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EXTENSIONS AND APPLICATIONS OF A SECOND-ORDER LANDSURFACE PARAMETERIZATION

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HYDROLOGY AND WATER RESOURCE SYSTEMS

Report No. 290

Prepared under the support of
The National Aeronautics
and Space Administration
Grant NAG 5-134

July 1983

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ABSTRACT

This work develops extensions and applications of a second-order land-surface parameterization, proposed by Andreou and Eagleson [1982]. Procedures for evaluating the near surface storage depth to be used in one-cell land-surface parameterizations are suggested and tested by using the model.

Sensitivity analysis to the key soil parameters is performed. A case study involving comparison with an "exact" numerical model and another simplified parameterization, under very dry climatic conditions and for two different soil-types, is also incorporated.

ACKNOWLEDGEMENTS

This work was completed with the support of the National Aeronautics and Space Administration (NASA) under Grant NAG 5-134.

The work was performed by Stefanos A. Andreou, research assistant in Civil Engineering at MIT and was supervised by Peter S. Eagleson, Professor of Civil Engineering.

Thanks are due to Antoinette DiRenzo, who performed all the necessary typing.

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NOTATION

Symbols	Definition and Dimensions	
A	albedo	[-]
C	volumetric heat capacity of a homogeneous medium	[cal $L^{-3}deg^{-1}$]
C_H	coefficient for the sensible heat transfer	[-]
C_w	coefficient for the water vapor transfer	[-]
C_p	specific heat of water vapor at constant pressure	[$L^2T^{-2}deg^{-1}$]
c	pore disconnectedness index	[-]
d	diffusivity index	[-]
D_e	disorption diffusivity	[L^2T^{-1}]
$D_{\psi v}$	Vapor conductivity	[LT^{-1}]
E_{P_A}	annual potential evapotranspiration	[L]
E_{T_A}	annual actual evapotranspiration	[L]
E	exfiltration parameter	[-]
E	evaporation rate	[LT^{-1}]
$\bar{e}_T$	average annual actual evapotranspiration rate	[LT^{-1}]
$\bar{e}_p$	average annual potential evapotranspiration rate	[LT^{-1}]
e_T	actual evapotranspiration rate	[LT^{-1}]
e_p	potential evapotranspiration rate	[LT^{-1}]
e_{T_v}	transpiration rate from vegetation	[LT^{-1}]

Symbols	Definition and Dimensions	
f_e	exfiltration capacity of soil	$[LT^{-1}]$
f_i	infiltration capacity of soil	$[LT^{-1}]$
G	heat flux into the soil	$[FL^{-1}]$
h_o	surface retention capacity	$[L]$
H	sensible heat	$[FL^{-1}]$
I_s	incoming shortwave radiation	$[FL^{-1}]$
I_{ℓ_d}	down long-wave radiation	$[FL^{-1}]$
i	rainfall rate	$[LT^{-1}]$
J	evapotranspiration efficiency	$[-]$
$K(1)$	saturated hydraulic conductivity	$[LT^{-1}]$
$k(1)$	saturated intrinsic permeability	$[L^2]$
k	Von Karman's Constant	$[-]$
k_v	plant coefficient	$[-]$
k_s	soil thermal diffusivity	$[L^2T^{-1}]$
L	latent heat of vaporization	$[L^2T^{-1}]$
L	Monin-Obukhov length	$[L]$
M	vegetal canopy density	$[-]$
M_o	equilibrium vegetal canopy density	$[-]$
m_τ	rainy season length	$[T]$

Symbols	Definition and Dimensions	
m_{t_b}	mean time between storms	[T]
m_{P_A}	mean annual precipitation	[L]
m_ν	mean number of storms per year	[-]
m_i	mean storm intensity	$[LT^{-1}]$
m_H	mean storm depth	[L]
m	pore size distribution index of soil	[-]
m	relative thickness of the atmosphere	[-]
N	cloud cover	[ ]
n	effective porosity of the soil	[-]
p	precipitation rate	$[LT^{-1}]$
P_A	annual precipitation	[L]
$\bar{p}$	mean storm intensity	$[LT^{-1}]$
p_a	atmospheric pressure	$[FL^{-2}]$
q^*	saturated atmospheric specific humidity	[-]
q_a	specific humidity of the atmosphere at screen elevation	[-]
$(R_i)_B$	bulk Richardson number	[-]
R_{g_A}	annual groundwater runoff	[L]
R_{S_A}	annual surface runoff	[L]
R	gas constant	$[L^2T^{-2}deg^{-1}]$

Symbols	Definition and Dimensions	
S_e	exfiltration "desorptivity"	$[LT^{-1/2}]$
S_i	infiltration "sorptivity"	$[LT^{-1/2}]$
s	average soil moisture at the surface layer	[-]
s_o	average annual soil moisture at the surface layer	[-]
s_k	soil moisture concentration at time k	[-]
$\bar{T}$	average annual atmospheric temperature	[deg]
T_a	air temperature at screen height	[deg]
T_1	near surface soil temperature	[deg]
T_2	deep mean soil temperature	[deg]
t_o	time when the surface becomes dry during an evaporation period	[T]
t_r	storm duration	[T]
t_b	time between storms	[T]
U_a	wind speed	$[LT^{-1}]$
w	upward capillary rise from the water table	$[LT^{-1}]$
$\bar{Y}_A$	average annual yield	[L]
y_{c_p}	cumulative yield	[L]
y_g	percolation rate	$[LT^{-1}]$
$\bar{y}_g$	average annual percolation rate	$[LT^{-1}]$
y_s	surface runoff rate	$[LT^{-1}]$
$\bar{y}_s$	average annual surface runoff rate	$[LT^{-1}]$

Symbols	Definition and Dimensions	
$y(s)$	total yield rate at soil moisture level s	$[LT^{-1}]$
Z_r	surface layer thickness (storage depth)	[L]
Z_a	screen height	[L]
Z_o	surface roughness	[L]
α	angle of the sun above the horizon	[rad]
a_1	molecular scattering coefficient	[-]
Θ	volumetric moisture content	[-]
Θ_{f_c}	field capacity	[-]
κ	shape factor of Gamma-distributed rainstorm depths	[-]
λ	thermal conductivity	$[cal\ L^{-1}T^{-1}deg^{-1}]$
λ	parameter of Gamma-distributed storm depth	[-]
ρ_α	density of water vapor in the air	$[FL^{-4}T^2]$
ρ_e	mass density of water	$[FL^{-4}T^2]$
ρ_{vg}	density of water vapor at the ground surface	$[FL^{-4}T^2]$
σ	capillary infiltration parameter	[ ]
σ_{P_A}	Variance of annual precipitation	$[L^2]$
τ_1	one day	[T]
ϕ_e	dimensionless desorption diffusivity of soil	[-]

Symbols	Definition and Dimensions	
ϕ_i	dimensionless sorption diffusivity of soil	[-]
Ψ	soil matric potential	[L]

Chapter 1

Introduction

This work consists of extensions and applications of the second-order Budyko-type parameterization of landsurface hydrology proposed by Andreou and Eagleson [1982]. It is based on a one-dimensional, short-term water balance model which can be coupled with a thermal balance model to obtain estimates of moisture and heat fluxes across the land surface. More specifically the objectives of this study were:

1. Perform a sensitivity analysis of the annual yield estimated by the model with respect to the soil properties $k(1)$ and c, about their "optimal" [Eagleson, 1982] values for contrasting climates.
2. Evaluate the sensitivity of the yield to the selection of the storage depth used by the model.
3. Establish an analytical procedure for making the above evaluation.
4. Propose a way of selecting the storage depth independently from calibrations using detailed numerical models.
5. Test the model under very dry climatic conditions and compare the results with those obtained by the parameterization of Milly and Eagleson [1982].

Chapter 2

Sensitivity of Cumulative Yield During the Rainy Season with Respect to Changes of the Optimal Soil Properties k(1) and c

In all tests of the short-term water-balance model, that appear in the Technical Report No. 280 [Andreou and Eagleson, 1982], for the catchments of Clinton, Massachusetts and Santa Paula, California, the values of soil intrinsic permeability k(1) and pore disconnectedness index c were set equal to their optimal values, as they were derived by applying Eagleson's [1982] ecological optimality hypotheses. In this section, a sensitivity analysis is performed, in order to find out the effect on the prediction of cumulative yield, of varying k(1) and c from those optimal values.

It is important to know how robust the results of the model are with respect to changes in those values of soil properties, since in reality many uncertainties will be encountered about their true value.

Thus, the short-term water-balance model, as described by Andreou and Eagleson [1982], was again applied at the two contrasting climates of Clinton, Massachusetts and Santa Paula, California. All climatic and soil parameters remained unchanged, as given in Table 2.1 except the values of k(1) and c that were varied one at a time.

The model was run for a period equal to the rainy season length. First, the value of k(1) was increased by 20 percent from its optimal value, everything else remaining constant, and the percentage change of cumulative yield from that corresponding to the optimal k(1) was calculated. The same procedure was also followed by increasing c by 20 per-

Table 2.1

Climatic and Soil Properties of Clinton, Massachusetts and Santa Paula, California

Clinton, Massachusetts		Santa Paula, California	
M_o	$= 0.912$	M_o	$= 0.424$
$\bar{e}_p$	$= 0.150$ cm/day	$\bar{e}_p$	$= 0.274$ cm/day
m_{t_b}	$= 3$ days	m_{t_b}	$= 10.42$ days
m_{t_r}	$= 0.32$ days	m_{t_r}	$= 1.43$ days
m_τ	$= 365$ days	m_τ	$= 212$ days
w/e_p	$= 0$	w/e_p	$= 0$
m_ν	$= 109$	m_ν	$= 15.7$
m_{P_A}	$= 94$ cm	m_{P_A}	$= 54$ cm
k_v	$= 1$	k_v	$= 1$
$\bar{T}_a$	$= 8.4°C$	$\bar{T}_a$	$= 13.8°C$
κ	$= 0.50$	κ	$= 0.25$
λ	$= 0.578$	λ	$= 0.0732$
$k(1)$	$= 5.57\text{x}10^{-11} \cdot \text{cm}^2$	$k(1)$	$= 12.27\text{x}10^{-11}\text{cm}^2$
c	$= 4.75$	c	$= 5.25$
n_e	$= 0.35$	n_e	$= 0.35$

[The values of M_o, k(1), and c were set equal to those corresponding to peak climatic values, according to the vegetal equilibrium hypothesis and the ecological optimality hypothesis, as they are described by Eagleson [1982.]]

cent from its optimal value and keeping everything else constant. It must be pointed out that when $k(1)$ or c was changed from its optimal value, the annual water balance [Eagleson, 1978] was solved in order to determine a new value of the annual average soil moisture s_o around which to linearize the evaporation and yield functions.

The test was repeated using different values of Z_r (the surface layer thickness) in the range of 20cm $\sim$ 120cm. Two different tests were performed for each climate; one assuming bare soil and one by setting the vegetation equal to its optimum value M_o^*.

The results are shown in Tables 2.2 and 2.3. As was expected, for the humid climate of Clinton, Massachusetts the cumulative yield is rather insensitive to changes in the soil properties $k(1)$ and c. There is no particular sensitivity to either of those two parameters. Also, there is very small difference between bare and vegetated soil. The explanation for this is that in the humid climate, evaporation is almost always at the potential rate (and for the linearized model used here, this happens all the time due to it's structure). Thus, the only changes that can occur in the yield by varying the soil properties will be due to changes in storage. So, as $k(1)$ increases less water is stored in the layer of depth Z_r and as c increases more water is held in storage. The differences between bare and vegetated soil are very small and are due to small numerical differences between the functions of $J(s_o, M, k_v)$ used for bare and vegetated soil respectively.

For the catchment of Santa Paula, we first observe a difference between bare and vegetated soil. That is, in the presence of vegetation, control passes to the soil for longer time periods, so the role of evapo-

Table 2.2

Sensitivity of Yield Due to Changes in Soil Properties and Z_r

<u>Clinton, Massachusetts</u> (Bare Soil)

	s_o=0.71	s_o=0.75
Z_r (m)	% increase of yield due to 20% increase of k(1)	% reduction of yield due to 20% increase of c
0.2	11.11	0.78
0.4	4.97	2.72
0.6	5.27	2.61
0.8	4.89	3.58
1.0	4.10	4.47
1.2	7.56	2.49

<u>Clinton, Massachusetts</u> (M_o^*=0.912)

	s_o=0.71	s_o=0.75
Z_r (m)	% increase of yield due to 20% increase of k(1)	% reduction of yield due to 20% increase of c
0.2	3.17	1.06
0.4	0.98	2.43
0.6	2.28	2.39
0.8	2.37	3.42
1.0	1.85	4.80
1.2	5.37	2.34

Table 2.3

Sensitivity of Yield Due to Changes in Soil Properties and Z_r

Santa Paula, California (Bare Soil)

Z_r (m)	s_o=0.52 % increase of yield due to 20% increase of k(1)	s_o=0.58 % reduction of yield due to 20% increase of c
0.2	5.07	8.19
0.4	4.72	20.33
0.6	8.76	27.85
0.8	3.68	23.36
1.0	2.79	22.16
1.2	1.35	21.45

Santa Paula, California (M_o^*=0.424)

Z_r (m)	s_o=0.55 % increase of yield due to 20% increase of k(1)	s_o=0.60 % reduction of yield due to 20% increase of c
0.2	4.51	9.72
0.4	12.16	18.47
0.6	18.71	31.45
0.8	11.95	24.52
1.0	10.87	22.70
1.2	11.69	22.64

ration is reduced and percolation becomes more important than when M=0. Thus, the yield becomes more sensitive to changes in k(1) in the presence of vegetation.

We also observe that yield is more sensitive to changes in c than to changes in k(1) for both cases (M=0 and $M=M_o^*$).

In both climates it turns out that knowing the true values of the soil properties k(1) and c is important in order to determine the soil moisture level in the layer near the surface. But for the humid climate, the accuracy of the estimates of those parameters does not significantly influence the estimates of the annual yield obtained by the model. On the contrary, for the semi-arid climate, deviations from the true values of k(1) and c can cause serious errors in the estimation of the annual yield.

Chapter 3

Sensitivity of Annual Yield to Changes of Storage Depth Z_r

In Chapter 7 (Section 7.2) of Andreou and Eagleson [1982] a sensitivity analysis of the annual yield derived from the model was performed with respect to changes in the parameter Z_r. The simulation period used was equal to the length of one rainy season (365 days for Clinton, Massachusetts and 212 days for Santa Paula, California). The differences between the statistics of the generated rainstorm events and interstorm periods and of the historical data were shown in Table 6.1 of Andreou and Eagleson [1982]. Here the model was run for a longer simulation period corresponding to 10 consecutive years of successive precipitation events and dry periods. The statistics of the generated events and the corresponding historical values are shown in Table 3.1. A very small discrepancy between the two is observed.

For every value of Z_r from 20cm to 200cm (using 20cm increments) the value of the average annual yield $\bar{Y}_A$ over the 10 year period was calculated.

The precipitation characteristics that were used to generate the rainy and dry periods in Clinton, Massachusetts were those of Boston, Massachusetts appropriately transformed, so that they corresponded to those of Clinton, Massachusetts. That was necessary to be done, since observations of annual yield, necessary for later comparison did not exist for Boston and on the other hand, hourly precipitation data from Clinton were not available for analysis.

Table 3.1

Statistical Properties of Storm Characteristics

Clinton, Massachusetts

	Historical (Boston, Massachusetts)	Generated
Storm depth [cm]	$E[h] = 0.86$	$E[h] = 0.88$
	$Var[h] = 1.50$	$Var[h] = 1.35$
Storm duration [days]	$E[t_r] = 0.32$	$E[t_r] = 0.32$
	$Var[t_r] = 0.10$	$Var[t_r] = 0.10$
Time between [days]	$E[t_b] = 3$	$E[t_b] = 3.11$
	$Var[t_b] = 9$	$Var[t_b] = 9.49$

Santa Paula, California

	Historical	Generated
Storm depth [cm]	$E[h] = 3.41$	$E[h] = 3.31$
	$Var[h] = 46.65$	$Var[h] = 37.35$
Storm duration [days]	$E[t_r] = 1.43$	$E[t_r] = 1.49$
	$Var[t_r] = 2.04$	$Var[t_r] = 2.38$
Time between [days]	$E[t_b] = 10.42$	$E[t_b] = 10.72$
	$Var[t_b] = 108.58$	$Var[t_b] = 107.89$

By using the sampled mean and variance of annual precipitation at Clinton, Massachusetts obtained from 30 years of data and assuming that the number of storms at Clinton is approximately the same as that in Boston (m_ν=109) we can obtain the values of the parameters κ and λ of the rainstorm characteristics in Clinton by using the following formulas [Eagleson, 1978]

$$\frac{\sigma^2_{P_A}}{m^2_{P_A}} = \frac{1}{m_\nu} [1 + \frac{1}{\kappa}] \tag{3.1}$$

and

$$m_H = \frac{\kappa}{\lambda} \tag{3.2}$$

where

$$m_H = m_{P_A}/m_\nu, \; m_{P_A} = 111.3 \text{ cm}, \; \sigma^2_{P_A} = 268.38 \text{ cm}^2, \; m_\nu = 109$$

The derived values for κ and λ at Clinton are 0.73 and 0.175 respectively.

Thus, using those values for κ and λ and assuming all other precipitation characteristics between Boston and Clinton to be the same, rainstorm events and interstorm durations were generated.

It was found that for the humid climate of Clinton, Massachusetts, the value of $\bar{Y}_A$ remained always almost constant at 54cm, for any value of Z_r in the range 40 $\sim$ 200cm. On the contrary, for the semi-arid climate of Santa Paula, California, there is a drastic change of $\bar{Y}_A$ as Z_r varies, which can be seen from Figure 1. More precisely, there is a rapid decrease of $\bar{Y}_A$ as Z_r increases, although the percentage change of the yield is reduced as Z_r becomes larger.

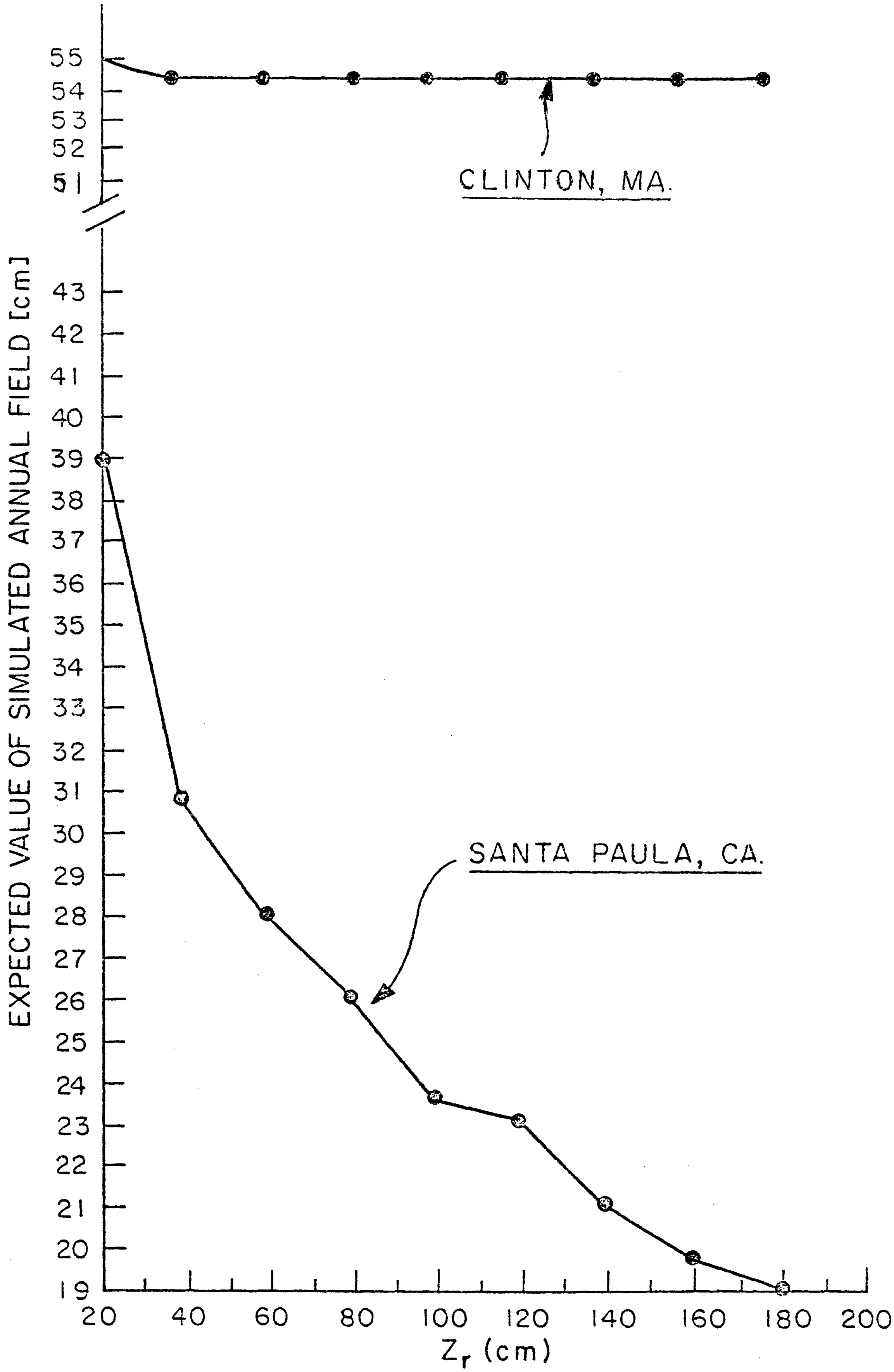

FIGURE 1

Expected value of simulated annual yield as a function of storage depth, Clinton, Massachusetts and Santa Paula, California

The declining value of $E[Y_A]$ with increasing Z_r may be understood qualitatively by first realizing that the larger the "bucket" Z_r, the smaller will be the fluctuation in soil moisture within the bucket due to a given climatic forcing.

In a dry climate where the evaporation rate is controlled by the soil, this reduced decay of soil moisture concentration during an evaporation period means that the volume of evaporative flux increases over that for a smaller bucket having the same average soil moisture. This causes a reduction in the water yield.

Of course the average soil moisture concentration in the bucket is itself dependent on the bucket size which may upset the above reasoning in a particular case.

In a wet climate, the evaporation rate is climate controlled and this sensitivity is not present.

It thus becomes important in dry climates at least to correctly define Z_r.

