes out at the weeping holes at the springing of the arches. To secure against leakage &c in the Croton aqueduct on one of the bridges of that work the conduit was lined with brick, lined with an iron casing inside of which was a lining again of brick.

The conduit - dimensions and manner of construction.

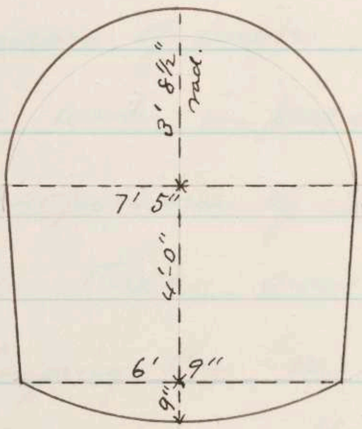
The conduit is of good size, much larger than the old Cochituate conduit; the sectional area being equivalent to a circle of 8½ ft. diameter. It is estimated that it will deliver at the rate of 70 millions gallons in 24 hours when the water is flowing at the assumed water line and in ordinary conditions of cleanliness.

Below is shown a section of the inside with the radii of the curves and also a section of the Croton conduit for comparison. It will be seen that the form is such as to give great strength.



SUDBURY RIVER.

Area = 56.5 sq.ft.



CROTON.

Area = 53.34 sq.ft.

Upon the covering stones of the spandrel walls is a bed of concrete and upon this the invert is laid. This invert is 8 in thick of the length of one brick on end. The curved side walls are the same; their interior angles are formed with moulded bricks which form a continuous skewback.

The interior of the curved side walls and invert are then pointed with cement. The covering arch is next constructed on centres and the latter having been removed the mortar projecting on the inside is scraped away. All the beds of the brick are laid in the direction of the radii of the curves. The covering arch is laid in three courses of brick being 12 in. thick. To make the work impervious to water a mortar of equal proportions of sand and cement is applied 3/8 in. thick;- over the concrete foundation before laying the invert; on the inside of the side walls before the 4 in. lining is built up; and on the outside of the covering arch; also over the whole inside of the conduit. When the bridge is completed there will also be an additional coating of pure Portland cement 1/4" thick over the inside. The side walls outside the conduit are of brick the least thickness of the wall being about 3 feet and are built with an air space the width of a brick, which air space serves as a further protection from the effects of frost. They are covered with a coping of smooth cut granite which is fastened by anchoring rods to them. The space between the coping stones and the top of the covering arch is filled with cement concrete, of a somewhat finer quality than that used for spandrel filling, and a layer of coal-tar concrete is laid over the top of all, forming with the coping stones which it overlaps, a water tight roof. The exterior of the conduit walls are of face brick and formed into panels by pilasters 2 ft wide and 8 ft. from centre to centre.

The terminal chambers at each end of the bridge are built of the same quality of stone as the exterior spandrel walls, the steps and coping being of the same quality as the string courses or coping.

There are wing walls at each side which are also of ashlar, the top of which are at the level of the top of the bridge while the top of the terminal chambers is 6½ ft. higher than the top of the bridge. A series of steps of smooth hammered stone leads down to the top of the bridge upon which the coal-tar concrete will form a wide foot path. An ornamental iron fence will serve as a protection and add to the beauty of the bridge. It is proper to add that the view from this bridge is exceedingly pleasant.

Entrances to the interior of the bridge are provided by the two manholes one in each terminal chamber by which access is had to the conduit and by the entrance at the foot of the roadway arch. These are shown in the drawings.

The grade of the conduit is 6 ft to the mile.

The floor of the passages in the top of spandrel for the purpose of conducting water fall 1 ft in 400 ft each way from the crown of the first small arch on the east side of the river.

Calculations - Stability &c.

In designing an arch of masonry many points have to be considered in order to assure sufficient strength and stability. The load to be supported is generally known, to which, having assumed the form of arch, semicircular, segmental, elliptical or otherwise we have to proportion the rise and span, which are both somewhat limited, and also to determine the thickness of the arch-ring, both at crown and springing; the hight to which the backing should be built, &c.
In this class of structures there is almost always some similar work already in existence which may be imitated or modified to suit the requirements.
But we ought to know especially in any new case, when we may not obtain from experience and analogy the necessary information, exactly what are the actions which are produced in an arch by different manners of loading &c. In other words we wish to know the form of the curve of pressures. There are several theories concerning this subject. The principle of the

"Least Resistance" first advanced by Mosely is generally accepted. Dr. Scheffler in his valuable treatise on the Stability of Constructions (of which there is a French translation) develops this theory in its application to arches. His method which is mainly graphical is very easy in working and is apparently founded on the true theory. The principal thing desired is to find the point of rupture where the curve of pressures passes near or beyond the edge of the voussoirs. At this point the horizontal thrust is a maximum, that is, the thrust at the crown required to keep the arch in equilibrium about the joint of rupture is greater than that required to produce equilibrium about any other joint. Accordingly, finding the load on the arch above different joints in succession, the thrust can be computed and and the joint at which it occurs taken as the joint of rupture. Otherwise the point of rupture may be found by solving, by a series of approximations, a complicated formula given in Rankine. In the present case where the relation between the form of the arch and the

load cannot be easily expressed, it is easier to find the point of rupture by a graphical construction.
The horizontal component of the thrust along the arch is found for a number of points as follows:—
By multiplying the load from the crown to the joint in consideration, including the weight of the voussoirs, by the cotangent of the angle of inclination of the arch at that point to a horizontal. $H = P \cot i$.
The spandrel being of different materials and therefore the load not being uniformly varying even, it is perhaps best to find the intensity of vertical load at different points from the crown to the springing.
1st taking a section at the crown. (of large arch)