In Chapter 4 a quantitative analysis is performed in order to explain the functional relation between the yield and the parameter Z_r. Analytical expressions, relating the cumulative yield to Z_r are derived and the trends and behavior of $\bar{Y}_A = \bar{Y}_A(Z_r)$ as they appear in Figure 1, are explained by using approximate solutions of those expressions.

Chapter 4

Sensitivity of Cumulative Yield to Storage Depth Z_r

In this chapter an attempt is made to develop an analytical relation between the cumulative yield and the parameter Z_r during a precipitation and an evaporation event. The impact that variations of Z_r have on the yield is investigated by examining the sign and the order of magnitude of the derivative of the yield with respect to Z_r.

Following the development described in Chapter 4 of Andreou and Eagleson [1982], the one-dimensional short-term water balance of a soil column of depth Z_r can be written in the form:

$$nZ_r \frac{ds}{dt} = i - e_T - y \tag{4.1}$$

where

e_T = evapotranspiration rate

y = yield rate

i = rainfall rate

n = effective porosity of soil

Z_r = storage depth

s = average soil moisture concentration within the layer of thickness Z_r

By linearizing the values of e_T and y around their annual average values, Equation (4.1) can be written in the form:

(i) During a precipitation event (assuming $e_T \approx 0$)

$$nZ_r \frac{ds}{dt} = i - \bar{y} - C(s-s_o) \tag{4.2}$$

(ii) During an evaporation event (i=0)

$$nZ_r \frac{ds}{dt} = -\bar{e}_T - \bar{y}_g - C'(s-s_o) \qquad (4.3)$$

where $\bar{y}$ is the average annual yield rate that equals the average surface runoff rate $\bar{y}_s$ plus the average percolation rate $\bar{y}_g$, and C and C' are the linearization coefficients which are given by:

$$C = \frac{m_{P_A}}{m_\nu m_{t_r}} (C_2 + C_3) \qquad (4.4)$$

$$C' = C_1 \bar{e}_p + \frac{C_3 m_{P_A}}{m_\nu m_{t_r}} \qquad (4.5)$$

and

m_{P_A} = mean annual precipitation

m_ν = mean number of storms per year

m_{t_r} = mean storm duration

$\bar{e}_p$ = average annual potential evaporation rate

C_1 = $\frac{\partial J}{\partial s}$, where J is the evapotranspiration efficiency

$$C_2 = \left. \frac{\partial(y_s/\bar{p})}{\partial s} \right|_{s=s_o} \qquad (4.6)$$

$$C_3 = \left. \frac{\partial(y_g/\bar{p})}{\partial s} \right|_{s=s_o} \qquad (4.7)$$

where

$\bar{p}$ = $m_{P_A}/(m_\nu . m_{t_r})$

s_o = average annual soil moisture concentration

Expressions for calculating C_1, C_2, and C_3 are given by Andreou and Eagleson [1982].

An analytical solution for the differential Equations (4.2) and (4.3) is now derived. We again distinguish between the following cases:

Precipitation

The solution of Equation (4.2) is given in the following form:

$$s(t) = \left(\frac{i - \bar{y} + C\, s_o}{C}\right)\left[1 - e^{\frac{C(t_o - t)}{nZ_r}}\right] + s_i\, e^{\frac{C(t_o - t)}{nZ_r}} \tag{4.8}$$

where

t_o = time that precipitation starts

s_i = initial soil moisture at time t_o

By using its linearized form, the yield rate y during precipitation can be written as follows:

$$y(s) = \bar{y}(s_o) + C(s - s_o) \tag{4.9}$$

The cumulative yield y_{c_p} produced from time t_o to time t can thus be written:

$$y_{c_p} = \int_{t_o}^{t} [\bar{y}(s_o) - C\, s_o + C.s(t)]dt$$

By setting $t_o = 0$ we obtain:

$$y_{c_p} = (\bar{y}(s_o) + C\, s_o)t + (i - \bar{y}(s_o) + C\, s_o)t$$
$$- \left[(i - \bar{y}(s_o) + C\, s_o) - s_i \cdot C\right] \cdot \left[\frac{nZ_r}{C} - \frac{nZ_r\, e^{\frac{-Ct}{nZ_r}}}{C}\right] \tag{4.10}$$

By differentiating y_{c_p} with respect to Z_r, we obtain a quantitative measure of the change that will occur in the cumulative yield during a precipitation event if we vary Z_r. Thus, by using Equation (4.10) we have:

$$\frac{dy_{c_p}}{dZ_r} = -\left[\frac{n}{c}\left(1 - e^{\frac{-Ct}{nZ_r}}\right) - \frac{te^{\frac{-Ct}{nZ_r}}}{Z_r}\right]\cdot\left[i - \bar{y}(s_o) + C\ s_o - s_i C\right] \qquad (4.11)$$

It is evident from Equation (4.11) that depending on the relative magnitude of the components appearing in it, the sign $\frac{dy_{c_p}}{dZ_r}$ can be either positive or negative. Here, an attempt will be made to evaluate this derivative for a particular value of Z_r by assuming an average storm intensity and duration for the catchments of Clinton, Massachusetts and Santa Paula, Calfiornia. The chosen value of Z_r was 100cm. By using the soil and climatic properties of those two catchments given in Table 2.1 we obtain:

Clinton, Massachusetts

$$i - \bar{y}(s_o) + C\ s_o - s_i\ C = 2.68 - 0.5 + (7.69\text{x}0.72) - 7.69s_i$$
$$= 7.71 - 7.69\ s_i > 0, \text{ since } s_i \leq 1$$

and

$$\frac{n}{c}\left(1 - e^{\frac{-Ct}{nZ_r}}\right) - \frac{te^{\frac{-Ct}{nZ_r}}}{Z_r} = \frac{0.35}{7.69}\left(1 - e^{\frac{-7.69\text{x}0.32}{0.35\text{x}100}}\right) - \frac{0.32}{100}\,e^{\frac{-7.69\text{x}0.32}{0.35\text{x}100}}$$

Substituting in Equation (4.7), we obtain:

$$\frac{dy_{c_p}}{dZ_r} = -0.0001\ [7.71 - 7.69\ s_i] < 0$$

Santa Paula, California

$$i - \bar{y}(s_o) + C\ s_o - Cs_i = 2.69 - 0.35 + (4.32x0.55) - 4.32s_i$$

$$= 4.71 - 4.32s_i > 0, \quad \text{Since } s_i \leq 1$$

and

$$\frac{n}{c}\left(1 - e^{\frac{-Ct}{nZ_r}}\right) - \frac{te^{\frac{-Ct}{nZ_r}}}{Z_r} = \frac{0.35}{4.32}\left(1 - e^{\frac{-4.32x1.43}{0.35x100}}\right) - \frac{1.43}{100}\ e^{\frac{-4.32x1.43}{0.35x100}}$$

$$= 0.0011$$

Thus $\frac{dy_{c_p}}{dZ_r} = -0.0011\ [4.71 - 4.32s_i] < 0$

We observe that in both cases, at least on the average and in the vicinity of Z_r = 100cm, cumulative yield is expected to decrease as Z_r increases.

Evaporation

The solution of Equation (4.3) is given in the following form:

$$s(t) = \left(\frac{-\bar{e}_T - \bar{y}_g + C's_o}{C}\right)\left[1 - e^{\frac{C'(t_o-t)}{nZ_r}}\right] + \left(\frac{i - \bar{y} + C\ s_o}{C}\right)\left[1 - e^{\frac{-Ct}{nZ_r}}\right] \cdot$$

$$\cdot e^{\frac{C'(t_o-t}{nZ_r}} + s_i\ e^{\frac{-Ct_o}{nZ_r}} \cdot e^{\frac{C'(t_o-t)}{nZ_r}} \qquad (4.12)$$

where evaporation starts at time $t_o \equiv t_r$, and the initial value of soil moisture at that time was calculated from Equation (4.8), by setting $t = t_r$.

The linearized function of the yield rate y_g during evaporation is given by:

$$y_g = \bar{y}_g(s_o) + C_3\bar{p}(s - s_o) \tag{4.13}$$

Thus the cumulative yield during an evaporation event will be given by:

$$y_{c_E} = \int_{t_o=t_r}^{t} [\bar{y}_g(s_o) + C_3.\bar{p}.s(t) - C_3.\bar{p}.s_o]dt \tag{4.14}$$

By substituting Equation (4.12) in Equation (4.14), we obtain the following expression for y_{c_E}:

$$\begin{aligned} y_{c_E} = {} & \bar{y}_g(t - t_o) - C_3\bar{p}\, s_o(t-t_o) + C_3\bar{p}\left[\left(\frac{(-\bar{e}_T - \bar{y}_g + C's_o)(t-t_o)}{C'}\right)\right. \\ & - \frac{nZ_r\left(1 - e^{\frac{C'(t_o-t)}{nZ_r}}\right)}{C'}\left(\frac{-e_T - \bar{y}_g + C's_o}{C} + \frac{i - \bar{y} + C\, s_o}{C}\right) \\ & \left. - \frac{nZ_r}{C'}\left(e^{\frac{Ct_o}{nZ_r}} - e^{\frac{C'(t_o-t) - Ct_o}{nZ_r}}\right)\cdot\left(\frac{i - \bar{y} + C\, s_o}{C} - s_i\right)\right] \end{aligned} \tag{4.15}$$

If we calculate the derivative of y_{c_E} with respect to Z_r, we find the following expression:

$$\frac{dy_{c_E}}{dZ_r} = -\frac{C_3\bar{p}n}{C'}\left[1 - e^{\frac{C'(t_o-t)}{nZ_r}} + \frac{C'(t_o-t)\, e^{\frac{C'(t_o-t)}{nZ_r}}}{nZ_r}\right] \cdot$$

$$\left(\frac{-\bar{e}_T - \bar{y}_g + C's_o}{C} + \frac{i - \bar{y} + C\, s_o}{C}\right)$$

$$- \frac{C_3\bar{p}n}{C'}\left[e^{\frac{-Ct_o}{nZ_r}}\left(1 + \frac{Ct_o}{nZ_r}\right) + e^{\frac{C'(t_o-t)-Ct_o}{nZ_r}}\left(\frac{C'(t_o-t) - Ct_o}{nZ_r} - 1\right)\right] \cdot$$

$$\cdot\left(\frac{i - \bar{y} + C\, S_o}{C} - s_i\right) \tag{4.16}$$

As we did for the precipitation event, here also the derivative of y_{c_E} with respect to Z_r will be evaluated at Z_r = 100cm and assuming $t_o = t_r = m_{t_r}$, $t - t_o = t_b = m_{t_b}$.

By substituting the parameters for Clinton, Massachusetts and Santa Paula, California in Equation (4.16), we get the following results:

Clinton, Massachusetts

We have: t_o = 0.32 days, $t - t_o$ = 3 days, $i = m_i = 2.69\ \frac{cm}{day}$, $C = 7.69$, $C' = 4.95$, Z_r = 100 cm

$$\frac{dy_{c_E}}{dZ_r} = -0.040 - 0.030\ (1.005 - s_i) < 0, \quad \text{since } s_i \leq 1$$

Santa Paula, California

We have: $t_o = 1.43$ days, $t - t_o = 10.42$ days, $i = m_i = 2.48 \frac{cm}{day}$, $C = 4.32$, $C' = 3.86$, $C_3\bar{p} = 3.41$, $Z_r = 100cm$

We obtain:

$$\frac{dy_{c_E}}{dZ_r} = -0.273 - 0.114\ (1.025 - s_i) < 0, \quad \text{since } s_i \leq 1.$$

Thus, in all cases we observe that at least on the average and for Z_r in the vicinity of 100cm, the cumulative yield decreases as Z_r increases. We also observe that at Santa Paula, California the yield is much more sensitive to changes in Z_r than it is at Clinton, Massachusetts. At Clinton, Massachusetts, the value of $\frac{dy_c}{dZ_r}$ is very close to zero.

Those analytically derived results are consistent with the results obtained by the model and shown in Figure 1.

Chapter 5

Selection of the Appropriate Value of Storage Depth

Up to this point, the model was tested by using an apriori selected value of the storage depth Z_r and different results were obtained by varying its value. In this chapter a way of determining Z_r through comparisons with the observed values of the annual yield is discussed.

To a first-order approximation, the observed expected value of the annual yield can be compared with the expected value of the annual yield obtained by the model after operating it for a certain number of simulation periods. The value of Z_r could then be fitted, so that the two expected yields match.

This comparison was made for the catchments of Clinton, Massachusetts and Santa Paula, California and the results can be summarized as follows. In Clinton, Massachusetts the expected value of annual yield obtained from 30 years of observations (1904 $\sim$ 1933) is $E[\bar{Y}_A]_{obs} = 55.4$cm. The expected value of the annual yield after a 10 year simulation period was found to be: $E[Y_A] \cong 54.30$cm. That value was found to be almost exactly the same for a range of values of Z_r between 40cm and 200cm. This result indicates the insensitivity of the expected annual yield to the value of Z_r for the humid climate of Clinton. This can be explained by the prevailing climate control conditions in this area, as it was argued in detail in Chapter 2. Nevertheless, the result does not help us to determine the appropriate value of Z_r for this catchment.

For Santa Paula, California, the expected value of the observed annual yield is $E[Y_A]_{obs} = 17.4$cm, and the value of the expected annual precipitation is $m_{P_A\ obs} = 54$cm. The simulated value of the expected annual yield obtained by the model, as we observe in Figure 1, is decreasing as Z_r increases and reaches a value of 19cm at $Z_r = 180$cm. The average annual precipitation produced by the generated rainstorm events is equal to $m_{P_{A\ gen}} = 63.58$cm which is considerably larger than the observed value for Santa Paula. This is due to the fact that the average number of storms per year is very small ($m_\nu = 15.7$) and also the variability of the rainy season length was not taken into account in those tests (i.e., τ was set equal to its average value m_τ).

Thus again, by only using expected values of the yield, we cannot obtain conclusive results for the appropriate value of Z_r.

A more accurate way of fitting Z_r to observations of yield would be, by comparing the observed CDF of the annual yield to the CDF of the annual yield obtained by the model through simulation. This type of comparison was performed for the catchments of Clinton, Massachusetts and Santa Paula California. It was considered that the best fit between the observed and simulated CDF was achieved if they had similar shapes and slopes. Possible over or under estimations of the yield by the simulated CDF are expected due to the finite length of the simulation.

In Figures 2 to 6 the values of the observed and simulated CDF's of the annual yield at Clinton, Massachusetts are plotted, for values of Z_r equal to 40, 100, 140, 160, 200cm, respectively. The precipitation characteristics of the rainstorm events were those at Clinton using the derived values of κ and λ from Equations (5.1) and (5.2). We can argue that the best fitting between the two is achieved at $Z_r \simeq 160$cm where a very good agreement with the observed values of yield exists. This result strongly indicates that a value of Z_r in the vicinity of 160cm will be appropriate for operating the model.

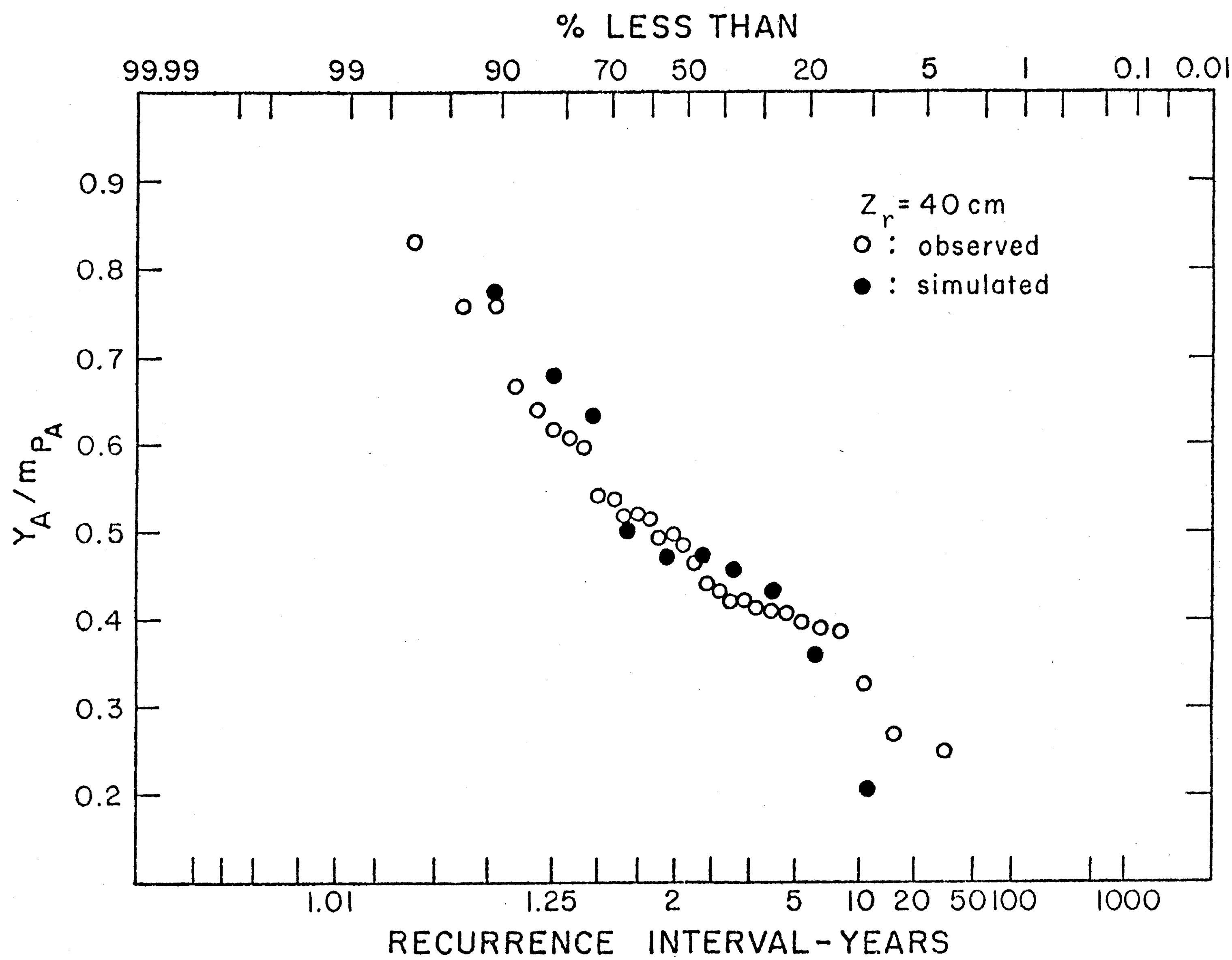

FIGURE 2

Comparison between simulated and observed CDF of annual yield, $Z_r = 40$cm, Clinton, Massachusetts

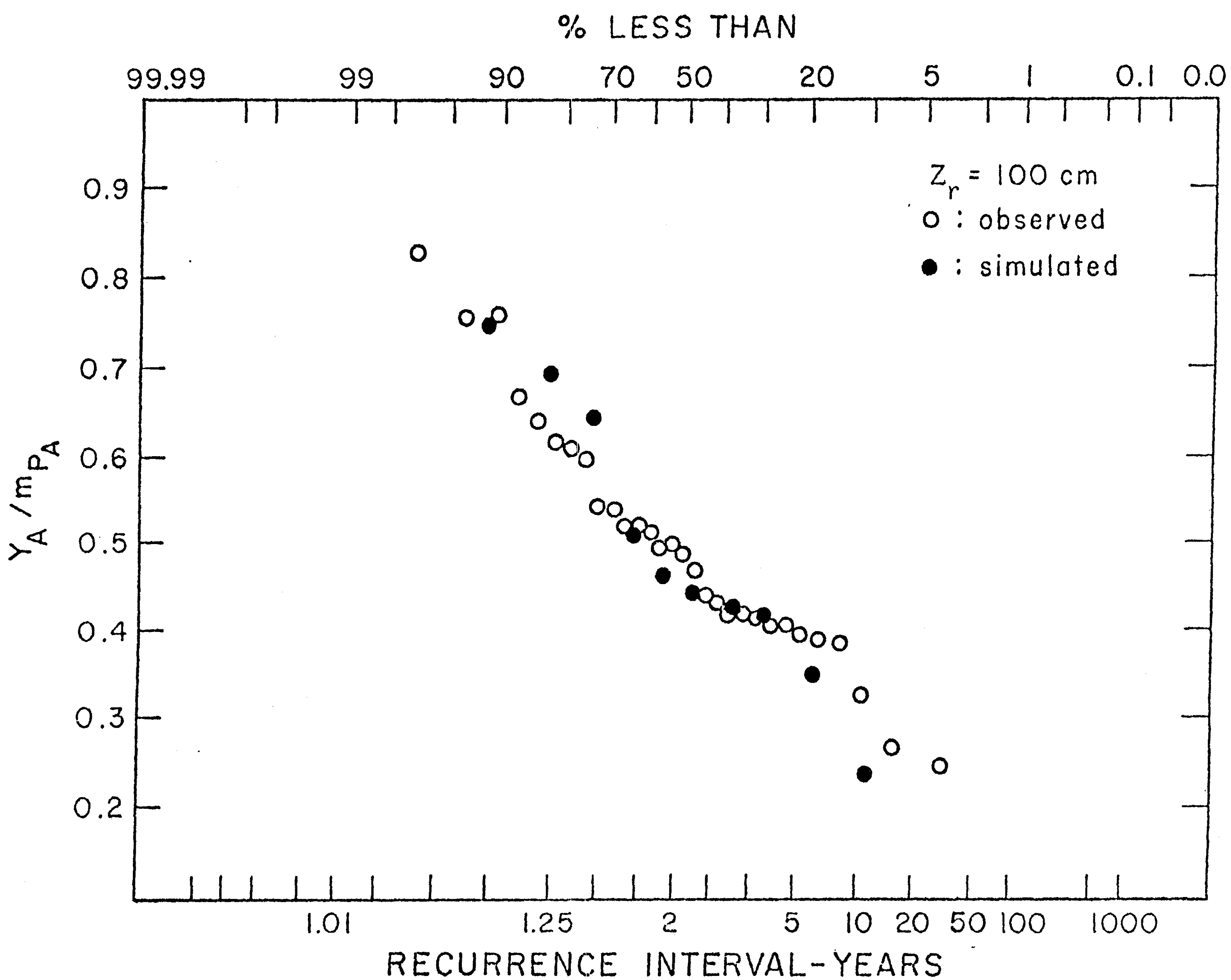

FIGURE 3

Comparison between simulated and observed CDF of annual yield, $Z_r = 100$cm, Clinton, Massachusetts