The section of brick is	78 sq. ft.	} nearly	
Stone	48 "		
Concrete	18 "		
when conduit is full of water	56.5 "		

These figures are also the number of cu. ft. for 1 ft in length. Then calling the weight of one cu. ft of Stone 160 lbs; brick masonry 120 lbs and concrete 150 lbs;

and multiplying together, the load for one ft. in length is obtained: thus. Stone, 48 × 160 = 7680 lbs
concrete, 18 × 150 = 2700 "
brick, 78 × 120 = 9360 "
water 56.5 × 62.4 = 3525 "
Total - - - 23265 "

Dividing by 18 ft. the width of the bridge at crown gives 1292.5 lbs. as the mean intensity of load on a slice of the arch 1 ft. thick. The above is a constant quantity for every point of the arch as it takes into account the materials only to the extrados of the arch at crown. — to the bottom of the lower string course. Besides this at the crown there are only the voussoirs to be added which being 5 ft. deep gives 5 × 160 = 800 lbs. intensity of load.

2d. Section taken where concrete backing begins, 32 ft from the crown. section of brickwork 26 sq. ft.
" Stone 68 "
weight of stone ~~58~~ 68 × 160 = 10880 lbs
" " brick 26 × 120 = 3120 "
Total - - 14000 "

Also voussoirs for 1 ft. wide 6 cu.ft = 960 lbs

$\frac{14000}{18.5} + 960 = 1717$ lbs intensity of load.

3^{d}. Section 60 ft. from crown at lowest point of concrete filling.

Section of brick 56 sq.ft.
" " Stone 200 "
" " Concrete 77 "

weight of brick $56 \times 120 = 6720$ lbs
Stone $200 \times 160 = 32000$ "
Concrete $77 \times 150 = 11550$ "
Total - - 50270 "

voussoirs 10 cu.ft. = 1600 lbs.

$\frac{50270}{21} + 1600 = 3994$ lbs. (21 ft width of arch.)

4th Section at springing.

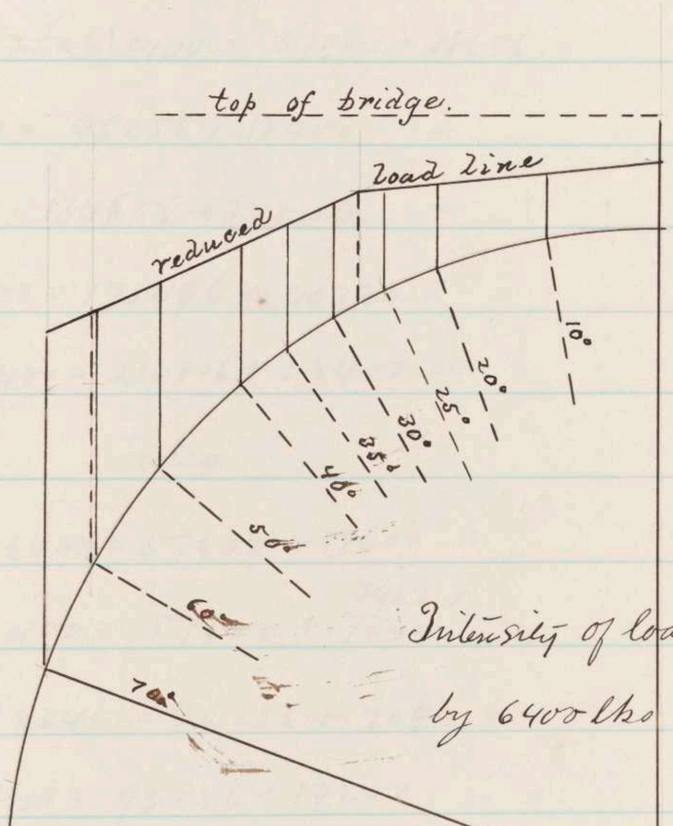
Section of stone 252 sq.ft.
brick 101 "
Concrete 155 "

weight of stone $252 \times 160 = 40320$ lbs
brick $101 \times 120 = 12120$ "
Concrete $155 \times 150 = 23250$
Total - - 75690 lbs

voussoirs 14 cu. ft. = 2240 lbs

$\frac{75690}{22} + 2240 = 5671$ lbs intensity of load.

We can now draw the line of reduced load.



Intensity of load represented by 6400 lbs to the inch.

The area of the trapezoids which now represent the different loads may be computed by multiplying the width by the mean depth and by the number of pounds taken to represent an inch. It is best to do this on a large scale for a greater accuracy.