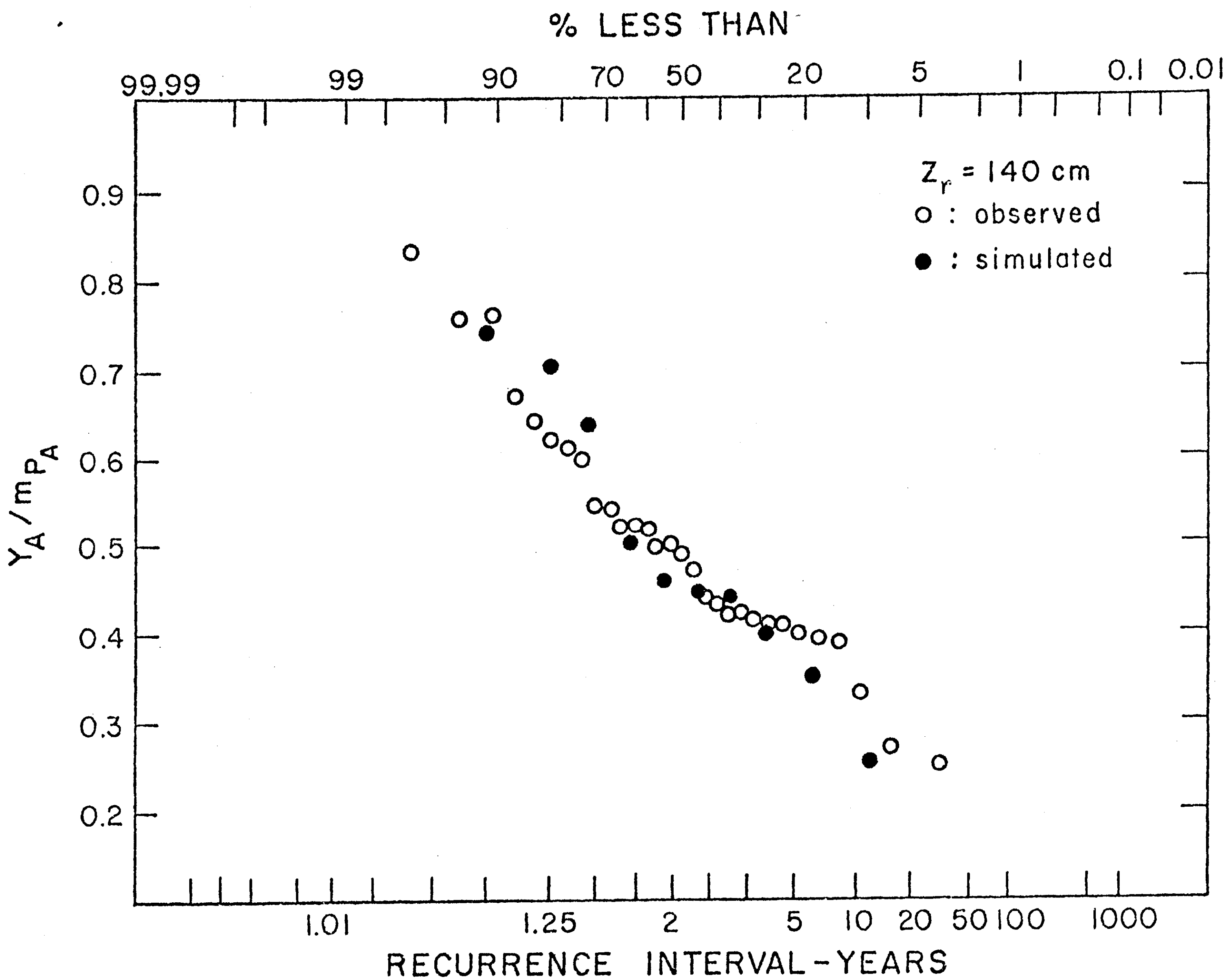

FIGURE 4

Comparison between simulated and observed CDF of annual yield, $Z_r = 140$cm, Clinton, Massachusetts

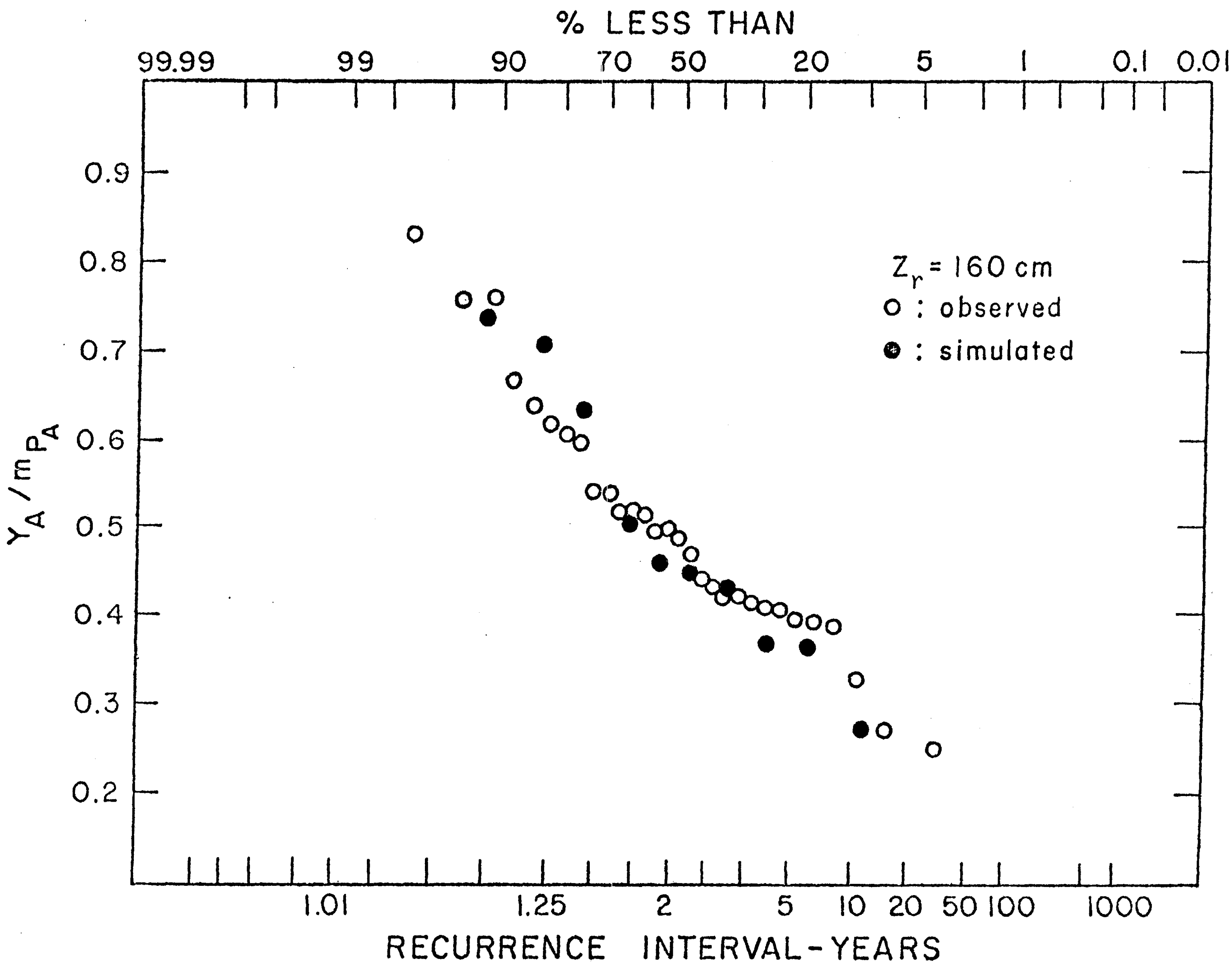

FIGURE 5

Comparison between simulated and observed CDF of annual yield, $Z_r = 160$cm, Clinton, Massachusetts

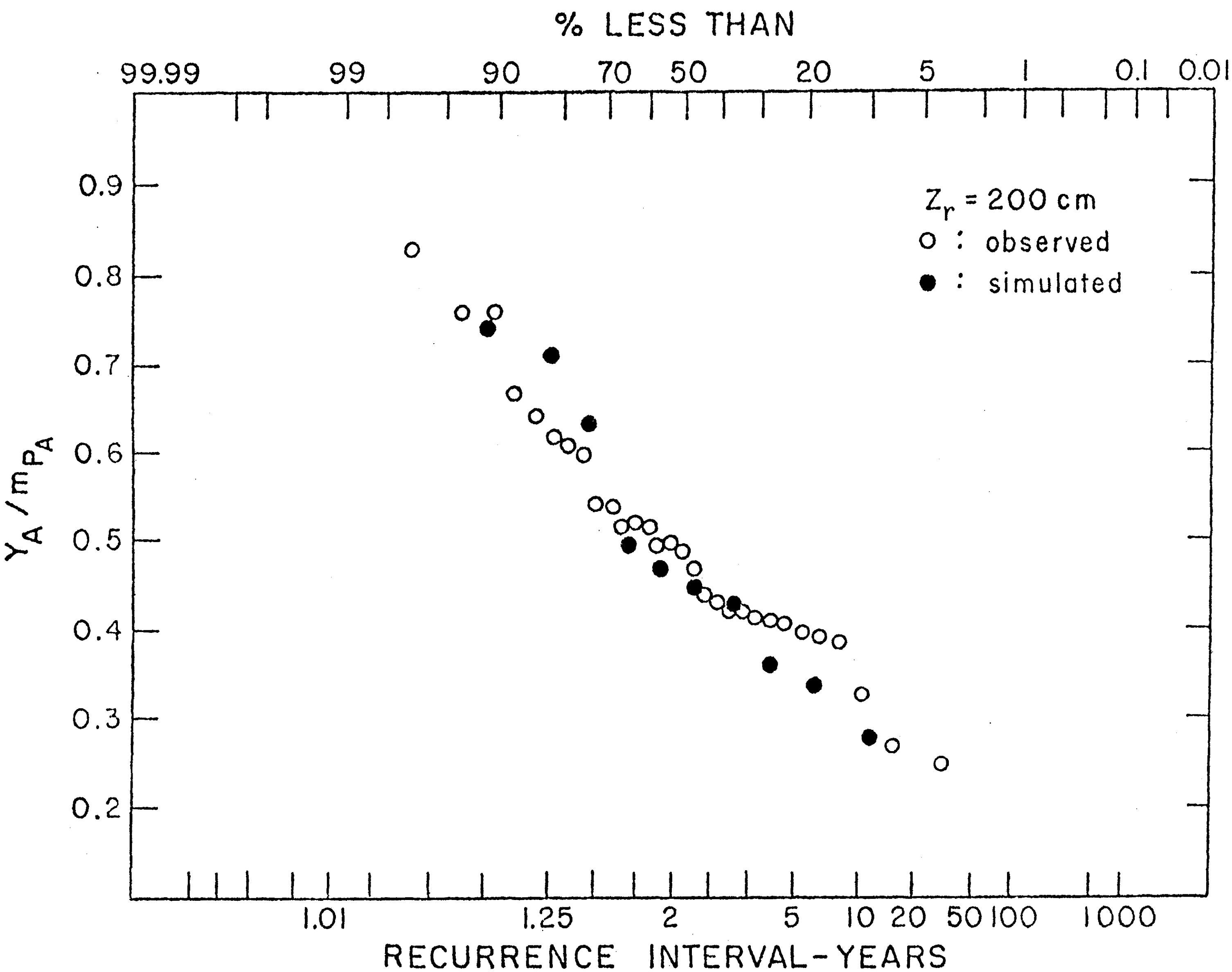

FIGURE 6

Comparison between simulated and observed CDF of annual yield, $Z_r = 200$cm, Clinton, Massachusetts

By using Z_r = 160cm, a comparison between the fluxes obtained by the analytical model and the isothermal version of C. Milly's [1980] numerical model was performed. This type of comparison is described with more details in Andreou and Eagleson [1982], where Z_r = 100cm was assumed. The results for the storage change and for the yield produced, are shown in Figures 7 and 8 respectively for Clinton, Massachusetts. It is evident that by using Z_r = 160cm, we clearly have an improvement of the analytical model.

The same test was also performed for Santa Paula, California. The simulated and observed CDF's of the annual yield are shown in Figures 9 through 13 for values of Z_r equal to 40, 100, 140, 160, 180cm, respectively. It appears that when Z_r is again about 160cm we obtain the best fitting between observed and simulated values of the yield CDF. Since the number of storms per year in Santa Paula is small, we expect to obtain even better results if we run the model for a longer simulation period, since we will approach even closer the historical statistics of the precipitation events. We must nevertheless, keep in mind that some of the discrepancies between observed and simulated values of the yield are due to the fact that the variability of the rainy season length was neglected during those simulations.

For Z_r = 160cm, where the best fitting was observed, the model was operated for a longer simulation period equal to 30 years. The obtained CDF of the simulated annual yield is compared with the observed in Figure 14. It can be argued that it gives a fairly good estimate of the actual CDF.

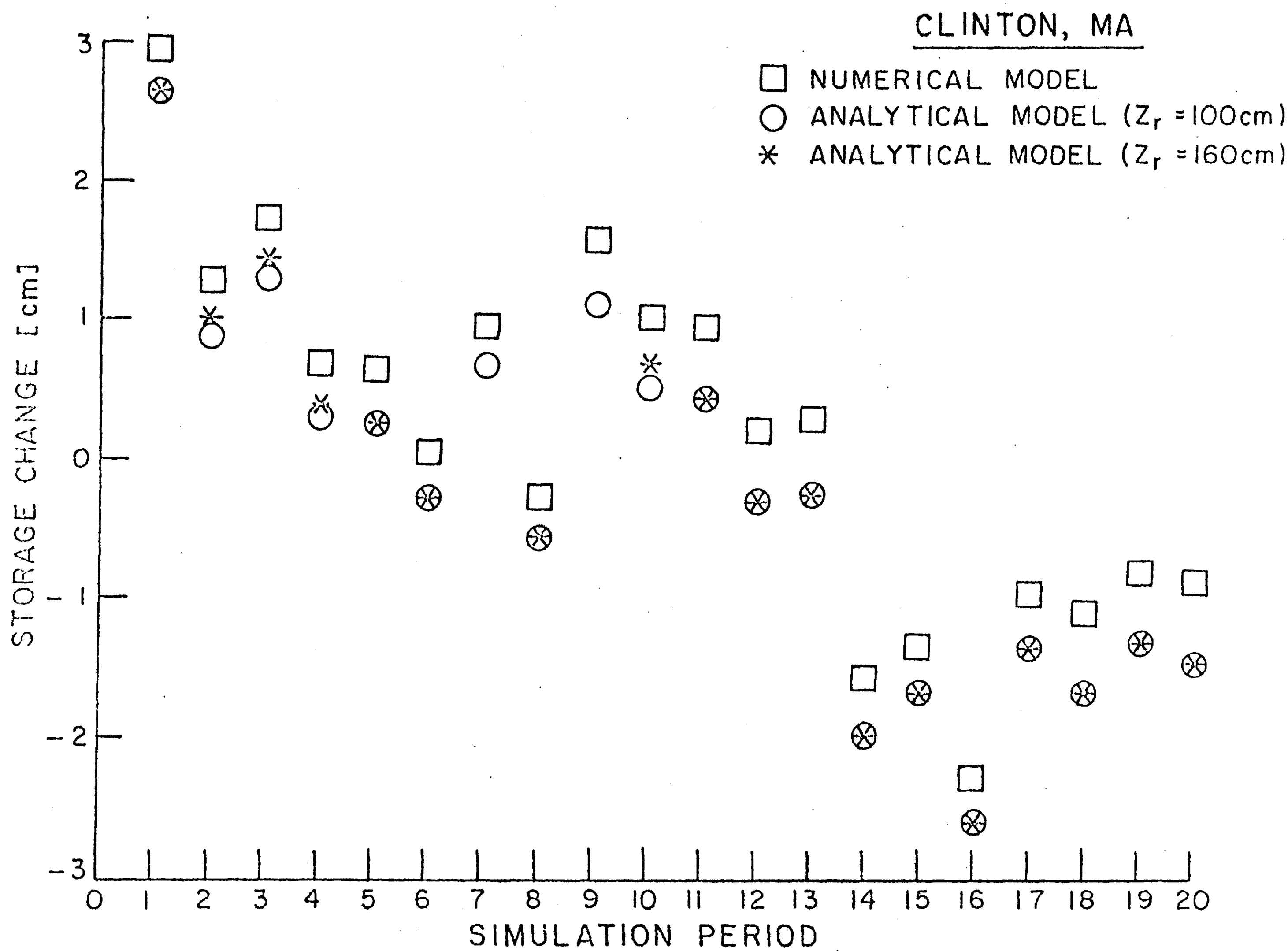

FIGURE 7

Comparison of storage change obtained from Milly's and Eagleson's [1980] numerical model, Clinton, Massachusetts

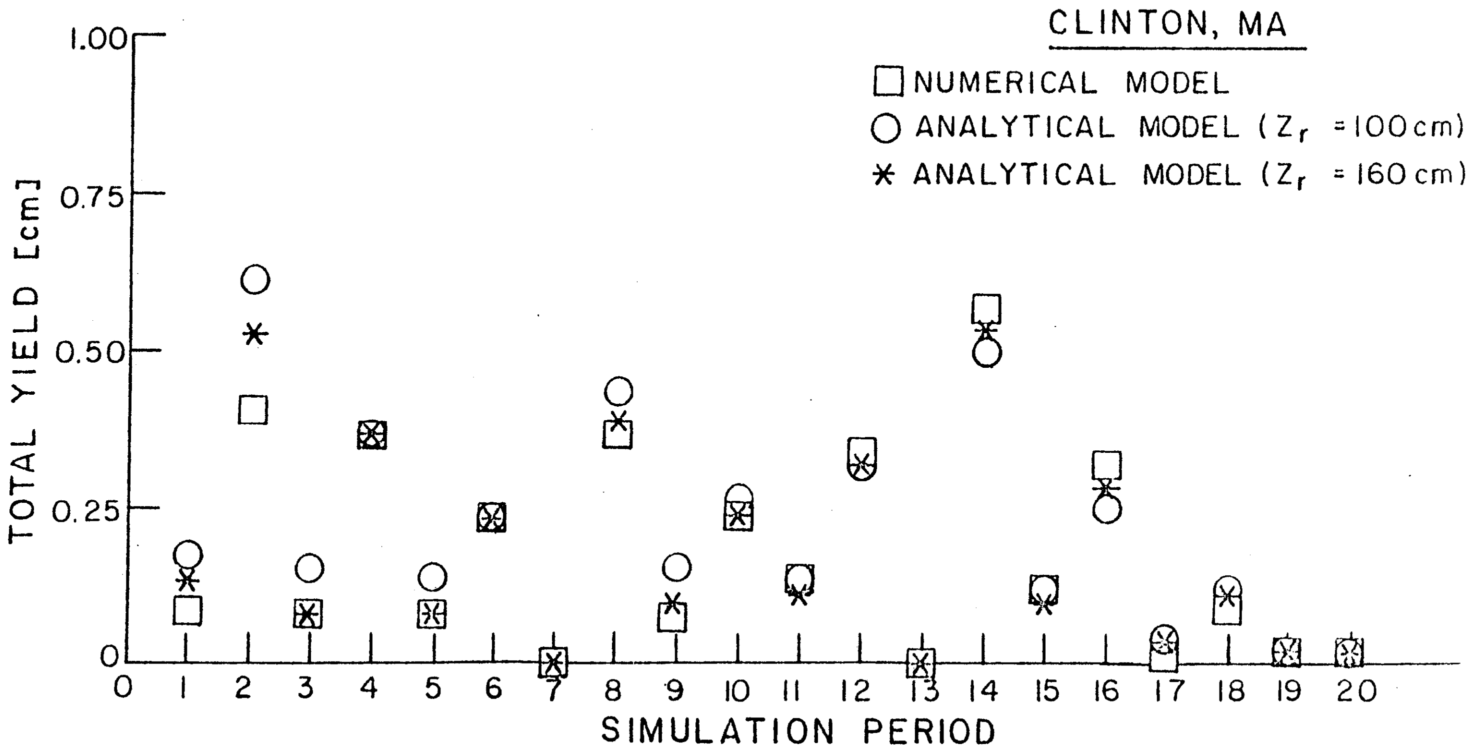

FIGURE 8

Comparisns of total yield obtained from Milly's and Eagleson's [1980] numerical model, Clinton, Massachusetts

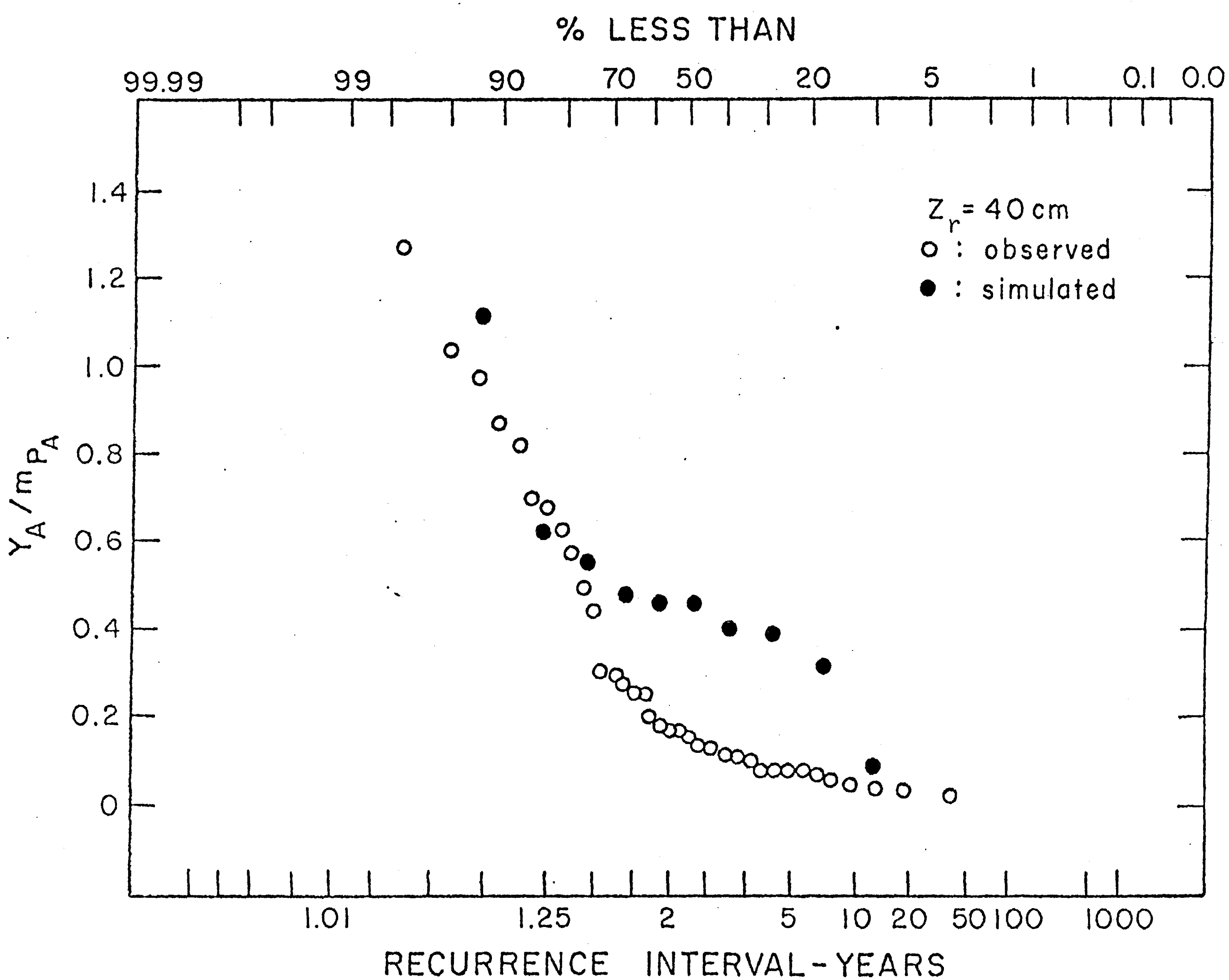

FIGURE 9

Comparison between simulated and observed CDF of annual yield, $Z_r = 40$cm, Santa Paula, California