Calling P_1 the load down to 10° P_2 down to 20° and so on, we have:—

$P_1 = 12 \times .35 \times 6400 = 26880$ lbs - - - 26880 lbs

$P_2 = P_1 + 12 \times .4 \times 6400 = 26880 + 30720 =$ 57600 lbs

$P_3 = P_2 + (7.5 \times .525 + 3.5 \times .6) 6400 = 57600 + 38656 =$ 96256 "

$P_4 = P_3 + 10 \times .6 \times 6400 = 96256 + 38400 =$ 134656 "

$P_5 = P_4 + 8 \times .7 \times 6400 = 134656 + 35840 =$ 170496 "

$P_6 = P_5 + 7 \times .9 \times 6400 = 170496 + 40320 =$ 210816 "

$P_7 = P_6 + 5 \times 1.2 \times 6400 = 210816 + 38400 =$ 249216 "

also

$P_{25^\circ} = P_2 + 5.5 \times .5 \times 6400 = 57600 + 17600 =$ 75200 "

$P_{28^\circ} = P_2 + 9 \times .6 \times 6400 = 57600 + \overset{34560}{\cancel{7040}} =$ 92160 "

$P_{32^\circ} = P_3 + .2 \times 5.5 \times 6400 = 96256 + 7040 =$ 103296 "

$P_{35^\circ} = P_3 + 5 \times .6 \times 6400 = 96256 + 19200 =$ 115456 "

Multiplying each value of P. by the corresponding value of cotan. i the thrusts are obtained as in the following table

i	P. (in lbs.)	cot. i.	P. cot i. = H.
10°	26880	5.67	152409.
20°	57600	2.75	158400
25°	75200	2.145 ~~1.73~~	161304
28°	92160	1.88	173260
30°	96256	1.73 ~~0.84~~	166522
32°	103296	1.6 ~~0.577~~	165273
35°	115456	1.428 ~~0.364~~	164871
40°	134656	1.19	160240
50°	170496	0.84	143216
60°	210816	0.577	121640
70°	249216	0.364	90714

The greatest thrust is here seen to be required below a point about 28° from the crown.

The concrete filling is carried up to about 25° from the crown and the only question remaining is, whether the curve of pressures can be drawn within the required limits that is if the thrust at the crown, the resultant of vertical load from crown to point of rupture, and the

thrust at the point of rupture are in equilibrium and accordingly intersect in one point; this by drawing on a larger scale is found to be true. Hence we may regard the arch as secure against turning about any joint and since the line of pressure at any point does not make with the normal to that joint an angle exceeding that of friction, as perfectly stable. The horizontal thrust at the springing is seen from the table preceding to be 90714 lbs = 45.3 tons.
This thrust is received directly by the solid natural rock of the river bank.

As to the stability of the abutment (east side) it would be most likely to give way by rotation about the springing of the small arch: it could not slide below 60° from crown on account of the direct support. Calling the horizontal thrust at this point 60 tons (see table), the arm is 15 ft, and hence the moment 60 × 15 = 900 ft tons. This is resisted by the weight of the abutment and half arch, the latter alone being 100 tons, acting about an arm nearly the same as that of the thrust and hence fully sufficient to resist it.

As an illustration of Dr. Scheffler's method, take the case of the arch stones themselves before the spandrel is built up or the centring struck.

Let the arch ring be divided into 5 fictitious voussoirs as shown in the figure on the next page. Let W. be the weight of a voussoir, c, the distance of its resultant from the crown, and m, its moment. $M = \Sigma m$, $S = \Sigma W$, $C = \frac{M}{S}$.

The results are tabulated as follows:-

No. of joint	W.	c.	m.	S.	M.	C.
1	17.4	8.4	140.7	17.4	140.7	8.4
2	17.4	24.4	424.5	34.8	565.2	16.2
3	18.0	39.5	711.0	52.8	1276.2	24.2
4	21.0	52.3	1098.3	73.8	2374.5	32.0
5	24.4	62.5	1523.0	98.2	3897.5	40.2

Combining the loads given in column S with the thrust obtained by taking moments about the point of rupture, produce the resultant to the joint under consideration and thus find the curve of pressures. The horizontal thrust is