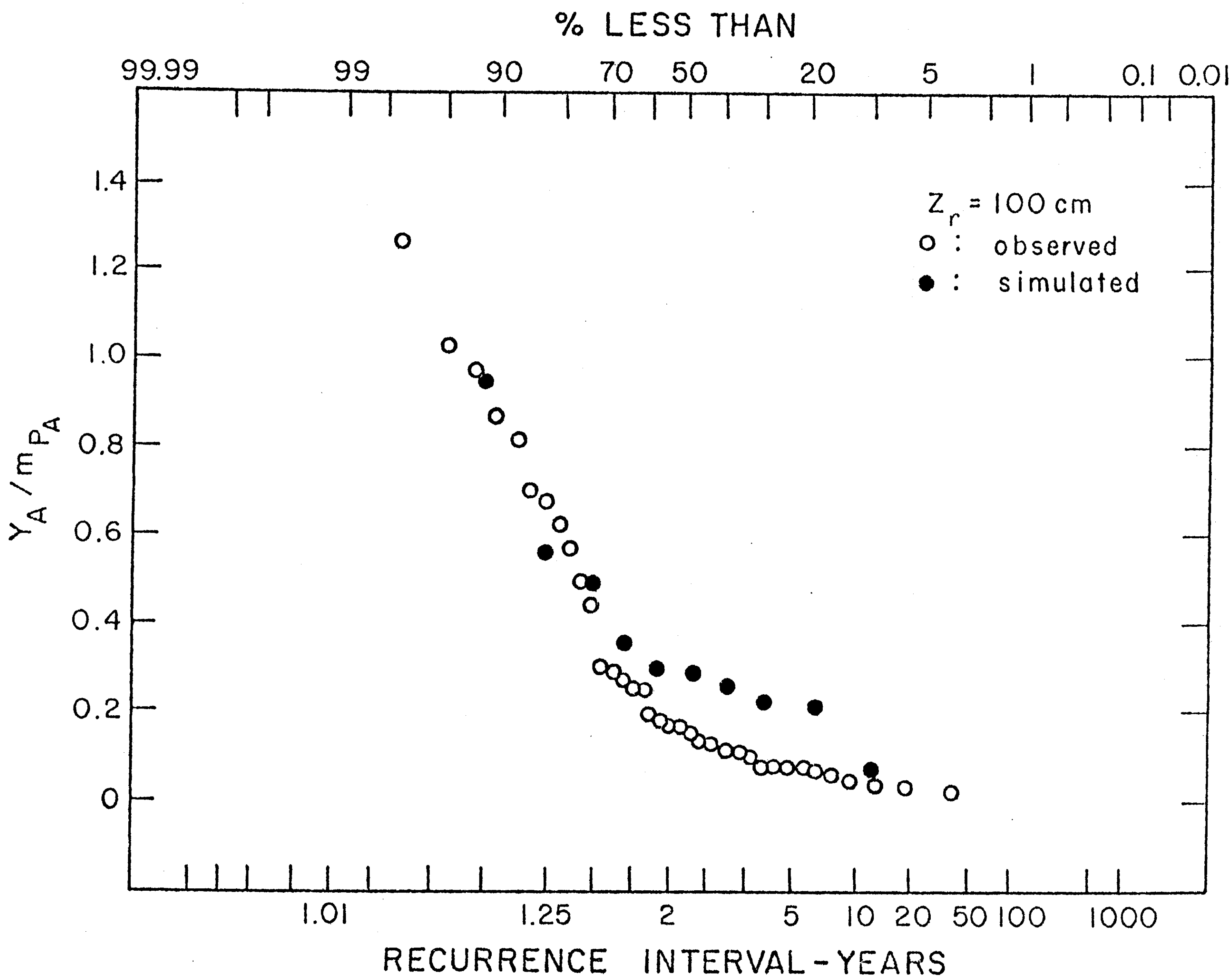

FIGURE 10

Comparison between simulated and observed CDF of annual yield $Z_r = 100$cm, Santa Paula, California

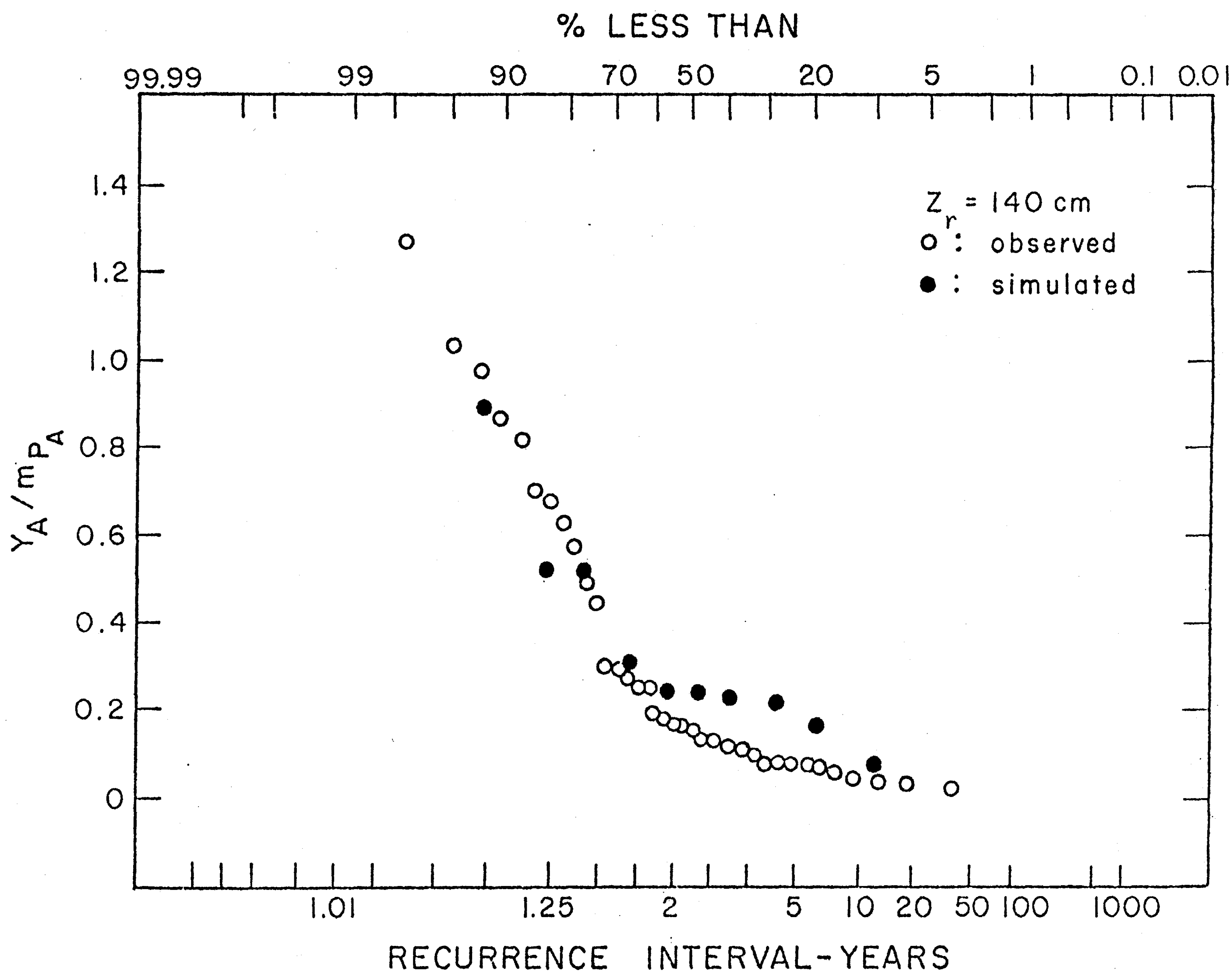

FIGURE 11

Comparison between simulated and observed CDF of annual yield, $Z_r = 140$cm, Santa Paula, California

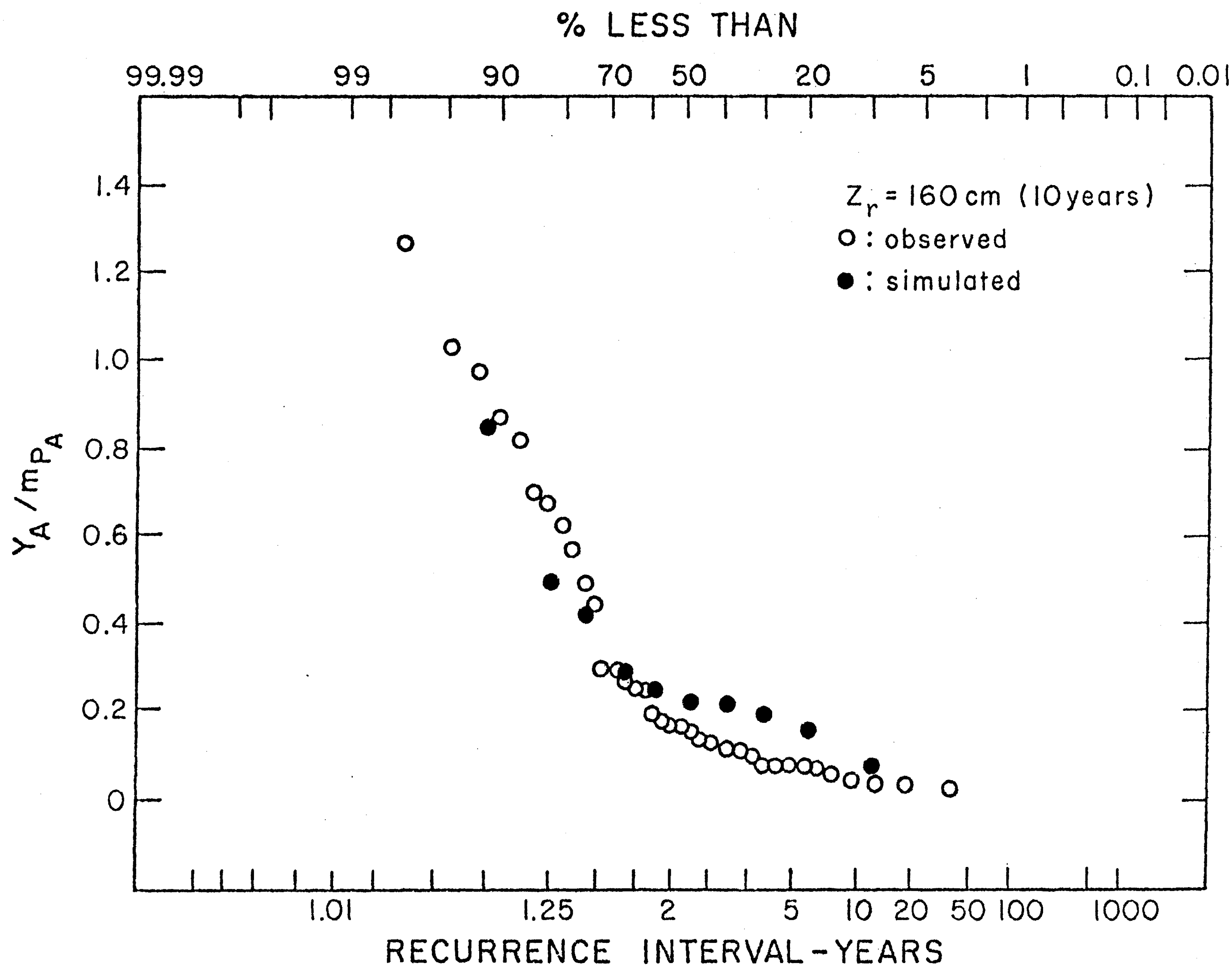

FIGURE 12

Comparison between simulated and observed CDF of annual yield $Z_r = 160$cm, Santa Paula, California

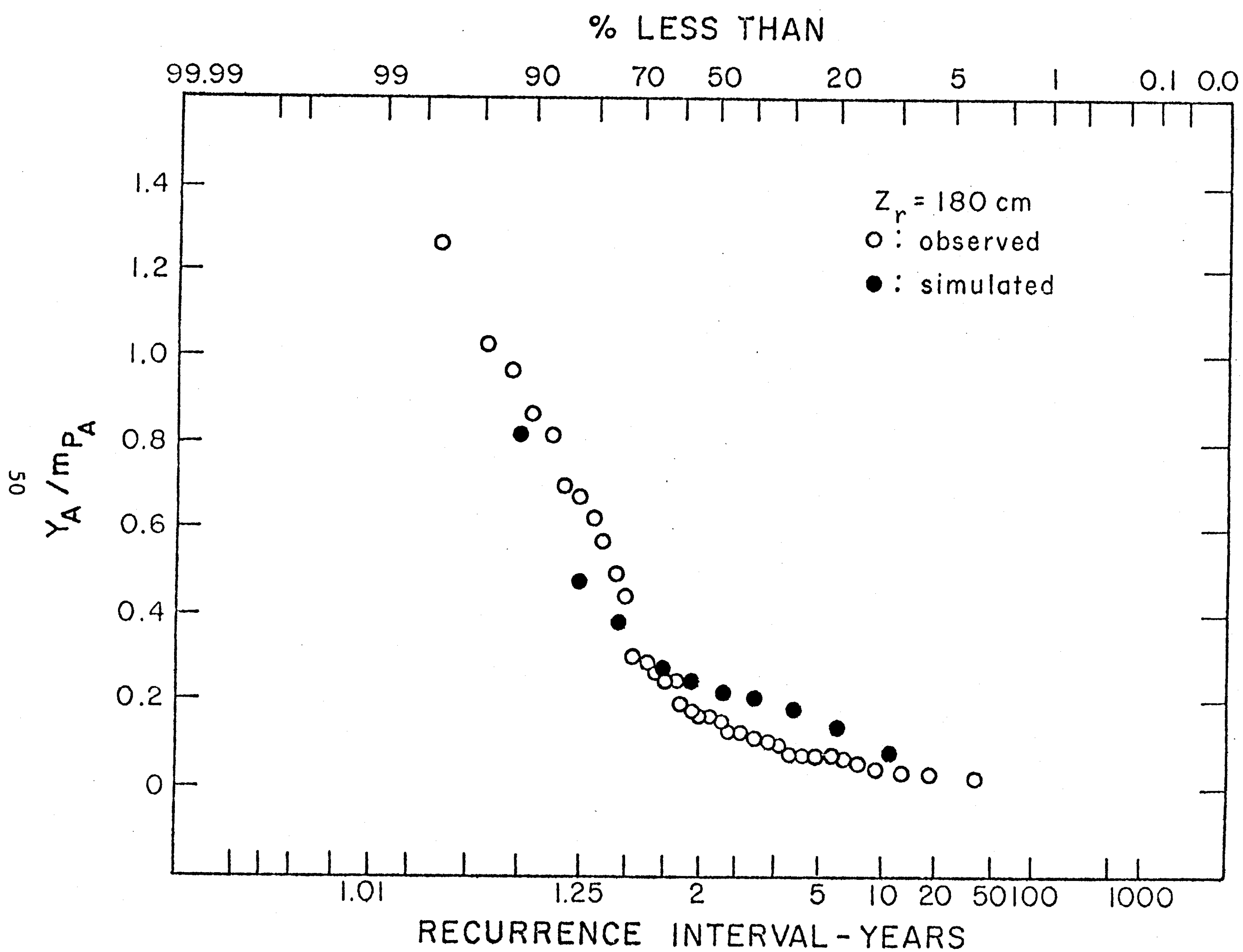

FIGURE 13

Comparison between simulated and observed CDF of annual yield, $Z_r = 180$cm, Santa Paula, California

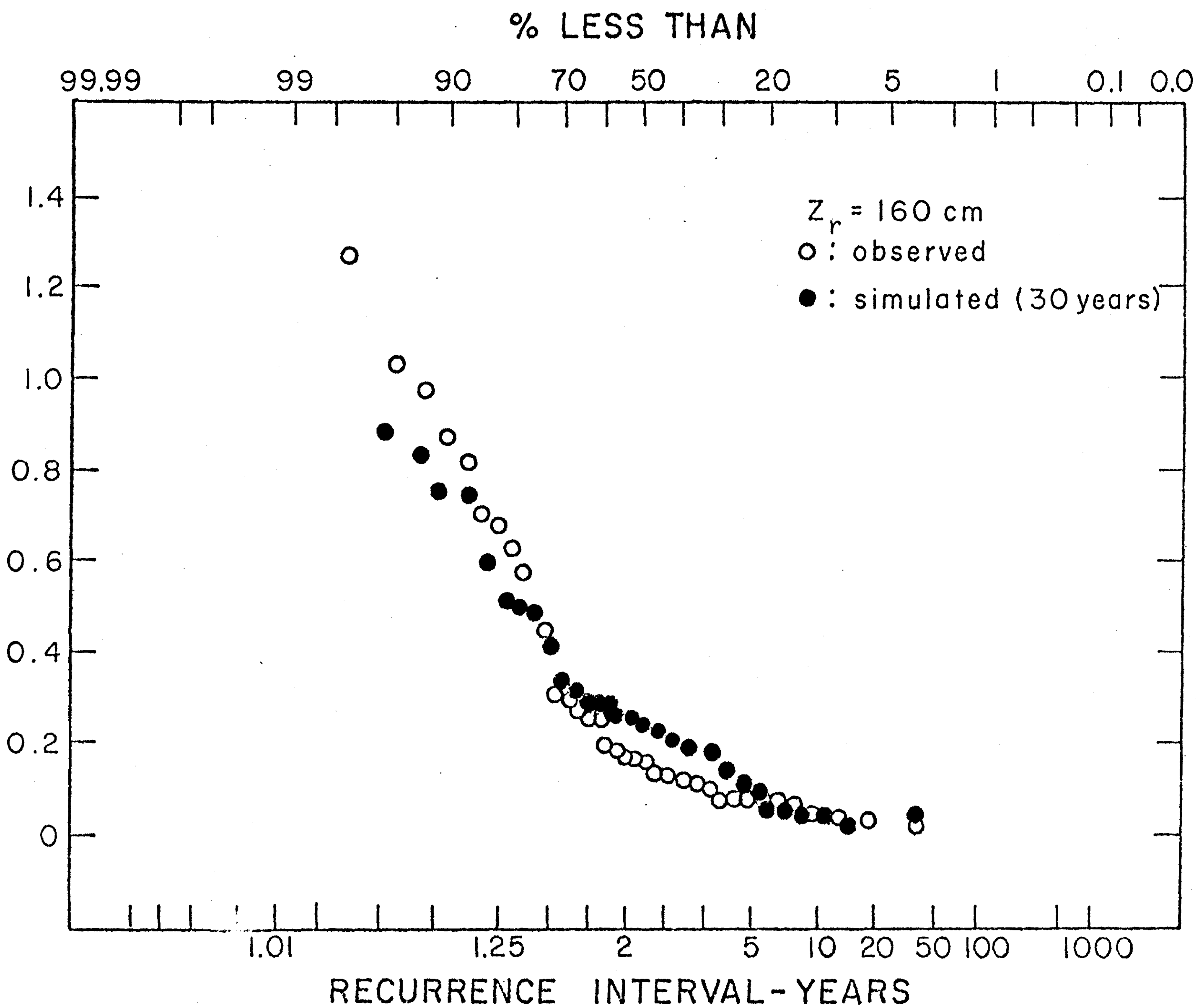

FIGURE 14
Comparison between simulated and observed CDF of
annual yield, $Z_r = 160$cm, 30 years of simulation, Santa Paula, California

By setting $Z_r = 160$cm, the fluxes obtained by the analytical model and by Milly's [1980] isothermal version of his numerical model are now compared again for Santa Paula, California. Storage changes are shown in Figure 15 and yield produced is shown in Figure 16. It is apparent that setting $Z_r = 160$cm produces an improvement over the previously obtained results where $Z_r = 100$cm.

The "best" value for Z_r obtained through simulation can now be compared with the value of the penetration depth of a step change in surface soil moisture corresponding to the average climatic and soil conditions of the region under investigation.

The value of the penetration depth, combining the diffusive component with the gravitational seepage component is given by [Eagleson, 1978]:

$$Z_{max} = 4(Dt)^{\frac{1}{2}} + \frac{tK(\Theta_o)}{n} \tag{5.1}$$

where D is the sorption diffusivity (D_i) or the desorption diffusivity (D_e), $t = m_{t_r}$ if $D = D_i$ and $t = m_{t_b}$ if $D = D_e$, $K(\Theta_o)$ is the hydraulic conductivity at the average soil moisture level Θ_o and n is the effective porosity.

Values of D, t, $K(\Theta)$ and n for Clinton, Massachusetts and Santa Paula, California are given in Table 5.1. By substituting in Equation (5.1) we find:

<u>Clinton, Massachusetts</u>

For infiltration: $Z_i = 65.53 + 0.33 = 65.86$cm.