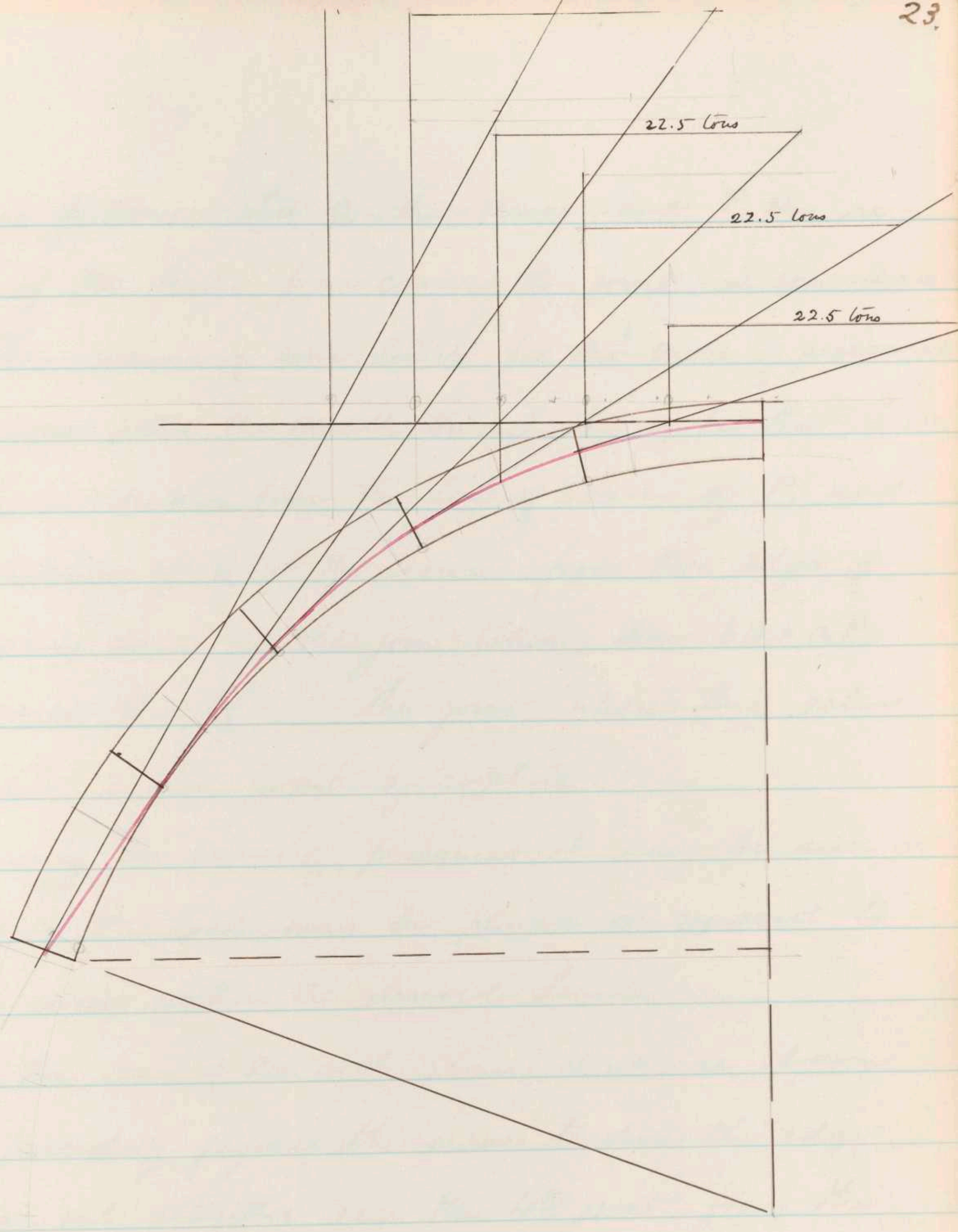


Diagram showing the curve of pressure, the only load being that of the voussoirs. Thrust at crown = 22,5 tons

Scale of figure 15 ft to the inch.

found as follows:- Let Q = that thrust and P, the resultant of the loads from crown to joint in consideration. Then assuming some limits for the curve of pressures, (Rankine says within the middle third,) let a, be the perpendicular distance from the line of action of P. and b, the distance of Q at the crown from the edge of the limiting curve at the joint taken; then $bQ = aP$. and hence $Q = \frac{aP}{b}$. The joint where this value is greatest is the point of rupture.

By drawing the curve of pressures it may be seen at once where the load may be placed or removed to keep the curve within the desired limits.

Thus in the case of the arch stones alone as shown in the preceding figure the curve touches the edge or passes out altogether near the 4th joint from the crown and consequently the spandrel had to built up above this point before the centring could be struck. To be more accurate the diagram must be drawn on a large scale. By finding the reduced load area to represent different cases of loading it may easily be seen when the work is in a stable condition.

Centring.

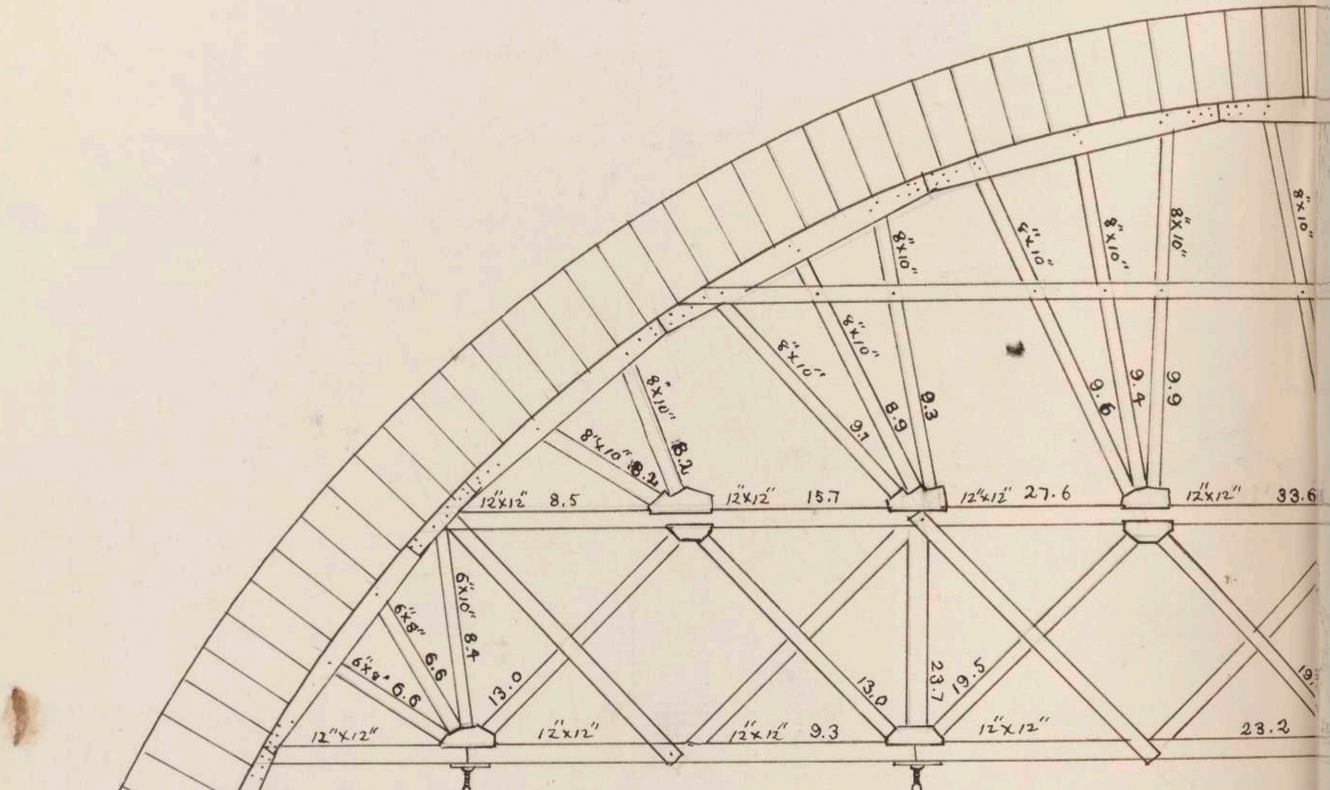
The arrangement of the different pieces in the centring is shown in the following plate which represents the front elevation of one of the four ribs. The laggings are supported by back-pieces about 16 ft long, which are themselves supported directly by three struts. These struts transfer the thrust to a girder and this girder is placed on screws which rest upon vertical timbers braced together transversely and standing on the river bed as described under "foundations" above.

Calculation of strains in the pieces.

In calculating the greatest normal pressure that can come upon any back piece of the arch with exactness, friction ought to be taken into account as also the fact that the normal pressure of any voussoir upon the centring is decreased the more the arch stones are laid above it, since the normal component of the tangential thrust due to the voussoirs above acts outward and opposite to the normal component of the weight of the voussoir considered. According to Rankine friction may be neglected as the action of the voussoirs when completely laid to the keystone is nearly the same whether friction acts or not. But what it is required to do is to find the greatest normal thrust on any one back-piece, which would not occur when the whole arch was complete. However for practical purposes, disregarding friction the calculation as Rankine says err on the safe side.

Then the only way to proceed appears to be to find the normal thrust for each stone separately from crown to springing as follows:—

Suppose all eight voussoirs laid on any back piece; θ, the angle between a tangent at its centre and a horizontal; and $\Delta\theta$, the angle subtended at the centre by one voussoir. Resolve the weight of the first voussoir w_1 into two components, $w_1 \sin\theta_1$ and $w_1 \cos\theta_1$: the first is resisted by the first joint and the second by the centring. The next load w_2 is resolved into $w_2 \sin\theta_2$ and $w_2 \cos\theta_2$;