For exfiltration: $Z_e = 50.04 + 3.11 = 53.11$cm

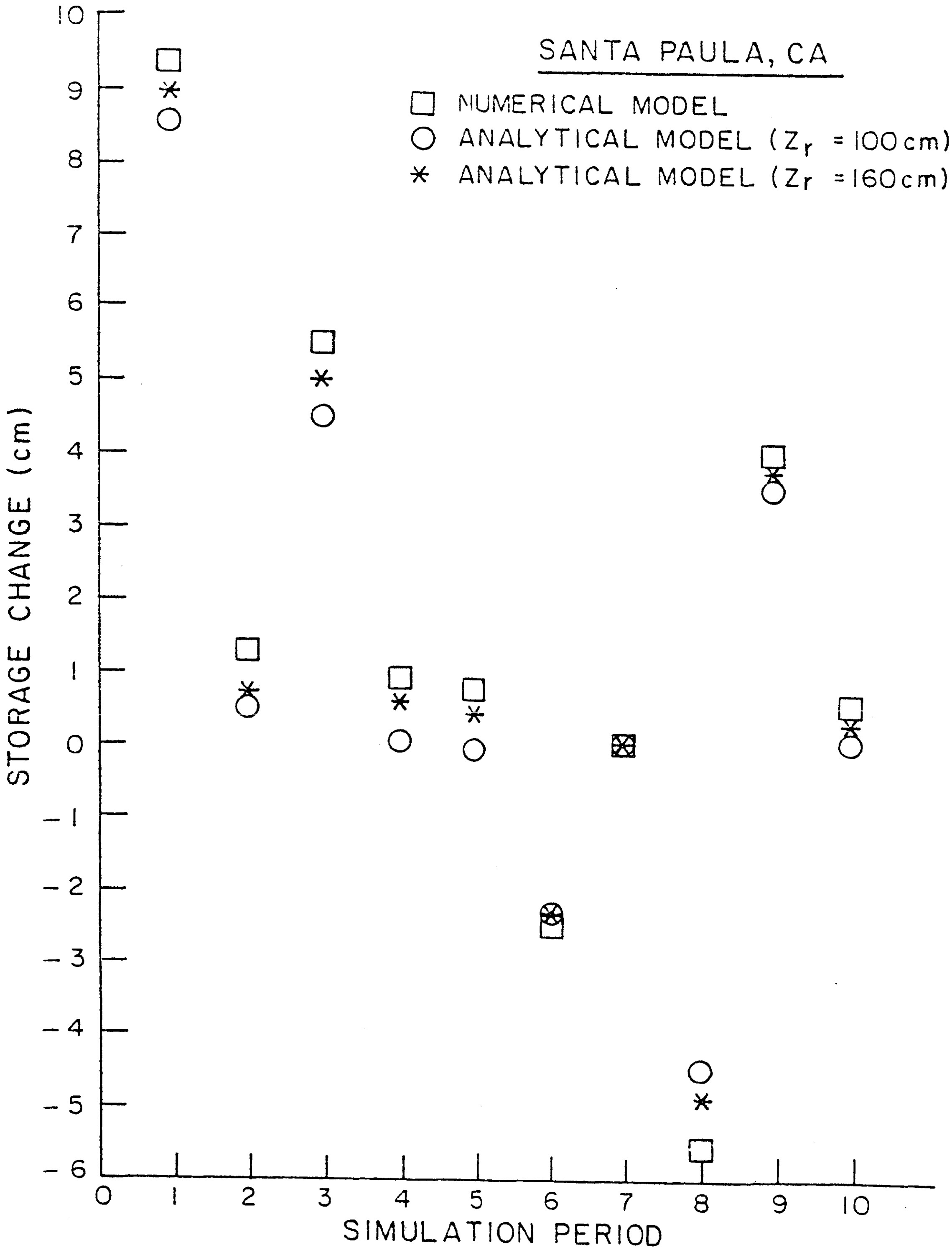

FIGURE 15

Comparison of storage change obtained from Milly's and Eagleson's [1980] numerical model, Santa Paula, California

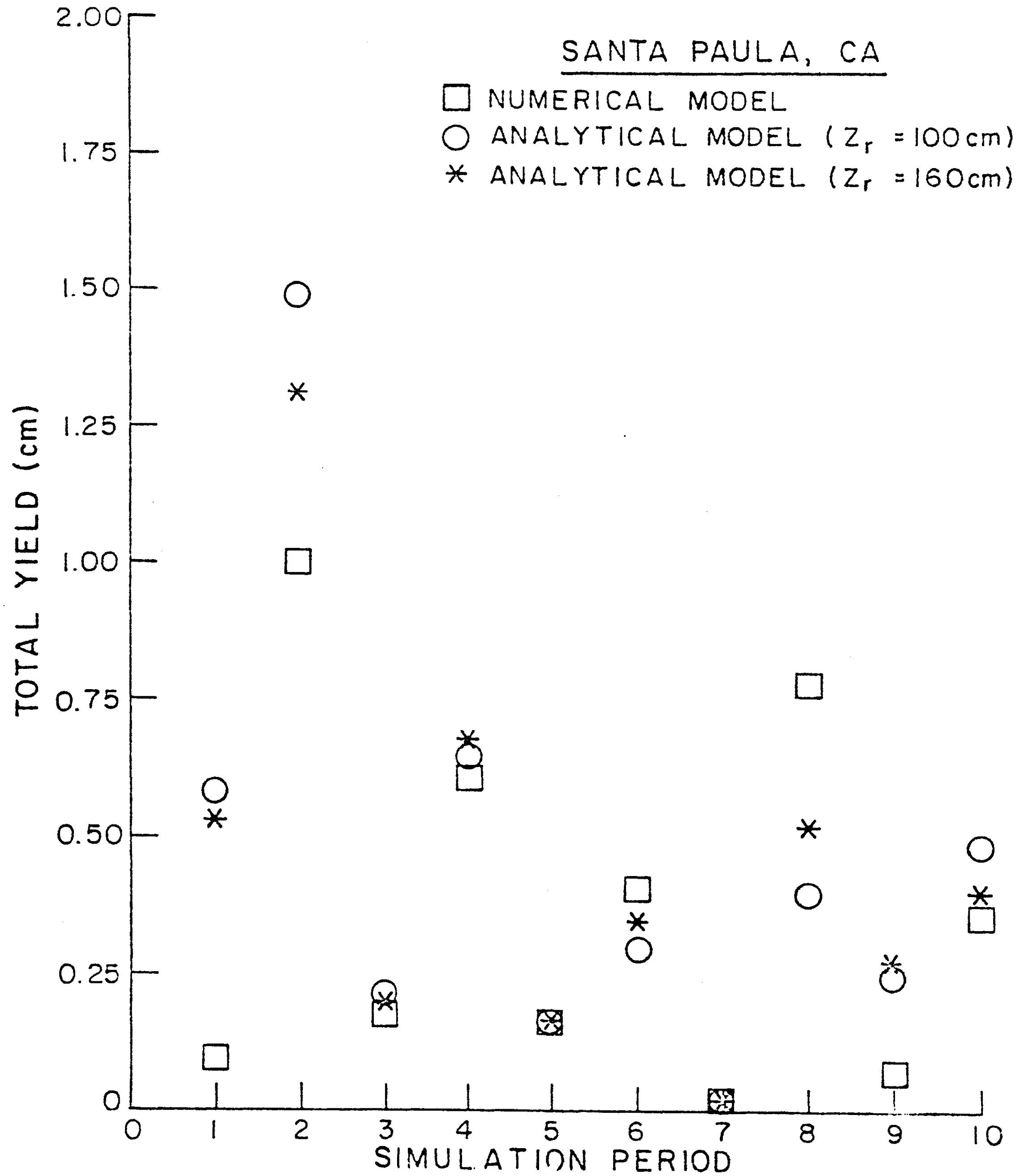

FIGURE 16

Comparison of total yield obtained from Mill'y and Eagleson's [1980] numerical model, Santa Paula, California

Table 5.1

Clinton, Massachusetts

$m_{t_r} = 0.32$ days , $D_i = 9.71 \times 10^{-3}$ cm^2/sec

$m_{t_b} = 3$ days , $D_e = 6.04 \times 10^{-4}$ cm^2/sec

$K(1) = 4.20 \times 10^{-6} \frac{cm}{sec}$, $n = 0.35$

Santa Paula, California

$m_{t_r} = 1.43$ days , $D_i = 1.18 \times 10^{-2}$ cm^2/sec

$m_{t_b} = 10.42$ days , $D_e = 3.456 \times 10^{-4}$ cm^2/sec

$K(1) = 9.25 \times 10^{-6} \frac{cm}{sec}$, $n = 0.35$

Santa Paula, California

For infiltration: $Z_i = 153 + 3.26 = 156.26\text{cm}$

For exfiltration: $Z_e = 70.55 + 23 = 93.55\text{cm}.$

Although the calculated value of the penetration depth almost coincides with the value obtained through simulation for the catchment of Santa Paula, there is a discrepancy of about 1m between the two for the catchment of Clinton, Massachusetts.

Observing Figures 2 to 6, we see that the selected value of Z_r at every simulation run, influences the fitting of the CDF's primarily during the very dry years at Clinton. Since the analytical model uses a linearization around the average annual soil moisture in order to estimate the evaporation flux, the tangent to the evapotranspiration efficiency curve at the point corresponding to s_o has a very small slope. Thus, the values of evaporation predicted by the model when the soil moisture level becomes very low are considerably overestimated. This results in less water stored in the bucket of depth Z_r and thus eventually in less percolation to the water table. This explains the fact that by choosing a value of Z_r on the order of 60cm as predicted by the penetration depth calculations an underestimation of the yield is obtained during the very dry years.

On the other hand, the value of Z_r does not play a significant role during the wet years for a humid climate, because the evaporation rate is usually equal to the potential evaporation rate. In any case, the differences in the CDF's for the different values of Z_r are not that pronounced as in the semi-arid climate of Santa Paula From the above observations it is found that for a semi-arid climate, such as that of Santa Paula, California, the value of Z_r obtained through penetration depth calculations is very close to that obtained by fitting the observed CDF of the annual yield to that predicted by the analytical model through simulation. It is also found that the values of Z_r obtained by the above mentioned procedures differ significantly for the humid climate of Clinton. Fortunately, however, knowing the appropriate value of Z_r for a humid climate becomes of some importance only for the case of very dry years.

As it will also be shown in Chapter 6, the value of Z_r is highly varying depending on both the climate and the soil of a region. Thus, it is not appropriate to select it arbitrarily as is done in some algorithms. For example, Budyko [1956] suggested a value of $Z_r \cong 100\text{cm}$, Arakawa [1972] assumed $Z_r \cong 30\text{cm}$, Gates et al. [1977] suggestes $\Theta_{f_c} Z_r \approx 30\text{cm}$, where Θ_{f_c} is the field capacity and Shukla [1977] proposed $\Theta_{f_c} Z_r \approx 10\text{cm}$.

In summary, this research demonstrates that the important climate and soil conditions of a region can be incorporated into a priori estimation of Z_r through computation of the penetration depth. Where long-term water yield data are available Z_r may be estimated by fitting simulated to observed cdf's of annual yield.

Chapter 6

Comparison with an Exact Numerical Model and Other Simplified Parameterizations Under Very Dry Conditions

6.1 Introduction

The latent heat flux obtained by the model is compared with that obtained by the numerical model for heat transfer and moisture flow in soils developed by Milly and Eagleson [1980]. A comparison is also made with other simplified parameterizations proposed by Milly and Eagleson [1982].

The climate chosen to demonstrate this comparison is that of Winslow, Arizona, which is characterized by very dry conditions. The model was tested for two types of soil; silty loam and sand. A periodic atmospheric forcing was applied for a period of ten days. The force-restore method was used in order to update the estimates of the near surface and the deep soil temperatures. Thus, a thermal balance model was operated conjunctively with the soil moisture model, the coupling between them occurring through the evaporation rate e_T and also through changes in the soil emissivity ε and surface albedo A due to changes in soil moisture.

6.2 The Periodic Atmospheric Forcing

The surface boundary layer is forced by six atmospheric variables, which are: the incoming shortwave radiation I_s, the down long-wave radiation from the clouds I_{ℓ_d}, the precipitation rate P, the air temperature T_a, the wind speed U_a, and the vapor pressure of the air ρ_{v_a}.

The type of periodic forcing chosen is the one described by Milly and Eagleson [1982] and it will be briefly repeated here for convenience.

a. Shortwave Radiation

The intensity of shortwave radiation reaching the ground is given by:

$$I_s = \begin{cases} W_{BO} \sin \alpha \, e^{-n' a_1 m} (1 - 0.65N^2) & \sin \alpha > 0 \\ 0 & \sin \alpha \leq 0 \end{cases} \tag{6.1}$$

where W_{BO} is the solar constant (2 cal cm^{-2} min^{-1}), α is the angle of the sun above the horizon, a_1 is the molecular scattering coefficient, n' is a turbidity factor, m is the relative thickness of the atmosphere and N is the proportion of the sky covered by clouds. The angle α is given by:

$$\sin\alpha = \sin\delta \cdot \sin\phi + \cos\delta \cdot \cos\phi \cos\left[\frac{\pi}{12} (t - 12)\right] \tag{6.2}$$

where δ is the solar declination, ϕ is the latitude and t is the time in hours since midnight.

$$a_1 = 0.128 - 0.054 \log m \tag{6.3}$$

and

$$m = (\sin\alpha)^{-1} \tag{6.4}$$

Representative values of the forcing parameters for Winslow in July, which were used in all applications of the model to be described later, are shown in Table 6.1.

Table 6.1

Representative Values of the Forcing parameters
for Winslow in July

Parameter	Winslow/July
$\bar{T}_a$	12.9°C
$\bar{P}$	18.6 cm yr^{-1}
ϕ	35° 01'
δ'	21° 30'
n'	2.5
N_o	0.45
t_b	94 hours
t_r	2 hours
t_i	20 hours
i	0.193 cm hr^{-1}
T_m	25.8°C
T_{di}	7.8°C
t_T	15 hours
U_m	360 cm s^{-1}
U_{di}	180 cm s^{-1}
t_u	18.5 hours
ρ_{va}	7×10^{-6} g cm^{-3}

The cloud cover ratio N is taken as follows:

$$N = \begin{cases} N_o & \text{if} \quad p = 0 \\ 1 & \text{if} \quad p > 0 \end{cases} \tag{6.5}$$

b. Down Longwave Radiation

The atmospheric longwave radiation is given by:

$$I_{\ell d} = \varepsilon_a \, \sigma(T_a + 273)^4 \, (1 + 0.17N^2) \tag{6.6}$$

where the atmospheric emissivity is

$$\varepsilon_a = 9.37\text{x}10^{-6} \, (T_a + 273)^2 \tag{6.7}$$

c. Precipitation

The precipitation rate P is expressed as a periodic function of time of the following forms:

$$P = \begin{cases} 0, & t_i - t_b + K(t_b + t_r) < t < t_i + K(t_b + t_r) \\ i, & t_i + K(t_b + t_r) < t < K(t_b + t_r) + t_r \end{cases} \tag{6.8}$$

where i represents the average storm intensity, t_b and t_r are representative values of the time between storms and storm duration respectively, t_i is the starting time of the first storm, and K is any integer. Values of the parameters appearing in Equation (6.8) are given in Table 6.1.

d. Air Temperature and Wind Speed

Monthly averages of the three-hourly, diurnally varying air temperature and windspeed for Winslow were fitted to the following cosine curves:

$$T_a = t_m + T_{d_i} \cos\left[\frac{\pi}{12}(t - t_T)\right] \quad (6.9)$$

$$U_a = U_m + U_{d_i} \cos\left[\frac{\pi}{12}(t - t_u)\right] \quad (6.10)$$

Values for T_m, T_{d_i}, t_T, U_m, U_{d_i}, and t_u are given in Table 6.1.

The value of ρ_{va} was assumed constant and is given in Table 6.1.

6.3 The Parameterization of Fluxes in the Surface Boundary Layer

a. Potential Evaporation Rate

The evaporation rate is calculated through the aerodynamic Equation:

$$E = -\frac{c_w U_a}{\rho_e}(\rho_{va} - \rho_{vg}) \quad (6.11)$$

where c_w is a bulk transfer coefficient, ρ_e is the density of liquid water, ρ_{vg} is the density of water vapor at the ground surface, and ρ_{va} is the density of the water vapor in the air.

Equation (6.11) was used in the model only to evaluate a changeing value of the potential evaporation rate. When the surface became dry, the evaporation rate E was calculated by the Equation:

$$E = \min(e_T, e_p)$$

$$\text{where: } e_T = \bar{e}_T + C_1 \bar{e}_p(s - s_o)$$

which was documented by Andreou and Eagleson [1982], and where $\bar{e}_T$ is the annual average evaporation rate, $\bar{e}_p$ is the annual potential evaporation rate and C_1 is a linearization coefficient.

b. Sensible Heat Transfer

The sensible heat transfer was expressed by:

$$H = -\rho_a \; c_p \; c_H \; U_a(T_a - T_g) \tag{6.13}$$

where ρ_a is the air density, c_p is the specific heat of water vapor at constant pressure, c_H is a bulk transfer coefficient, T_a is the air temperature and T_g is the ground temperature.

Under conditions of neutral stability, the transfer coefficients become:

$$(c_H)_N = (c_w)_N = \frac{k^2}{\left(\ell n \frac{Z_a}{Z_o}\right)^2} \tag{6.14}$$

where k is Von Karman's constant (=0.4), Z_a is the screen height and Z_o is the surface roughness.

Under unstable conditions the transfer coefficients are functionally related to their neutral values through [Anderson, 1976].

$$\frac{c_H}{(c_H)_N} = \frac{c_w}{(c_w)_N} = \left\{1 - \frac{1}{\ell n\left(\frac{Z_a}{Z_o}\right)}\left[\ell n\left(\frac{1+x^2}{2}\right) + 2\,\ell n\left(\frac{1+x}{2}\right) - 2\tan^{-1}(x) + \frac{\pi}{2}\right]^{-1} \cdot \left[1 - \frac{2}{\ell n\left(\frac{Z_a}{Z_o}\right)}\,\ell n\left(\frac{1+x^2}{2}\right)\right]^{-1}\right. \tag{6.15}$$

where

$$x = (1 - 16\; Z_a/L)^{\frac{1}{4}} \tag{6.16}$$

and L is the Monin-Obukhov length, which is related to the bulk Richardson number

$$(R_i)_B = \frac{2 g Z_a (T_a - T_g)}{(T_a + T_g) U_a^2} \tag{6.17}$$

through the expression:

$$\frac{Z_a}{L} = \frac{k \dfrac{c_H}{(c_H)_N} \cdot (R_i)_B}{(c_H)_N^{\frac{1}{2}} \left\{ 1 - \dfrac{(c_H)_N^{\frac{1}{2}}}{k} \left[\ln \left(\dfrac{1+x^2}{2} \right) + \ln \left(\dfrac{1+x}{2} \right) - 2 \tan^{-1}(x) + \dfrac{\pi}{2} \right] \right\}} \tag{6.18}$$

Equations (6.15) - (6.18) were solved iteratively and a table relating $(R_i)_B$ to c_H and c_w was created.

For stable conditions the following relation holds:

$$\frac{c_H}{(c_H)_N} = \frac{c_w}{(c_w)_N} = \begin{cases} \left(1 - \dfrac{(R_i)_B}{R_{i_{cr}}} \right) & , (R_i)_B > R_{i_{cr}} \\ 0 & , (R_i)_B \geq R_{i_{cr}} \end{cases} \tag{6.19}$$

where $R_{i_{cr}}$ is the critical Richardson number, equal to 0.2.

In all applications that follow $Z_a = 200$cm, $Z_o = 0.1$cm and $(c_H)_N = (c_w)_N = 0.00277$.

6.4 The Soil Moisture Model

The model for updating the soil moisture within a surface layer of thickness Z_r, is the one developed by Andreou and Eagleson [1982], and the linearized equations of the short-term water balance were given in Chapter 4 (Equation 4.2 and 4.3). The only difference here is that during a precipitation event (Equation 4.2), the evaporation rate, is set equal to the changing potential rate e_p, in order to be consistent with the parameterization of Milly and Eagleson [1982] with which the comparison is made.

In order to locate more accurately the passage from climate control to soil control after a precipitation event, it is necessary to calculate the time t_o after precipitation ceases at which the surface becomes dry. This time is given by:

$$t_o = \frac{S_e^2}{2\bar{e}_p^{\,2}} \tag{6.20}$$

where the desorptivity S_e is given by:

$$S_e = 2\, s^{1+d/2} \left[\frac{nK(1)\psi(1)\phi_e(d)}{\pi m}\right]^{\frac{1}{2}} \tag{6.21}$$

and s is the average soil moisture concentration within the layer of thickness Z_r, immediately after the precipitation ends. For a more detailed reasoning of this procedure see Andreou and Eagleson [Chapter 7, Section 7.5, 1982].

6.5 The Force-Restore Method for Soil Temperature Prediction

The linear differential equation for estimating the surface temperature T_1 is given [Deardorff, 1978] by:

$$\frac{dT_1}{dt} = c_1\, G - c_2(T_1 - T_2) \tag{6.22}$$

The values of c_1 and c_2 are given by:

$$c_1 = 2\left[\frac{\pi}{\lambda C \tau}\right]^{\frac{1}{2}} \tag{6.23}$$

where λ is the thermal conductivity and C is the volumetric heat capacity of the homogeneous medium.

The period τ is equal to one day.

The heat flux into the soil G, is expressed from a surface energy balance as

$$G = (1 - A)I_s + \varepsilon I_{\ell d} - I_{\ell u} - \rho_e(L + c_\ell T_1)E - H + \rho_\ell c_\ell T_a P \qquad (6.25)$$

where A is the surface albedo, I_s the incoming shortwave radiation, ε the surface emmissivity, $I_{\ell d}$ the downward longwave radiation, $I_{\ell u}$ the upward longwave radiation ($= \varepsilon\sigma(T|_{z=o} + 273)^4$), E the evaporation rate, ρ_ℓ and c_ℓ the density and specific heat of water, H the sensible heat loss, T_a the air temperature and P the precipitation. The heat loss due to surface runoff and surface detention storage are neglected as not important.

The deep soil temperature T_2, which varies slowly due to the annual cycle of forcing is obtained from [Deardorff, 1978]

$$\frac{dT_2}{dt} = (\lambda\ C\ N_d\ \tau)^{-\frac{1}{2}} \cdot G \qquad (6.26)$$

The value of N_d used in the simulations described by Milly and Eagleson [1982] was set equal to 20. For the reasoning behind this, see Milly and Eagleson [1982, Section 4.4].

6.6 The Coupling with the Soil Moisture Model

The coupling between the thermal and water balance models occurs not only through the value of the evaporation rate E which was discussed in more detail earlier, but also through changes in the moisture content which influences the albedo, the emissivity, the thermal conductivity and the heat capacity of the soil.