8"x10"
8"x10"
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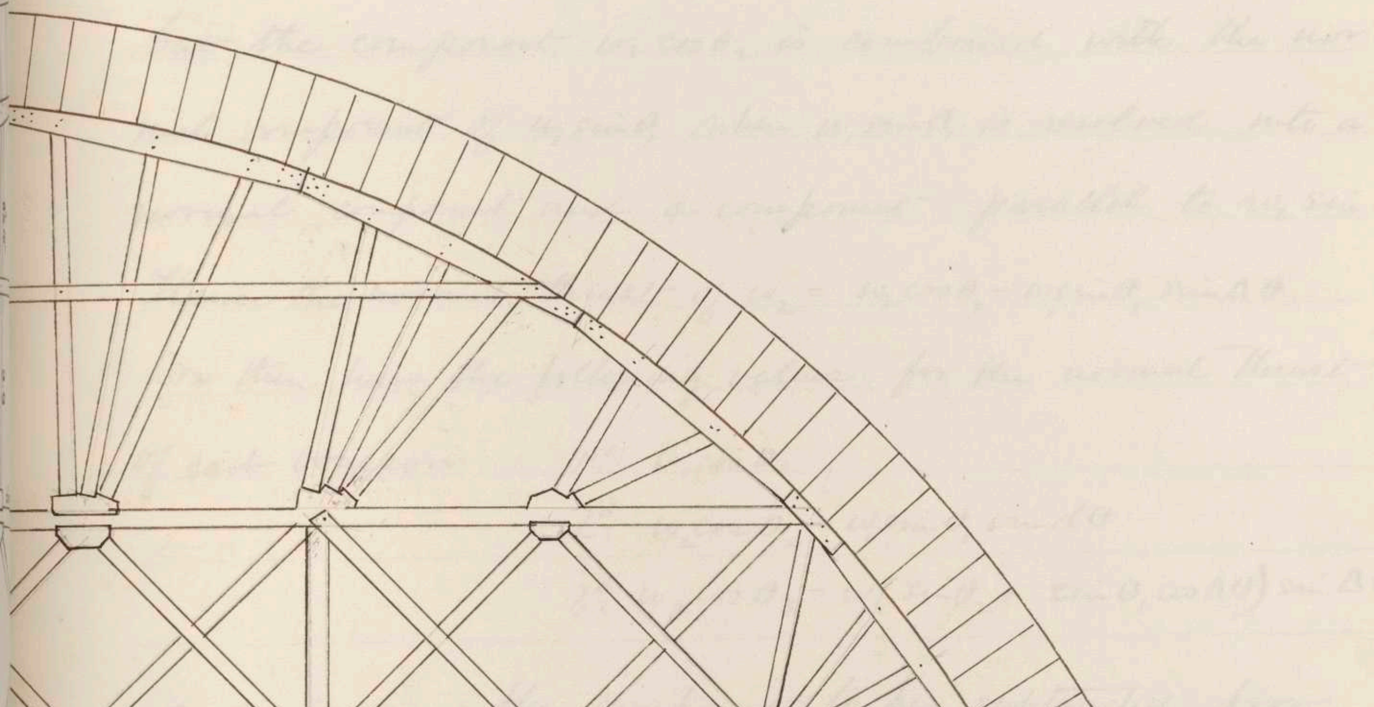
8"x10"
8"x10"
8"x10"
9.6
9.4
9.9
9.1
8.9
9.3
8.2
8.2
12"x12"
8.5
15.7
27.6
33.6
6"x10"
8.4
6"x8"
6.6
6"x8"
6.6
13.0
13.0
23.7
19.5
12"x12"
9.3
23.2
33.7
46.9
2 Pieces 10"x10"
2 Piece 12"x12"



but the component $w_2 \cos\theta_2$ is combined with the normal component of $w_1 \sin\theta_1$ when $w_1 \sin\theta_1$ is resolved into a normal component and a component parallel to $w_2 \sin\theta_2$

Hence the normal thrust of $w_2 = w_2\cos\theta_2 - w_1 \sin\theta_1 \sin\Delta\theta$.

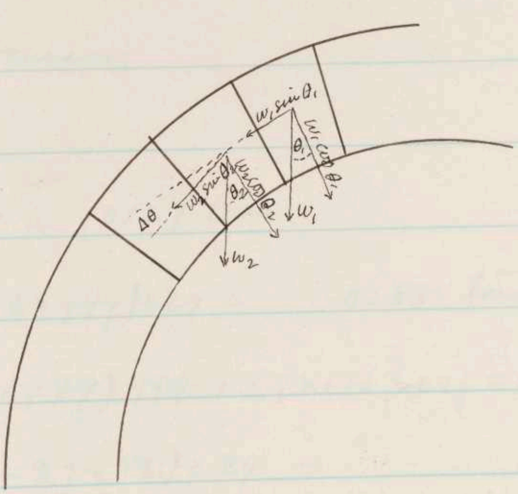
We then have the following values for the normal thrust of each voussoir

1^{st} $w_1 \cos\theta_1$

2^{d} $w_2 \cos\theta_2 - w_1 \sin\theta_1 \sin d\theta$

3^{d} $w_3 \cos\theta_3 - w(\sin\theta_2 + \sin\theta_1 \cos\Delta\theta) \sin\Delta\theta$.

and so on, the component to be subtracted from any $w\cos\theta$, being equal to:- the factor that is multiplied by $\sin\Delta\theta$ in the stone above $\times \cos.\Delta\theta$ + the weight of the stone considered multiplied by the corresponding $\sin\theta$. - all multiplied by $\sin\Delta\theta$.



The load on the back piece at the crown is 30 tons for one rib, so proceeding to the next backpiece the load is 29.6 tons $29.6 \times \cos 13° 20'$ (angle to the middle) = 28.8 tons. From this is to be subtracted the following quantities; —

$$\sin d\theta = \sin 1° 40' = .029 \quad \cos = .999$$

$\theta_1 = 6° 48$ sin = .1185 9

$\theta_2 = 8° 28'$ " = .147

$\theta_3 = 10\ 8'$ " = .176

$\theta_4 = 11° 48'$ " = .205

$\theta_5 = 13° 28'$ " = .233

$\theta_6 = 15° 8'$ " = .261

$\theta_7 = 16° 48'$ " = .289

$\theta_8 = 18° 28'$ " = .317

Wts of single voussoirs on

2^d backpiece	3.7 tons
3^d "	3.9 "
4^th "	4.2 "
5^th "	4.4 "

Taken off from voussoir

1. 0 tons

2. $3.7 \times .1186 \times .029 = .0127$ tons

3. $3.7 \times (.147 + .1186 \times .999) .029 = .0288$ tons

4. $\{3.7 (.147 + .1186 \times .999) .999 + 3.7 \times .176\} .029 =$ .0474 tons

5. $(1.6333 \times .999 + 3.7 \times .2) .029 =$ .0687 "

6. $(2.371 \times .999 + 3.7 \times .233) .029 =$ .0910 "

7. $(3.132 \times .999 + 3.7 \times .261).029 =$ .1188 tons

8. $(4.0977 \times .999 + 3.7 \times .289).029 =$ .1496 tons

Total .5170 "

Hence normal pressure on 2^d back-piece = 28.8 − .5 = 28.3 tons.