Since the soil moisture model used does not predict the (volumetric) soil moisture at the surface Θ_1, but an average soil moisture $\bar{\Theta}$ within the layer of thickness Z_r, the value of Θ_1 will be approximated as

$$\Theta_1 = \begin{cases} 0 \text{ if } e_T < e_p \text{ and } t > t_o \\ \bar{\Theta} \text{ if } t < t_o \end{cases} \qquad (6.27)$$

where t is the time passed after precipitation has ended.

The same type of approximation is also made by Milly and Eagleson [1982, Section 4.4.2] in their parameterization.

The volumetric heat capacity of soil C is expressed as a weighted average of the capacities of its components [de Vries, 1966]:

$$C = \sum_{i=1}^{5} c_i \, \Theta_i \qquad (6.28)$$

where Θ_i and c_i are the volumetric fraction and the volumetric heat capacity of the i'th soil constituent. The five soil components are (1) water, (2) air, (3) quartz, (4) minerals, (5) organic matter. The heat capacity of each constitutent is given in Table 6.2. The volumetric fractions for silty loam and sand were given in Table 6.3. The effective thermal conductivites λ for silty loam and sand, as a function of Θ and T were calculated by Milly and Eagleson [1982] and are shown in Figure 17. The product λC appearing in Equation (6.23) and (6.26) of the force-restore method was evaluated in the manner described by Milly and Eagleson [1982] and will be repeated here for convenience. (The subscript "2" is used when we refer to the prediction equation for T_2). Thus, we have:

$$(\lambda C)_2 = \lambda(\Theta_i) \; c(\Theta_i) \qquad (2.29)$$

Table 6.2

(After DeVries, 1966)

Volumetric Heat Capacity of Soil Constituents

Constituent	i	c_i
liquid water	1	1.0
air	2	3×10^{-4}
quartz	3	0.46
other minerals	4	0.46
org. matter	5	0.6

c_i in cal cm$^{-3°}$ K^{-1}

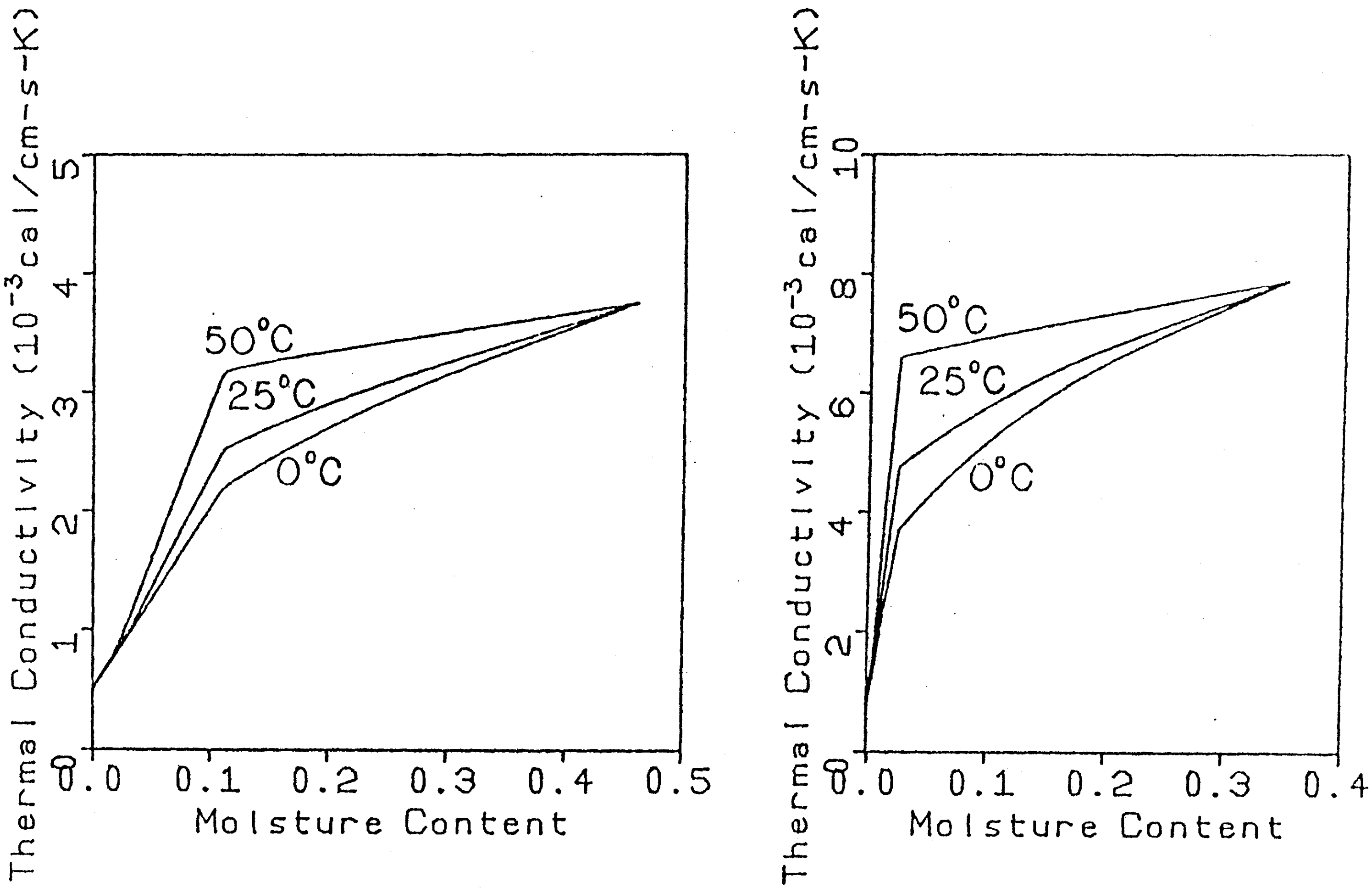

FIGURE 17

Effective thermal conductivity as a function of Θ and T. left: SILT LOAM. Right: Sand, After Milly and Eagleson [1980] where Θ_i is the initial moisture content, since due to the short duration of the simulations performed, the departure from initial conditions will be small.

When λC is evaluated for use in Equation (6.23) however it is important to account for the time-varying surface moisture. Thus we have:

$$(\lambda C)_1^{\frac{1}{2}} = 0.3\ [\lambda(\Theta_1)\ c(\Theta_1)]^{\frac{1}{2}} + 0.7(\lambda C)_2^{\frac{1}{2}} \tag{6.30}$$

where the subscript "1" refers to the prediction equation for T_1 and the value of Θ_1 is given by Equation 6.27. Equation (6.30) is the one applied in Milly's and Eagleson's [1982] parameterizations and consists of a slight simplification of a procedure proposed by Deardorff [1978].

The value of changing albedo is calculated as follows:

$$A = \begin{cases} A_d + (A_w - A_d)\dfrac{2\Theta_1}{n} & 2\Theta_1 < n \\ A_w & 2\Theta > n \end{cases} \tag{6.31}$$

Values of A_d and A_w are given in Table 6.3. For the soil emmissivity ε, we will use a value equal to 0.95 if $\Theta_1 \neq 0$ and a value of 0.9 if $\Theta_1 = 0$.

The soil-moisture and the force-restore equations were solved simultaneously using an explicit numerical procedure. The time step of integration was equal to a quarter of an hour.

6.7 Evaluation of Results

The latent heat flux calculated by the proposed parameterization was compared with that obtained by using the numerical model developed by Milly and Eagleson [1980] and with other simplified parameterizations proposed by Milly and Eagleson [1982]. The climatic variables and soil properties used are shown in Table 6.3.

Table 6.3

Climatic and Soil Parameters

Winslow, Arizona

$\bar{e}_p$	=	0.449 cm/day
m_{t_b}	=	4.58 days
m_{t_r}	=	0.10 days
m_τ	=	365 days
h_o	=	0.1 cm
$w/\bar{e}_p$	=	0
$\bar{T}_a$	=	12.9°C
m_ν	=	74
m_{p_A}	=	22.33 cm
κ	=	0.32

For Silty Loam

$n = 0.46$

$\kappa(1) = 1.24 \times 10^{-9}\ cm^2$

$c \simeq 5$

$A_d = 0.20$

$A_w = 0.10$

For Sand

$n = 0.35$

$\kappa(1) = 2.48 \times 10^{-8}\ cm^2$

$c \simeq 5$

$A_d = 0.35$

$A_w = 0.25$

The simulation period lasted 10 days and the initial soil temperature was set equal to 24°C while the initial soil moisture concentration was set equal to 0.083 for the silty loam and 0.091 for the sand. The results for the silty loam are shown in Figure 18. The solid line represents the solution obtained using the "exact" numerical model. This model is fully documented by Milly and Eagleson [1980]. The open circles represent the solution obtained using the numerical model, but where vapor flow is neglected as described by Milly and Eagleson [1982; Section 6]. The black dots represent the results obtained using the parameterization described in this report. The value of the storage depth Z_r was fitted in order to obtain the best approximation with the numerical model. The optimal value of Z_r was found to be equal to 2.97cm. We observe that the estimates of latent heat are in very good agreement with those of the numerical model up to the time that control passes to the soil. This occurs about nine hours after the end of the precipitation. We observe that when control passes to the soil there is a sudden drop in the latent heat flux, which is now considerably less than the one predicted by the numerical model. The reason for that is that vapor flow plays an important role in the early stages of exfiltration, when control passes to the soil and also when the soil moisture level in the surface layer is very low, as it is here. The effect of neglecting vapor flow in such a case can be seen very clearly from the solution of Milly's [1982] numerical model, as it is plotted with the open circles in Figure 18. Here the plotted circles represent the value of latent heat by the numerical

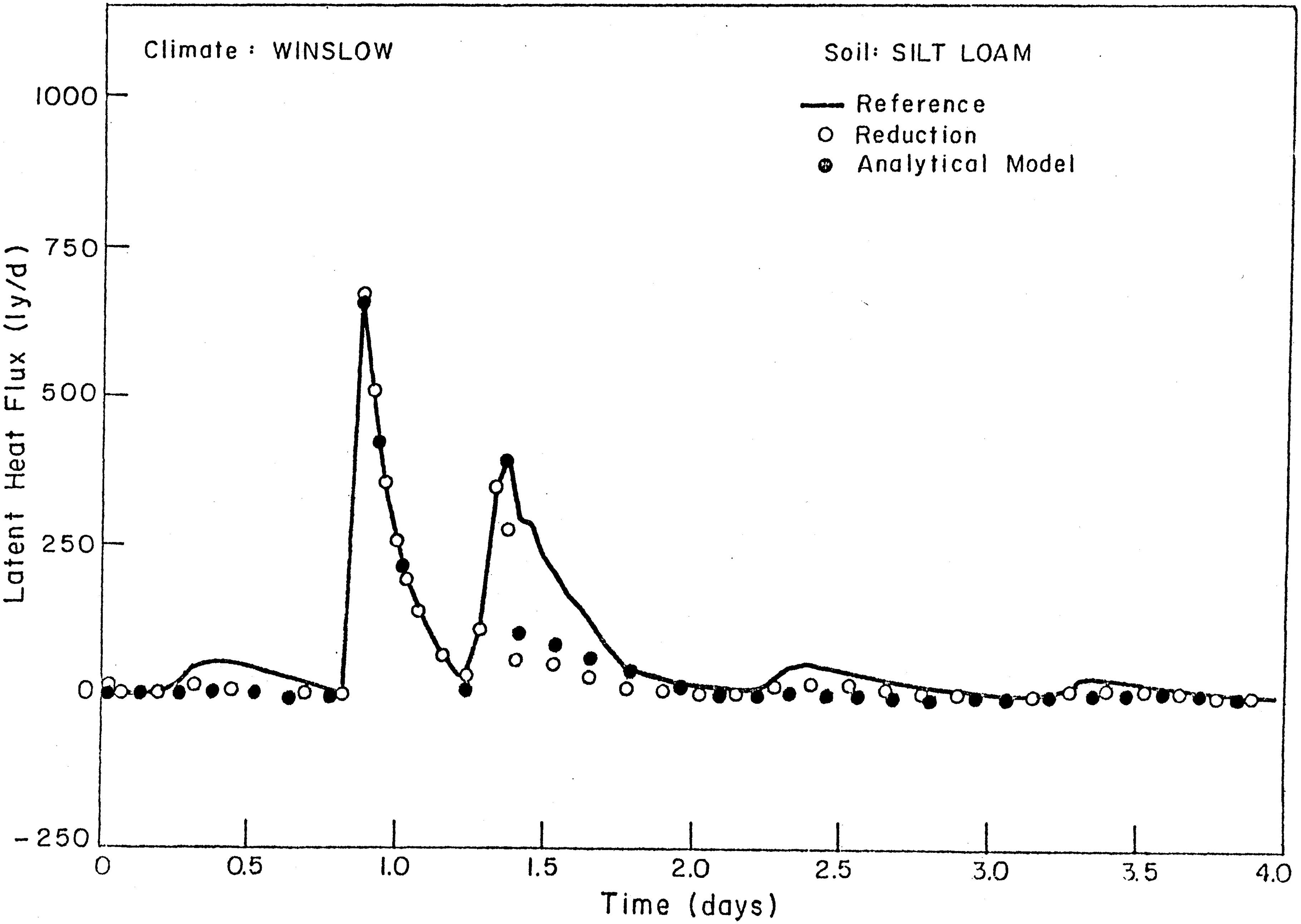

FIGURE 18
Comparison of latent heat fluxes obtained by the numerical model (Milly and Eagleson, 1982) and those obtained by the analytical and numeical models when vapor flow is ignored

model when vapor flow is neglected. We observe that the results are very similar to those of our parameterization (as shown by the solid circles in Figure 18), where vapor flow is not included in the equations modeling the moisture dynamics. The importance of vapor flow under very dry conditions can also be seen from Figure 19, where we observe that the values of hydraulic conductivity $K(\Theta)$ and vapor conductivity $D_{\psi v}(\Theta)$ are of the same order to magnitude when Θ is in the vicinity of 0.1, which is the case in our experiment. It is evident that vapor flow will be important only under very arid conditions and for particular types of soil, as is easily seen from Figure 19. If we want to take vapor flow into account in our model, a modification is necessary. As is proposed by Milly and Eagleson [1982], an effective value of the diffusivity D_e can be calculated, in which vapor flow is explicitly considered. The exfiltration capacity of the soil $f_e(t)$ can then be evaluated through the selection of an appropriate formula. Here, we will evaluate the exfiltration capacity by using the Philip [1960] equation

$$f_e(t) = \frac{1}{2} S_e t^{-\frac{1}{2}} - \frac{1}{2}[K(\Theta_1) + K(\Theta_o)] \tag{6.32}$$

where

$$S_e = 2(\bar{\Theta}-\Theta)[\frac{D_e(\bar{\Theta},t)}{\pi}]^{\frac{1}{2}} \tag{6.33}$$

and

$$D_e(\bar{\Theta}, t) = 1.85\ \bar{\Theta}^{-1.85} \int_{-\infty}^{\bar{\psi}} [\bar{\Theta}(t) - \Theta_H(\psi)]^{0.85} \cdot$$

$$\cdot \left\{K[\Theta_H(\psi)] + D_{\psi v}[\Theta_H(\psi), \psi] \right\} d\psi \tag{6.34}$$

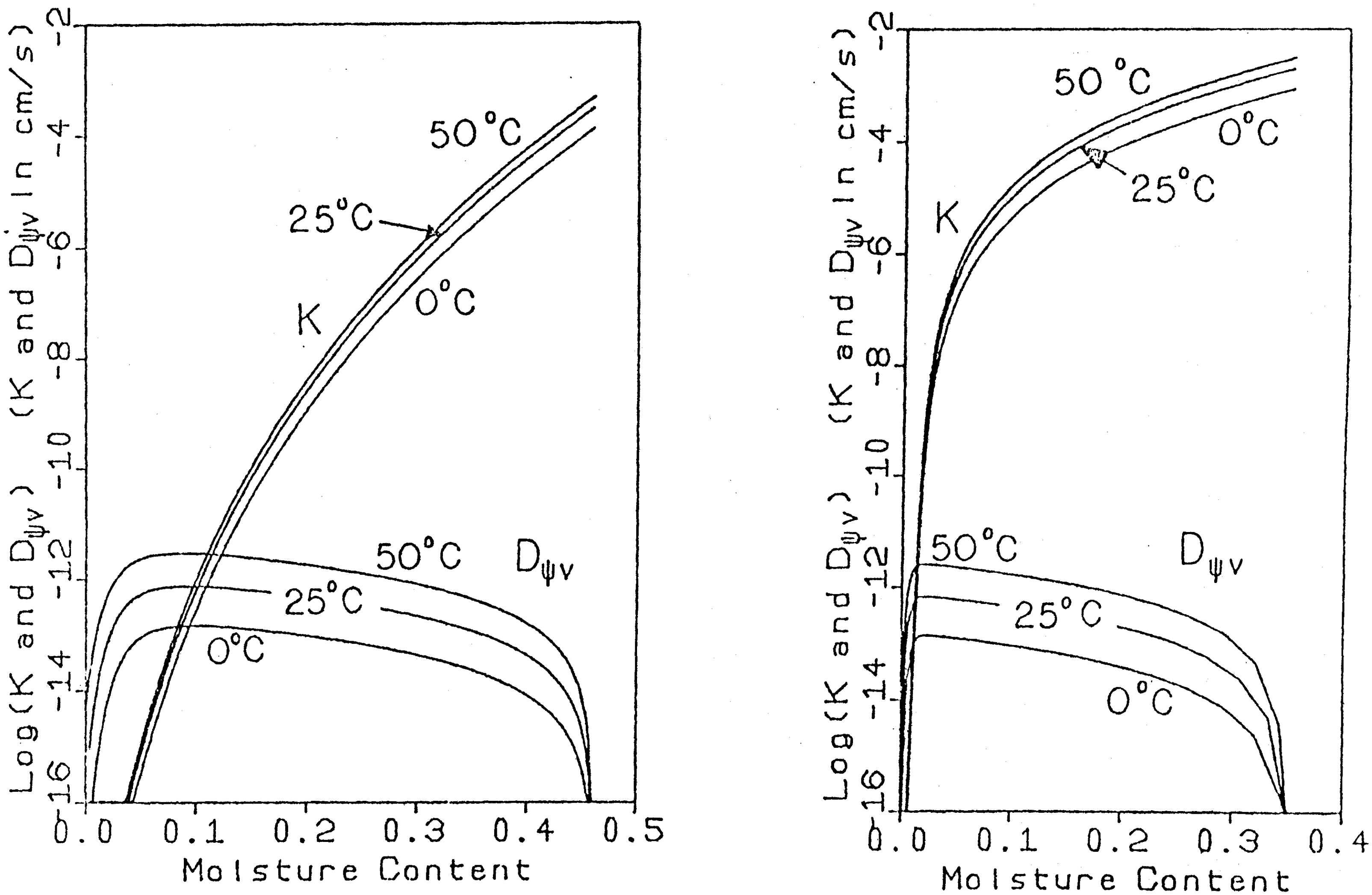

FIGURE 19

Liquid hydraulic conductivity, K and vapor conductivity, D_{ψ_v} as functions of Θ and T.

Left: SILT SOAM Right: SAND. After Milly and Eagleson [1982]

Equation (6.32) was applied to estimate the evaporation rate only from the time that the surface became dry ($\Theta_1=0$), up to the time when the evaporation rate obtained by (6.32) was of the same order of magnitude with the evaporation rate obtained from our original model. As we see from Equation (6.34) the effective diffusivity is updated at every time step. The integral appearing in Equation (6.34) is approximated using the functions of hydraulic conductivity, vapor conductivity and matrix potential, shown in Figures 19 and 20.

The results obtained when this procedure is applied, are shown in Figure 21, by plotted circles. As we see those results are in almost perfect agreement with those obtained by Milly's and Eagleson's [1982] parameterization. It should be noted that the computational burden introduced by these modification does not exceed that of Milly's simplified parameterization, although depends upon the form of the $K(\Theta)$, $D_{\psi v}(\Theta)$ and $\psi(\Theta)$ functions chosen.

The results obtained for the sandy soil are shown in Figure 22. Vapor flow was neglected in this case, since as we can see from Figure 19, the vapor conductivity is much smaller than the hydraulic conductivity for the sandy soil and for $\Theta > 0.01$. Again here we observe fairly good agreement between the proposed parameterization and that by Milly and Eagleson [1982]. The initial discrepancies of both from the numerical solution, are due to a transient error because of non-equivalence of initial conditions. In this case, the optimal value of the Z_r was found to be equal to 15 cm.

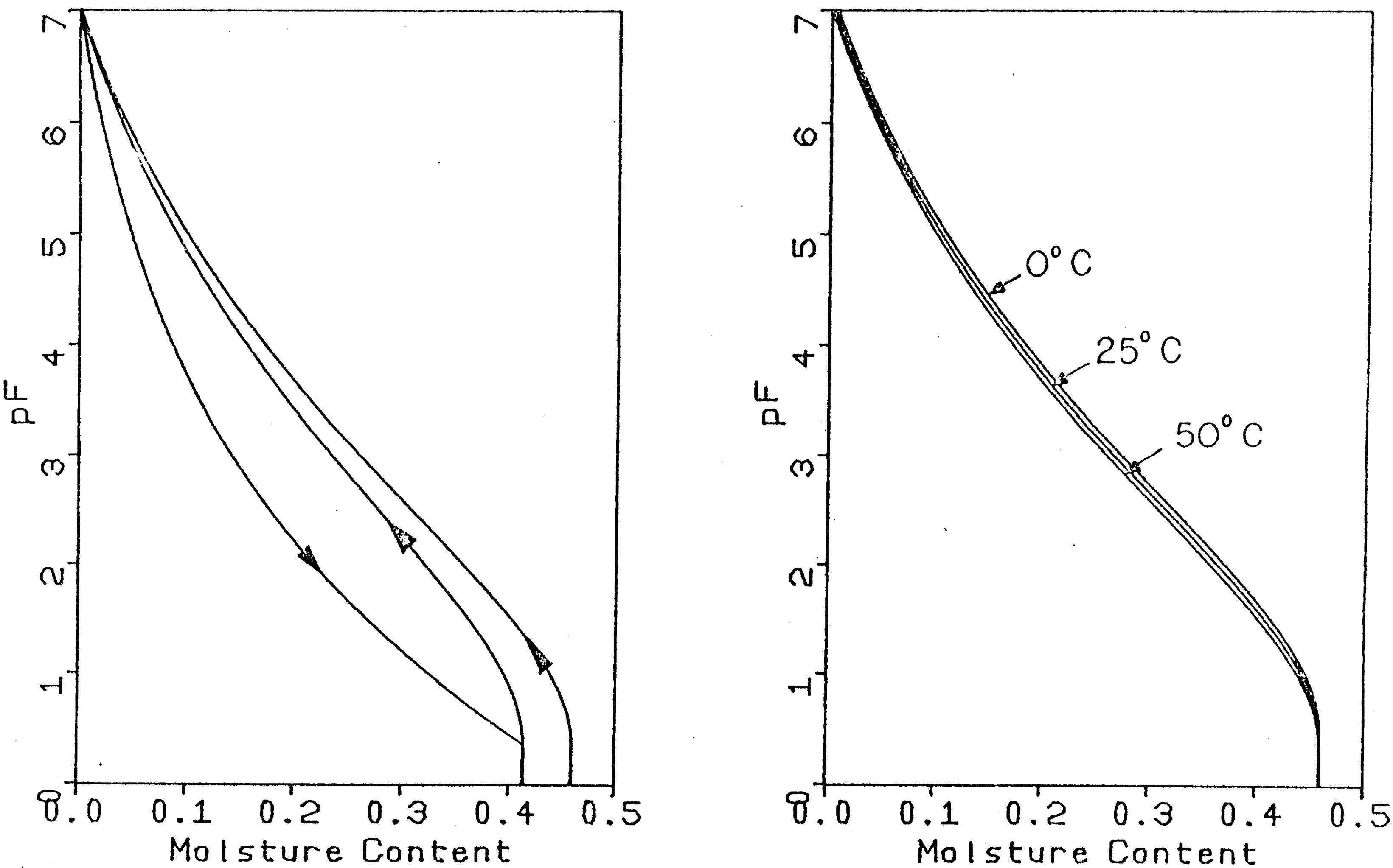

FIGURE 20

Moisture Retention in the SILT LOAM. Left: Main Branches of the hysteretic retention function. Right: Dependence of the first desorption curve on temperature (After Milly and Eagleson, 1982)

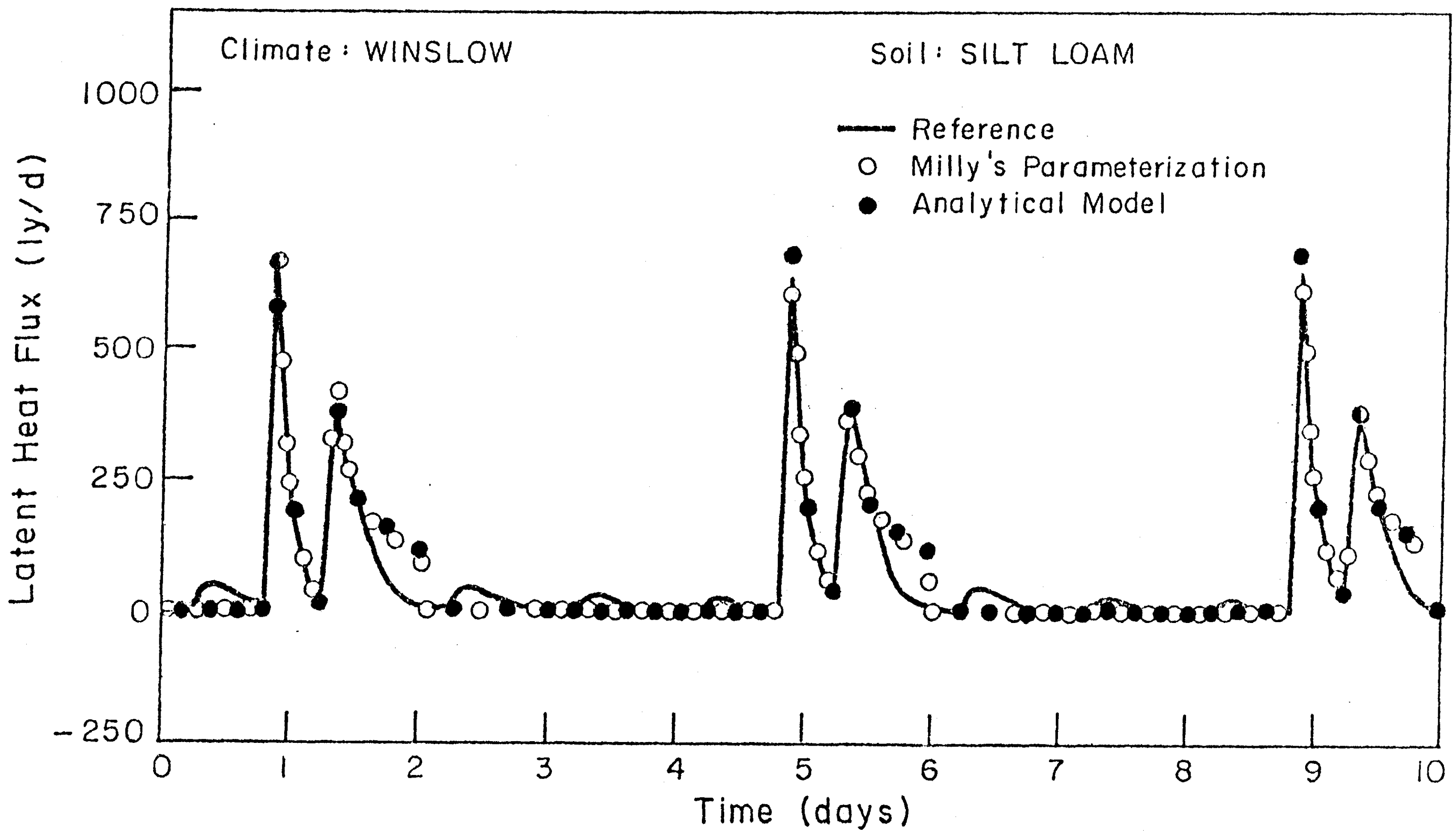

FIGURE 21

Comparison of latent heat fluxes computed from Milly's and Eagleson's [1982] numerical model, Milly's and Eagleson's [1982] simplified one-cell parameterization and the analytical model, SILT LOAM

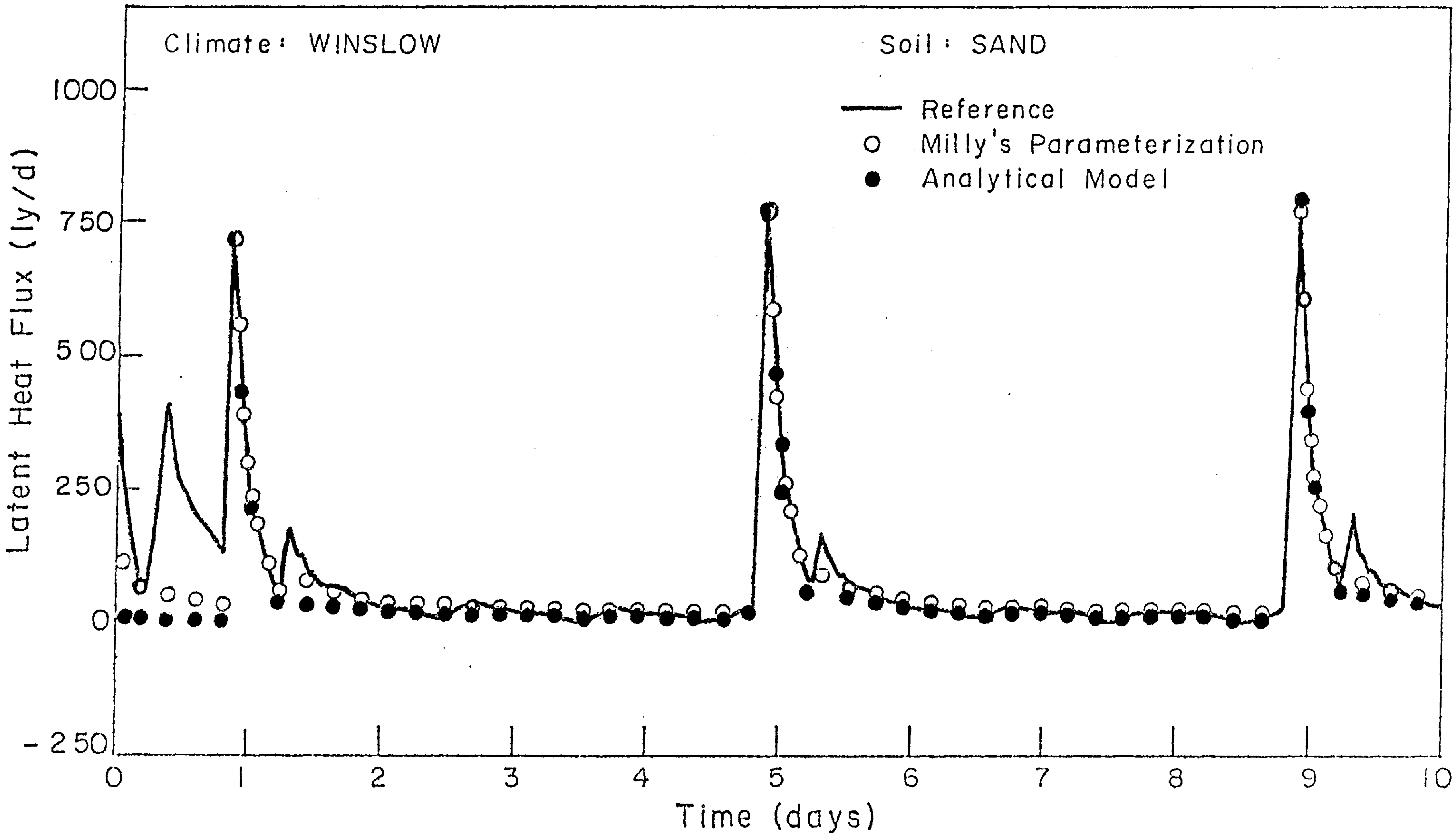

FIGURE 22

Comparison of latent heat fluxes computed from Milly's and Eagleson's [1982] numerical model, Milly's and Eagleson's [1982] simplified one-cell parameterization and the analytical model, SAND

Chapter 7

7.1 Summary

In this study, extensions and further applications of a second-order Budyko-type landsurface parameterization [Andreou and Eagleson, 1982] were made.

A sensitivity analysis was first performed, in order to examine the changes of the cumulative yield predicted by the model, caused by changes of the soil properties k(1) and c from their values derived by applying the ecological optimality hypotheses. [Eagleson, 1982]

A procedure was developed for obtaining analytically the sensitivity of the yield to the near-surface storage depth as defined by the model. Thus, for the proposed model and by using the soil and climatic properties of a given region, it is possible to derive analytically a measure of the sensitivity of the yield to the near surface storage depth.

A methodology of assessing the "best" value of the storage depth is proposed. The CDF of the annual yield obtained through simulation by the model was compared to the CDF of the observed values of annual yield, and the value of storage depth that gave the best fitting between the two was selected. The validity of the above methodology was tested through comparisons of the results obtained by the model and those obtained by applying Milly's [1980] numerical model. Having established a value of the storage depth by applying the previous method, the results of the comparison were always better than those obtained through setting the storage depth at its "nominal" value of 1m (as is suggested by several investigators).

Finally, comparisons of the results obtained by the proposed model were made with results obtained by Milly and Eagleson [1982] using other simplified parameterizations. The importance of vapor flow under very dry conditions and for certain types of soils was investigated. Necessary modifications of the model, in order to handle those conditions, were suggested and tested.

7.2 Conclusions

The conclusions derived from this research are the following

1. The annual yield obtained by the model, is sensitive to the values of k(1) and c derived from the ecological optimality hypotheses [Eagleson, 1982]. But for the humid climates, the accuracy of the estimates of those parameters does not affect significantly the estimates of the yield. On the contrary, for the semi-arid climates, it was found that the yield was very sensitive to those parameters. For the tested climates, the yield was also found to be much more sensitive to the value of the pore disconnectedness index c than to the value of the saturated intrinsic permeability k(1).

2. For the two contrasting climates of Clinton, Massachusetts and Santa Paula, California it was found by the model and also verified analytically, that the yield was much more sensitive to the selected value of storage depth for the semi-arid climate than for the humid climate.

3. If observations of the annual yield are available, the appropriate value of the storage depth to be used by the model, can be assessed through comparisons of the CDF's of annual yield, observed and simulated. The validity of obtaining the value of storage depth by applying this technique was verified, when the model was operated in real time and comparisons were made with Milly's numerical model. The possibility of a priori selecting Z_r by setting it equal to the penetration depth was also indicated by this study.

It was found, from one application, that for a semi-arid climate the value of Z_r determined with the above method was very close to the value of the penetration depth, thus providing the possibility for a priori selection of Z_r for such a climate. Although for a humid climate, the same result was not found, it was established that knowning the accurate value of Z_r is of importance in humid climates only during the very dry years.

4. It was found that the model with its present structure could not handle extremely dry situations for certain types of soils, where vapor flow is important during exfiltration. However, if the vapor conductivity dependence upon soil moisture is known, then they can be incorporated into the model, as suggested in Chapter 6. If this is done, it was found that the model can give very statisfactory results, when calibrated with Milly's [1980] numerical model. These results where very close to those obtained by the simplified parameterization of Milly and Eagleson [1982] which is calibrated to the numerical model through a fitted moisture redistribution parameter.

5. From this research it was found that a wide range of the appropriate value of storage depth exists in reality, depending on the type of climate and soil for every region. Thus, for one-cell models of landsurface parameterization, we must be careful in the selection of the storage depth. Choosing it to be uniformly equal to 1m, as is very often done, can yield large errors in the computed surface fluxes.

7.3 Suggestions for Further Research

In addition to further tests to verify the model, more extensive research is needed, to study the interrelation of storage depth, climate and soil for a variety of climates and soils.