3^d Backpiece.

$\theta_1 = 18°\ 28'$ sin = .317 cos. = .948

$\theta_2 = 20°\ 8$ " .344 " .939

$\theta_3 = 21°\ 48$ .37 .928

$\theta_4 = 23°\ 28$ .398 .917

$\theta_5 = 25°\ 8$ .425 .91

$\theta_6 = 26°\ 48$ .45 .893

$\theta_7 = 28°\ 28$ .477 .879

$\theta_8 = 30°\ 8$ .502 .865

Normal thrust due to each voussoir:—

1. $w\cos\theta_1 = 3.9 \times .948 =$ 3.697 tons

2. $3.9 \times .939 - 3.9 \times .317 \times .029 =$ 3.626 "

3. $3.9 \times .928 - 3.9(.939 + .317 \times .999).029 =$ 3.477 "

~~5~~4. $3.9 \times .917 - (4.9 \times .999 + 3.9 \times .398).029 =$ 3.389 "

5 $3.9 \times .91 - (6.45 \times .999 + 3.9 \times .425).029 =$ 3.314 "

6. $3.9 \times .893 - (8.107 \times .999 + 3.9 \times .45).029 \quad = \quad 3.197$ tons.

7. $3.9 \times .879 - (9.86 \times .999 + 3.9 \times .477).029 \quad = \quad 3.088$ "

8. $3.9 \times .865 - (11.72 \times .999 + 3.9 \times .502).029 \quad = \quad \underline{2.977}$

Total 26.765

4^{th} Back piece.

$\theta_1 = 31^\circ \ 48'$ sin = .527 cos = .85

$\theta_2 = 33^\circ \ 28'$ " .55 " .834

$\theta_3 = 35^\circ \ 8'$ " .575 " .818

$\theta_4 = 36^\circ \ 48'$ " .599 " .8

$\theta_5 = 38^\circ \ 28'$ " .622 " .783

$\theta_6 = 40^\circ \ 8'$ " .645 " .765

$\theta_7 = 41^\circ \ 48'$ " .666 " .745

$\theta_8 = 43^\circ \ 28'$ " .688 " .726

Normal thrust due to each voussoir. —

1. $w\cos\theta = 4.2 \times .85 =$ 3.570 tons

2. $4.2 \times .834 - 4.2 \times .527 \times .029 =$ 3.439 "

3. $4.2 \times .818 - (2.213 \times .999 + 4.2 \times .55).029 =$ 3.305 "

4. $4.2 \times .8 - (4.522 \times .999 + 4.2 \times .575).029 =$ 3.159 "

5. $4.2 \times .783 - (6.936 \times .999 + 4.2 \times .599).029 =$ 3.015 "

6. $4.2 \times .765 - (9.451 \times .999 + 4.2 \times .622).029 =$ 2.863 "

7. $4.2 \times .745 - (12.062 \times .999 + 4.2 \times .645).029 =$ 2.701 tons

8. $4.2 \times .726 - (14.77 \times .999 + 4.2 \times .666).029 =$ 2.546 "

Total 24.588 "

5th Backpiece.

$\theta_1 = 45° \; 8'$ sin $= .709$ cos $= .705$

$\theta_2 = 46° \; 48'$ " $.729$ " $.685$

$\theta_3 = 48° \; 28'$ " $.749$ " $.663$

$\theta_4 = 50° \; 8'$ " $.768$ " $.641$

$\theta_5 = 51° \; 48'$ " $.786$ " $.618$

$\theta_6 = 53° \; 28'$ " $.804$ " $.595$

$\theta_7 = 55° \; 8'$ " $.82$ " $.572$

$\theta_8 = 56° \; 48'$ " $.837$ " $.548$

Normal thrusts due to each voussoir:—

1. $w\cos\theta = 4.4 \times .705 =$ 3.102 tons

2. $4.4 \times .685 - 4.4 \times .709 \times .029 =$ 2.924 "

3. $4.4 \times .663 - (4.4 \times .729 + 3.12 \times .999).029 =$ 2.733 "

4. $4.4 \times .641 - (6.33 \times .999 + 4.4 \times .749).029 =$ 2.541 "

5. $4.4 \times .618 - (9.626 \times .999 + 4.4 \times .768).029 =$ 2.342 "

6. $4.4 \times .595 - (13.004 \times .999 + 4.4 \times .786).029 =$ 2.141

7. $4.4 \times .572 - (16.461 \times .999 + 4.4 \times .804).029 =$ 1.937

8. $4.4 \times 548 - (19.997 \times .999 + 4.4 \times .82) .029 =$ $\underline{1.726}$ tons

Total 19.446 "

Direct supports to Back pieces.

1st. Load 30 tons. Stresses in each piece 10.4, 10.0 and 10.4 tons.

The dimensions of the timbers are found by use of Gordon's formula $P = \dfrac{fS}{1 + a\dfrac{l^2}{h^2}}$

Taking for the value of f, the resistance to crushing in lbs. per. sq. in. 6000 lbs and allowing a factor of safety of 10; Also taking for a, $\frac{1}{250}$ we have

$P = \dfrac{600\,S}{1 + \frac{1}{250}\left(\frac{l}{b}\right)^2}$

For this first case, assuming $b = 8''$ and taken the length as the greatest unsupported length we find the dimensions to be $8'' \times 9''$, $8'' \times 8''$ and $8'' \times 9''$.

2d. case, supports of 2d. back piece.

Loads are 9.9, 9.4 and 9.6 tons.

9.9 tons = 19800 lbs $l = 10\text{ft} = 120''$

assume $b = 8''$, then $\left(\frac{l}{b}\right)^2 = 225$ and $\dfrac{P}{h} = \dfrac{4800 \times 250}{475}$

From this $h = \frac{475 \times 19800}{4800 \times 250} = 8''$

This gives $8'' \times 8''$ and the other pieces are so nearly the same in amount of stress sustained and length that the same dimensions may be used.

Take the 3rd back-piece, The longest strut here is 13.5 ft long = 162" $P = 9.7$ tons = 19400 lbs

assume $b = 8''$ and $\frac{P}{h} = \frac{4800 \times 250}{650}$ $\therefore h =$

$\frac{19400 \times 650}{4800 \times 250} = 7''$

hence we may use $8'' \times 7''$. and these same dimensions would do for the others of the same back-piece.

4th back-piece.

$P = 8.2$ tons = 16400 lbs $l = 7' = 84''$ assume $b = 6''$

$\frac{P}{h} = \frac{3600 \times 250}{1 + \cancel{250}\frac{196}{250}}$ $\therefore \frac{P}{h} = \frac{3600 \times 250}{446}$ and $h = 8''$

hence dimensions are $6'' \times 8''$.

5th back-piece. $P = 8.4$ tons = 16800 lbs $l = 10$ ft = 120 in.

assume $b = 6''$ then $\left(\frac{l}{b}\right)^2 = 400$ and

$\frac{P}{h} = \frac{3600 \times 250}{650}$ $\therefore h = \frac{16800 \times 650}{3600 \times 250} = 12''$

Here we find the dimensions larger than really used in construction, which is due to the stress having been found less by taking into account the action of friction. For the other two pieces to support this back-piece we have:-

$P = 6.6$ tons $= 13200$ lbs $l = 8' = 96''$

assume $b = 6''$ then $\left(\frac{l}{b}\right)^2 = (16)^2 = 256$

and $\frac{P}{h} = \frac{3600 \times 250}{506}$. From this $h = \frac{506 \times 13200}{3600 \times 250} = 7''$

giving scantling of $6'' \times 7''$.

Next we have to calculate the members in the truss. The stress in the middle post is evidently 30 tons = 60000 lbs. $l = 12' = 144''$ assume $b = 12''$ then $\left(\frac{l}{b}\right)^2 = 144$ and $\frac{P}{h} = \frac{7200 \times 250}{394}$ $\therefore h =$

$\frac{394 \times 60000}{7200 \times 250} = 13''$. giving $12 \times 13''$ those used were $12'' \times 14''$.

The next post has a stress of 23.7 tons = 47400 lbs

$l = 12' = 144''$ assume $b = 12''$ $(l/b)^2 = 144$

$\frac{P}{h} = \frac{7200 \times 250}{394}$ $\therefore h = 10''$

giving scantling of $12'' \times 10''$

To find the thrust in each of the diagonals in the bay next the middle, we resolve the weight transfered from the 2d. back-piece, which is 28.8 acting normally ~~vertically~~ into W cos of 13° 20' and W sin 13° 20'. The first 28.8 × .97 gives the vertical component = 27.9 tons and 28.8 × sin 13°20' = 28.8 × .23 = 6.6 tons the compression added to the upper chord.

For the load on 3d. back piece:-

26.7 sin 26°40' = 26.7 × .449 = 11.9 compression added to upper chord and

26.7 cos 26° 40' = 26.7 × .89 = 23.9 tons which gives the compression in 2d. post from middle.

Load on 4th. backpiece 24.5 tons, resolved gives 24.5 sin 40° = 24.5 × .64 = 15.7 tons, compression added to upper chord and

24.5 cos 40° = 24.5 × .76 = 18.6, half of which goes down each of the struts in the 2d bay.

To get the stress in each of these we must multiply $\frac{18.6}{2}$ by cosec. 45° ~~the sin of 45°~~. 9.3 × 1.4 = 13. tons stress.

Adding the vertical components of the stresses acting at the foot of the middle post we get 57.8 tons. stress in the middle vertical foundation timber.

For the other foundation timbers we obtain in the same way 46.9 tons and 33.7 tons

The centring was struck by means of the screws in the fall of 1876 after the spandrel had been built up about half way and the crown of the arch was loaded with several tons of granite. The deflection was about 1/2 inch