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APPENDIX

PROGRAM TAYLOR.FORTRAN

```
C ***********************************************************

C THIS PROGRAM GENERATES RAINSTORM EVENTS, STORM DURATIONS
C AND INTERSTORM PERIODS WHICH PRESERVE THE HISTORICAL STATISTICS.
C IT CALCULATES THE SOIL MOISTURE  OVER A DEPTH CLOSE TO THE SURFACE
C  EVERY HALF HOUR .IT PLOTS THE EVAPOTRANSPIRATION
C FUNCTION,THE SURFACE RUNOFF AND PERCOLATION FUNCTIONS.IT ALSO
C PLOTS THE DAILY SOIL MOISTURE DURING THE RAINY SEASON LENGTH.
C IT CALCULATES THE TOTAL STORAGE CHANGE, THE CUMULATIVE
C EVAPORATION AND YIELD AT THE END OF EVERY RAINY OR
C INTERSTORM PERIOD
C IT HAS THE OPTION OF USING MANABE'S MODEL
C TO CALCULATE THE MOISTURE FLUXES
C THE VALUE OF THE POTENTIAL EVAPOTRANSPIRATION RATE
C IS SET EQUAL TO ITS ANNUAL AVERAGE VALUE
C THIS PROGRAM ALSO CALCULATES VALUES OF
C ANNUAL YIELD FOR A SPECIFIED SIMULATION
C PERIOD GREATER THAN ONE YEAR.

C ******************************************************
C CLIMATIC AND SOIL VARIABLES
c epr=average annual evapotranspiration rate(cm/day)
c mtb=mean time between storms(days)
c mtr=mean storm duration(days)
c mpa=mean annual precipitation(cm)
c mtau=mean rainy season length(days)
c ta=average annual air temperature(C)
c mnu=mean number of storms per year
c n=soil porosity
c k1=saturated intrinsic permeability(cm2)
c c=pore disconectedness index
c Zr=surface layer tkickness(cm)
c Mo=vegetation cover
c Kv=plant coefficient
c k=parameter of gamma distibuted storm depths
c Lamda=parameter of gamma distributed storm depths
C ***********************************************************

real min,mo,m,n,nu,k1,mtb,mtr,mh,in
real sjk(20),yi(20),soj(20),a77(20),b77(20),b78(20)
real da(365),SKP(365),st(365),b79(20),a79(20),ys(20),yg(20)
real a78(20),day(365)
fii(d,so)=1./(d*(1.-so)**(1.45-.0375*d)+5./3.)
external plot_$setup (descriptors)
external plot_$scale (descriptors)
external plot_ (descriptors)
 character*10 xaxis,yaxis
fi(em)=10.**(.66+.55/em+.14/em**2.)
kl1=1
ran=1.
print,'To use Manabes parameterization type 2 , otherwise 1'
input,mnb
if(mnb.eq.1) go to 3020
print,'Input the initial soil moisture so'
input,so
go to 3021
 3020 print,'Input the average annual soil moisture so'
input,so
print,'Input Time step (in days) '
input,tis
```

```
C NR=Number of rainstorm events you want to generate

3021 print,'Input NR'
input,NR
print,'Input storm properties k and Lamda'
input,xk,aml
print,'If you want a simulation period greater than mtau print 2, otherwise 1'
input,SPE
if(SPE.eq.2) go to 4056
go to 4057
4056 print,'Input the number of simulation periods'
input,NSPE
4057 if(mnb.eq.2) go to 3040
print,'For daily fluxes type 1,for half hour fluxes type 2 '
input,fl
print,'To plot S(t) type 2,otherwise 1'
input,lot
print,'For cumulative fluxes after each storm and interstorm period type 2, otherwise 1'
input,ucu
print,'To plot S(t) for different values of Zr type 2 ,otherwise 1'
input,szr
print,'To print the cumulative fluxes only at the end of the rainy season type 2 , otherwise
input,fcu
3040 print,'To print the rainstorm events type 2 . otherwise 1'
input,rae
i1=1
3003 print,'epr,mtb,mtr,mpa,mtau,ta,mnu,n'
input,epr,mtb,mtr,mpa,mtau,ta,mnu,n
if(mnb.eq.2) go to 3022
2020 if(SPE.eq.1) go to 2021
if(SPE.eq.2.and.ran.eq.2) go to 4053
2021 print,'Mo,Kv,k1,c,Zr'
input,vg,vk,k1,cs,zr
if(vg.eq.1) stop
if(ran.eq.2) go to 3004
if(dif.eq.2) go to 3004
C J(s)=evapotranspiration efficiency function
C  Ys(s)=surface runoff function
C Yg(s)=ground water percollation function

1000 print,'To plot J(s) and y(s) type 2 , otherwise 1'
input,pl
if(pl.eq.1)  go to 3004
if(k11.eq.2) go to 3004
print,'To draw different curves for J(s) for different climates type 2, otherwise 1'
input,dif
 double precision sum1,mean1,mean2,mean3,B28
double precision sum2
double precision sum3
3004 if(ran.eq.2) go to 807
3022 if(rae.eq.1) go to 42
print,'STORM DEPTH STORM DURATION TIME BETWEEN '
print,'   (cm)           (days)          (days)   '

42 i1=1

C **************************************************************
C GENERATION OF RAISTORM EVENTS
C **************************************************************
```

```
C R1(I)=storm depth(cm)
C R2(I)=storm duration(days)
C R3(I)=interstorm duration(days)

 real R(3000),WK(6000),R1(3000),R2(3000),R3(3000)
double precision DSEED
DSEED=123765.0D0
A=xk
B=1./aml
 call ggamr(DSEED,A,NR,WK,R)
do 5 I=1,NR
R(I)=B*R(I)
5 continue
do 41 I=1,NR
R1(I)=R(I)
41 continue
DSEED=3478758.0D0
A=1.
B=mtr

 call ggamr(DSEED,A,NR,WK,R)
do 7 I=1,NR
 R(I)=B*R(I)
7 continue
do 21 I=1,NR
R2(I)=R(I)
21 continue
DSEED=649853.0D0
A=1.
B=mtb
 call ggamr(DSEED,A,NR,WK,R)
do 9 I=1,NR
 R(I)=B*R(I)
9 continue
do 30 I=1,NR
R3(I)=R(I)
30 continue
if(ran.eq.2) go to 807
if(rae.eq.1) go to 3023
go to 3024
3023 if(mnb.eq.2) go to 3025
go to 807
3024 do 11 I=1,NR
write(6,17) R1(I),R2(I),R3(I)
17 format(f10.6,4x,f10.6,4x,f10.6)
11 continue

807 m=2./(cs-3.)
d=cs-1./m-1
dE=2.+1./m
fied=fie(dE)
c *************************************************************
c COMPUTE WATER CONSTANTS
c *************************************************************

 call WATCN(ta,sut,nu,gamsw)
c **************************************************************
c COMPUTE CLIMATIC PARAMETERS
c *************************************************************
```

```
delta=1./mtr
mh=mpa/(mtau/(mtb+mtr))
amnu=mtau/(mtb+mtr)
mi=mh/mtr
eta=1./mh
alpha=1./mi
pi=3.14159
beta=1./mtb
c ***************************************************************
c COMPUTE DERIVATIVE OF J WITH RESPECT TO so
c ****************************************************************
den=(1.425-0.0375*d)
if(pl.eq.1) go to 802
k=0
so=0.
805 so=so+0.05
go to 802
802 ds=(1.-so)**den
dds=ds*d
deno=dds+(5./3.)
denom=deno**(4./3.)
soo=1.-so
so1=soo**(-4./3.)
denos=2*soo*deno
dt=(2.425-0.0375*d)
so2=soo**dt
deos1=so2*d*den
nom=-denos-deos1
nom1=nom*so1
der=nom1/(denom*3)
fic=fi(m)
si1=sqrt(n/(k1*fic))*sut/gamsw
si11=si1*so**(-1./m)
bk1=k1*gamsw/nu
sigc=n*eta**2.*bk1*si1/(pi*m*delta)*72000.
sigc1=sigc**0.3333333
dersig=sigc1*der
sia=5*n*bk1*86400*si1/(3*m*pi)
sigma=(sigc/deno*(1.-so)**2.)**.333333
g=alpha*bk1*86400*.5*(1.+so**cs)
gl=alog10(sigma)
xp=(1.766*gl)+(0.980*(gl**2.))
xp1=-.806-xp
 CSI=10.**xp1
xp2=(1.96*gl)+1.766
U=-dersig*xp2/sigma
 co=alpha*86400*bk1/2.*cs*so**(cs-1.)
 co1=U-co
 C2=co1*CSI*exp(-g)
 C38=mtau*86400*bk1*cs/mpa*so**(cs-1.)
 C3=C38/2.
if(vg.eq.0) go to 80
go to 90


 80 E=2.*beta*n*bk1*si1*fied/(pi*m*epr**2.)*86400*so**(d+2.)
if(E.ge.88.) E=88.
  z1=(1.+E*sqrt(2.))*exp(-E)
z2=gamma(1.5)-gamt(1.5,E)
```

```
z2=z2*sqrt(2.*E)
sj=1.-z1+z2
if(pl.eq.1) go to 803
k=k+1
sjk(k)=sj
if(k.eq.20) go to 804
go to 805
803 ag=gamma(1.5)-gamt(1.5,E)
g1=exp(-E)*sqrt(2.)
g2=E*sqrt(2.)+1.
g2=g2*exp(-E)
g3=ag*sqrt(2.)/(2.*sqrt(E))
g4=exp(-E)*sqrt(E)*sqrt(2*E)
gg=-g1+g2+g3-g4
E11=2.*beta*n*bk1*si1*fied/(pi*m*epr**2.)*86400
E12=(d+2.)*so**(d+1.)
derij=gg*E11*E12
 C1=derij
if(C1.le.0.01) C1=0.0
go to 100
90 B=(1.-vg)/(1.+(vg*vk))
B=B+(vk*vg**2.)/(2.*(1.+(vg*vk))**2.)
 C=1./(2.*(vg*vk)**2.)
E1=2.*beta*n*bk1*si1*fied/(pi*m*epr**2.)*86400
E=2.*beta*n*bk1*si1*fied/(pi*m*epr**2.)*86400*so**(d+2.)
o1=B*((vg*vk)+1)
o1=-o1+sqrt(B*2.)
o11=B*E*sqrt(2.*B)
o1=o1-o11
o1=o1*exp(-B*E)
o1=o1*E1*(d+2.)
o1=o1*(so**(d+1.))
o2=-vg*vk*C
o2=o2+sqrt(2*C)
o2=o2-(C*sqrt(2*C)*E)
 C88=C*E
 if(C88.ge.88) C88=88.
o2=o2*exp(-C88)*E1*(d+2.)
o2=o2*(so**(d+1.))
 CE=C*E
BE=B*E

 a1=(vg*vk)+1.
 a2=E*sqrt(2.*B)
 a3=a1+a2
 if(BE.ge.88.) BE=88.
 a3=a3*exp(-BE)
 a4=vg*vk
 a4=a4+(E*sqrt(2.*C))
 if(CE.ge.88.) CE=88.
 a4=a4*exp(-CE)
 a5=gamt(1.5,CE)-gamt(1.5,BE)
 a5=a5*sqrt(2.*E)
 a6=a3-a4-a5
 a6=a6*(1.-vg)/(1.-vg+(vg*vk))
 sj=1.-a6
 if(pl.eq.1) go to 806
 k=k+1
 sjk(k)=sj
 if(k.eq.20) go to 804
```

```
go to 805
806 o3=gamt(1.5,CE)-gamt(1.5,BE)
o3=o3*sqrt(2.*E1)
o3=(1.+d/2.)*o3*(so**(d/2.))
o31=-C*E1*(so**(d+2.))
o31=(C**1.5)*exp(o31)
o32=-B*E1*(so**(d+2.))
o32=(B**1.5)*exp(o32)
o33=o31-o32
o33=o33*(E1**1.5)
o33=o33*(2.+d)
o33=o33*(so**((1.5*d)+2.))
o33=o33*sqrt(2.*E)
o3=o3+o33
derj=o1-o2-o3
derj=derj*(1.-vg)
derj=-derj/((vg*vk)+1.-vg)
 C1=derj
if(C1.le.0.01) C1=0.0
B28=mtau*bk1*86400/mpa*so**cs
C ****************************************************************

C C1=Derivative of J with respect to s
C C2=Derivative of Ys with respect to s
C C38=Derivative of Yg with respect to s
C  sj=J(so)
C si11=psi evaluated at so
C bk1=saturated hydraulic conductivity (cm/sec)

C ****************************************************************

100 print101,C1,C2,C38,sj,si11,bk1
101 format(3hC1=,f10.6,4x,3hC2=,f10.6,4x,3hC3=,f10.6,4x,2hJ=,f10.6,4x,3hMH=,f10.2,4x,f20.1(
SK=so
804 p=mpa/(mnu*mtr)
 C1=C1*epr
if(sj.ge.0.99) sj=1.0
B1=sj*epr

if(pl.eq.1) go to 808
so=0.
k=0
811 so=so+0.05
ds=d*(1.-so)**den
deno=ds+(5./3.)
sigma=(sigc/deno*(1.-so)**2.)**.333333
808 B22=sigma**(-sigma)
sigm=sigma+1.
B22=B22*gamma(sigm)
B2=B22*exp(-g-(2*sigma))
B28=mtau*bk1*86400/mpa*so**cs
B4=B2*p
B5=B28*p*mnu*mtr/mtau
if(pl.eq.1) go to 809
k=k+1
ys(k)=B4
yg(k)=B5
soj(k)=so
if(k.eq.20) go to 810
go to 811
```

```
809 if(ucu.eq.2) go to 1816
 print,'  S(t)      i(cm/day)       Et(cm/day)       yield(cm/day)        DAY '
go to 1815
C ***************************************************************
C CALCULATE THE SOIL MOISTURE CONCENTRATION AND THE
C CUMULATIVE EVAPORATION AND YIELD AT THE END OF
C EVERY RAISTORM AND INTERSTORM PERIOD
C ***************************************************************

1816 print,'SOIL.MOIST. CUM.EVAP. CUM.YIELD'
1815 if(pl.eq.1) go to 812
810 if(kl1.eq.2) go to 3001
C ***************************************************************
C PLOT J VERSUS s
C ***************************************************************

 call plot_$setup(' ','s','J',1,0,0,0)
 call plot_$scale(0.,1.,0.,1.)
3001 i=0
kl1=2
do 813 j=1,20
i=i+1
b77(i)=sjk(j)
a77(i)=soj(j)
813 continue
 call plot_(a77,b77,20,1,' ')
if(dif.eq.1) go to 3002
go to 3003
3002 read(5,)
C ***************************************************************
C PLOT Ys AND Yg VERSUS s
C ***************************************************************

 call plot_$setup(' ','SOIL MOISTURE','SURFACE RUNOFF',1,0,0,0)
 call plot_$scale(0.,1.,0.,2.)
i=0
do 814 j=1,20
i=i+1
b78(i)=ys(j)
a78(i)=soj(j)
814 continue
 call plot_(a78,b78,20,1,' ')
read(5,)
 call plot_$setup(' ','SOIL MOISTURE','GROUNDWATER RUNOFF',1,0,0,0)

 call plot_$scale(0.,1.,0.,2.)
i=0
do 834 j=1,20
i=i+1
b79(i)=yg(j)
a79(i)=soj(j)
834 continue
 call plot_(a79,b79,20,1,' ')
go to 1000
812 if(szr.eq.1) go to 817
do 2001 i1=1,2
print,'Input Zr(cm)'
input,zr
817 a=n*zr
Dt=tis
```

```
LS=0
4053 if(SPE.eq.2.and.ran.eq.2) go to 4054
I=0
go to 4055
4054 LS=LS+1
if(LS.ge.NSPE) stop
4055 LM=0

K=0
KP=0
tsc=0.0
SK3=0.0
SK2=0.0
LMM=0
yieldc=0.0
evapc=0.0
400 if(ucu.eq.1) go to 401
if(szr.eq.2) go to 401
if(fcu.eq.2) go to 401
 write(6,1701)SK,evapc,yieldc
1701 format(f8.5,4x,f8.5,4x,f8.5)
c ****************************************************************
C CALCULATE THE VALUE OF SOIL MOISTURE EVERY HALF HOUR
C DURING A PRECIPITATION EVENT
C ****************************************************************


 401 Dt1=0.
yt=0.0
sia1=sia*fii(d,SK)
sia2=2*(1.-SK)*sqrt(sia1)
Ao=bk1*86400/2.
if(SK.le.0)  go to 215
ao1=Ao*(1.+(SK**cs))
go to 216
215 ao1=Ao
216 I=I+1
r2=R2(I)
in=R1(I)/r2
To1=2*in*(in-ao1)
to2=sia2**2./To1
to3=2.*(in-ao1)
to4=1.+(ao1/to3)
To=to2*to4
300 Dt1=Dt1+Dt
if(Dt1.ge.r2) go to 200
LM=LM+1
if(Dt1.ge.To) yt=1
if(SK2.lt.SK3) yt=0.0
SK1=SK+(in-p*((B2*yt)+(B28*mnu*mtr/mtau))-p*(SK-so)*((C2*yt)+(C3*mnu*mtr/mtau)))*Dt/a
SK2=SK1
SK3=SK
if(SK1.ge.0.999) go to 211
go to 212
211 SK1=0.999
yield=in
yieldc=yieldc+(in*tis)
go to 213
212 yield=p*((B2*yt)+(B28*mnu*mtr/mtau))+p*(SK-so)*((C2*yt)+(C3*mnu*mtr/mtau))
yieldc=yieldc+(yield*tis)
```

```
213 SK=SK1
if(fl.eq.1) go to 250
if(szr.eq.2) go to 300
write(6,210) SK,in,yield
210 format(f8.5,4x,f8.5,22x,f8.5)
go to 300
250 tiss=1./tis
 if(LM.ge.tiss) go to 251
go to 300
251 LM=0
LMM=LMM+1
if(LMM.lt.mtau) go to 907
write(6,908) SK,evapc,yieldc,I,tsc
908 format(f8.5,4x,f8.5,4x,f8.5,4x,i10,4x,f10.3)
go to 900
907 KP=KP+1
SKP(KP)=SK
da(KP)=LMM
if(ucu.eq.2) go to 300
if(szr.eq.2) go to 300
write(6,252) SK,in,yield,LMM
252 format(f8.5,4x,f8.5,22x,f8.5,9x,i5)
go to 300
200 if(ucu.eq.1) go to 201
if(fcu.eq.2) go to 201
 write(6,1700) SK,yieldc,yt
1700 format(f8.5,16x,f8.5,4x,f3.1)
C *****************************************************************
C CALCULATE THE VALUE OF SOIL MOISTURE EVERY HALF HOUR
C DURING AN INTERSTORM PERIOD
C **************************************************************

201 Dt1=0.
500 Dt1=Dt1+Dt
r3=R3(I)
if(Dt1.ge.r3) go to 400
LM=LM+1
evap=B1+(C1*(SK-so))
if(evap.ge.epr) go to 600
tsc=tsc+tis
evapp=evap/epr
if(evapp.le.vg) go to 701
SK1=SK-(evap+(B28*p*mnu*mtr/mtau)+(C3*p*mnu*mtr*(SK-so)/mtau))*Dt/a
if(SK1.le.0) SK1=0.0
evapc=evapc+(evap*tis)
go to 700
600 evap=epr
evapc=evapc+(evap*tis)
SK1=SK-(epr*Dt/a)-((B28*p*mnu*mtr/mtau)+(C3*p*mnu*mtr*(SK-so)/mtau))*Dt/a
if(SK1.le.0) SK1=0.0
go to 700
701 evap=epr*vg
evapc=evapc+(evap*tis)
SK1=SK-(evap*Dt/a)-((B28*p*mnu*mtr/mtau)+(C3*p*mnu*mtr*(SK-so)/mtau))*Dt/a
if(SK1.le.0)  SK1=0.0
700 yield=(B28*p*mnu*mtr/mtau)+(C3*p*mnu*mtr*(SK-so)/mtau)
if(yield.le.0.0000001) yield=0.0000001
yieldc=yieldc+(yield*tis)
SK=SK1
if(fl.eq.1) go to 750
```

```
if(szr.eq.2) go to 757
write(6,220) SK,evap,yield
220 format(f8.5,16x,f8.5,10x,f8.5)
757 K=K+1
if(K.ge.1000) stop
go to 500
750 tiss=1./tis
 if(LM.ge.tiss) go to 751
go to 500
751 LM=0
LMM=LMM+1
if(LMM.le.mtau) go to 901
write(6,905) SK,evapc,yieldc,I,tsc
905 format(f8.5,4x,f8.5,4x,f8.5,4x,i10,4x,f10.3)
go to 900
901 KP=KP+1
SKP(KP)=SK
da(KP)=LMM
if(ucu.eq.2) go to 500
if(szr.eq.2) go to 500
write(6,752) SK,evap,yield,LMM
752 format(f8.5,16x,f8.5,10x,f8.5,9x,i5)
go to 500
900 if(szr.eq.2) go to 2008
if(ran.eq.2)  go to 5000
C **************************************************************
C CALCULATE THE STATISTICAL PROPERTIES OF THE GENERATED
C RAINSTORM EVENTS
C **************************************************************

 print,'Statistical properties of the simulated rainstorm characteristics'
5000 sum1=0.0D0
sum2=0.0D0
sum3=0.0D0
do 1001 IL=1,I
sum1=sum1+R1(IL)
sum2=sum2+R2(IL)
sum3=sum3+R3(IL)
1001 continue
mean1=sum1/(float(I))
mean2=sum2/(float(I))
mean3=sum3/(float(I))
var1=0.0
var2=0.0
var3=0.0
do 1002 IL=1,I
var1=var1+((R1(IL)-mean1)**2.)
var2=var2+((R2(IL)-mean2)**2.)
var3=var3+((R3(IL)-mean3)**2.)
1002 continue
vari1=var1/float(I-1)
vari2=var2/float(I-1)
vari3=var3/float(I-1)
if(ran.eq.1) go to 5001
NSPP=NSPE-1
if(ran.eq.2.and.LS.eq.NSPP) go to 5001
go to 2020
5001 print,'AVER.h(cm)      AVER.tr(days)     AVER.tb(days) '
write(6,1003) mean1,mean2,mean3
1003 format(f10.6,6x,f10.6,6x,f10.6)
```

```
print,'   VAR.h        VAR.tr            VAR.tb'
print 1004,vari1,vari2,vari3
1004 format(f8.2,4x,f8.2,10x,f8.2)
ran=2.
2031 if(lot.eq.2) go to 2030
go to 2020
2030 read(5,)
2008 if(il.gt.1) go to 2003

C *************************************************************
C PLOT THE SOIL MOISTURE CONCENTRATION WITHIN THE
C LAYER OF THICKNESS Zr  VERSUS TIME DURING THE
C RAINY SEASON LENGTH
C *************************************************************

  call plot_$setup(' ','DAYS','SOIL MOISTURE',1,0,0,0)
 call plot_$scale(1.,220.,0.,1.)
2003 i=0
do 910 j=1,LMM
i=i+1
st(i)=SKP(j)
day(i)=da(j)
910 continue
if(il.eq.1) go to 2004
if(il.eq.2) go to 2005
2005 call plot_(day,st,mtau,3,'.')
go to 2001
2004 call plot_(day,st,mtau,1,' ')
if(szr.eq.1) go to 2000
2001 continue
C *************************************************************
C CALCULATE THE MOISTURE FLUXES USING MANABE'S PARAMETERIZATION
C *************************************************************

3025 if(mnb.eq.1) go to 2000
print,'S(t)      CUM.EVAP.     CUM.YIELD'
SK=so
Dt=1./48.
I=0
yieldc=0.0
evapc=0.0
Dt11=0.0
3031 write(6,3033) SK,evapc,yieldc
3033 format(f8.5,4x,f8.5,4x,f8.5)
Dt1=0.0
I=I+1
r2=R2(I)
in=R1(I)/r2
3028 Dt1=Dt1+Dt
Dt11=Dt11+Dt
if(SK.ge.0.42) go to 3029
SK1=SK+in*Dt/(n*100)
SK=SK1
if(Dt1.ge.r2) go to 3027
go to 3028
3029 yield=(in-epr)*Dt
yieldc=yieldc+yield
if(Dt1.ge.r2) go to 3027
go to 3028
3027 write(6,3030) SK,evapc,yieldc
```

```
3030 format(f8.5,4x,f8.5,4x,f8.5)
Dt1=0.0
r3=R3(I)
3032 Dt1=Dt1+Dt
Dt11=Dt11+Dt
evap=epr
if(SK.lt.0.315) evap=epr*SK/0.315
SK1=SK-evap*Dt/(n*100)
evapc=evapc+(evap*Dt)
SK=SK1
if(Dt11.ge.mtau) stop

if(Dt1.ge.r3) go to 3031
go to 3032
2000 read(5,)
 stop
end
C *****************************************************************

 subroutine WATCN(ta,sut,nu,gamsw)

C *****************************************************************

real nu,nut
dimension sutt(11),nut(11),gamst(11)
data sutt/75.6,74.9,74.2,73.5,72.80,72.1,71.4,70.7,70.0,69.3,68.6/
data nut/17.93e-3,15.18e-3,13.09e-3,11.44e-3,10.08e-3,8.94e-3,
& 8.e-3,7.2e-3,6.53e-3,5.97e-3,5.94e-3/
data gamst/0.99987,0.99999999,0.99973,0.99913,0.99823,0.99708,
& 0.99568,0.99406,0.99225,0.99025,0.98807/
if(ta.gt.50.)go to 10
ita=ifix(ta*.2)+1
frac=ta-float(5*(ita-1))
ita1=ita+1
sut=(sutt(ita1)-sutt(ita))*0.2*frac+sutt(ita)
nu=(nut(ita1)-nut(ita))*0.2*frac+nut(ita)
gamsw=((gamst(ita1)-gamst(ita))*.2*frac+gamst(ita))*980.
return
10  sut=sutt(11)
nu=nut(11)
gamsw=gamst(11)
return
end
c *****************************************************************
c this function computes the gamma incomplete function
c *****************************************************************
function gamt(a,x)
if(x.eq.0)go to 40
if(x.gt.100)go to 50
sum=1./a
an=1.0
old=sum
33 old=old*x/(a+an)
if(old/sum-1.e-6)20,10,10
10 an=an+1.
sum=sum+old
if(an-300.)33,33,12
12 continue
20 xxx=(a*alog(x)+alog(sum)-x)
if(xxx.lt.-80.)go to 40
```

```
gamt=(exp(xxx))
go to 60
40 gamt=0.0
go to 60
50 gamt=gamma(a)
60 return
end
c ****************************************************************
c This function computes the gamma function by a Stirling approx.
c ****************************************************************
function gamma(y)
x=y+1.
pi=3.14159
stir1=1./(12.*x)
stir2=1./(288.*x**2.)
stir3=-139./(51840.*x**3.)
stir4=-571./(2488320.*x**4.)
stir=1+stir1+stir2+stir3+stir4
gamma=exp(-x)*x**(x-.5)*sqrt(2.*pi)*stir/y
end
function fie(d)
dimension y(6)
data y/0.18,0.11,0.077,0.056,0.044,0.034/
if(d.gt.7.)go to 10
x=d-1.
i=ifix(x)
frac=x-float(i)
y1=alog(y(i))
y2=alog(y(i+1))
fie=exp((y2-y1)*frac+y1)
return
10 fie=.034
return
end
```

PROGRAM WINSLOW.FORTRAN

```
C THIS PROGRAM CALCULATES THE LATENT HEAT
C FLUX OBTAINED FROM THE ANALYTICAL MODEL
C USING EXACTLY THE SAME PERIODIC ATMOSPHERIC
C FORCING SPECIFIED BY MILLY AND EAGLESON(1982,
C TR.279) FOR THE CLIMATE OF WINSLOW,ARIZONA.
C THE FUNCTIONS OF HYDRAULIC AND VAPOR CONDUCTIVITY
C USED ARE APPROXIMATIONS OF THOSE SPECIFIED
C IN TR.279 FOR SILTY LOAM AND SAND.
C THE MODEL WAS ALSO MODIFIED TO TAKE INTO ACCOUNT
C VAPOR FLOW FOR THE SILTY LOAM.
C ***************************************************
C
C CLIMATIC AND SOIL PARAMETERS
C
C ep=average potential evaporation rate(cm/day)
C mtb=mean time between storms(days)
C mtr=mean storm duration(days)
C mt=mean rainy season length(days)
C ta=average annual temperature(deg.Celcius)
C mv=mean number of storms per year
C mpa=mean annual precipitation(cm)
C n=effective porosity of soil
C k1=saturated intrinsic permeability(cm2)
C c=pore disconnectedness index
C so=average annual soil moisture
C yg=average annual percolation rate(cm/day)
C ys=average annual surface runnoff rate(cm/day)
C mi=mean storm intensity(cm/day)
C c1,c2,c3=linearization coefficients of the annual
C          water balance as obtained from the prog-
C          ram Taylor.fortran.
C et=actual average annual evapotranspiration rate(cm/day)
C ad,aw=coefficients of the albedo function as specified
C       in TR.279
C Si=initial soil moisture
C T1i=initial surface temperature(deg.celcius)
C T2i=initial deep soil temperature(deg.celcius)
C Zr=near surface storage depth(cm)
C
print,"ep,mtb,mtr,mt,ta,mv,mpa"
input,epr,mtb,mtr,mt,ta,mv,mpa
print,"n,k1,c,so,yg,ys,mi,c1,c2,c3,et,ad,aw"
input,un,ak1,c,so,yg,ys,ami,c1,c2,c3,et,ad,aw
100 print,"Si,T1i,T2i"
input,sk,t1k,t2k
sini=sk
print," Input Storage Depth Zr "
input,Zr
print,"If silty loam type 1,if sand type 2"
input,soil
if(soil.eq.1.) spr=0.0064426
if(soil.eq.2.) spr=0.0243819
to=0.0
prec=0.0
toc=0.0
tc=0.0
t=0.0
k=0
 c3=c3/2.
Dt=0.25/24.
```

```
m=0
13 kt1=-74+(k*96)
kt2=20+(k*96)

if(tc.gt.kt1.and.tc.le.kt2) go to 11
kt3=22+(k*96)
15 if(tc.gt.kt2.and.tc.le.kt3) go to 14
k=k+1
to=spr*2.*(sk**2.165)
to=86400*(to**2.)
to=to/(2.*(epr**2.))
to=to*24.
print,"To"
write(6,400) to
400 format(f10.4)
toc=0.0
prec=1.
tim=to
go to 13

c FINITE DIFFERENCE EQUATION FOR EVALUATING SOIL MOISTURE
c DURING A DRY PERIOD.
11 l=1
yt=0.0
 cv=0.45
toc=toc+0.25
if(to.gt.toc) sic=1.
if(to.le.toc) sic=0.
eta=et+(c1*epr*(sk-so))
if(eta.le.0.0) eta=0.0
yga=yg+((sk-so)*c3*ami)
if(yga.le.0.0) yga=0.0
if(to.ge.toc) eta=0.00119*0.622*ch*Ua*3600.*(es-9.67)*24./1013.25
if(to.le.toc.and.prec.eq.1.) go to 600
go to 601
600 if(soil.eq.2) go to 601
 th1=un*sk
if(th1.le.0.0) go to 601
yim=(-30.*th1)-5.5
De=(10.**(-5.5))
De=De-(10.**yim)
De=De/1.5
De=De*3600.*1.85*(th1**(-1.85))
Se=2.*th1*(sqrt(De/3.14))
tim=tim+0.25
fet=Se/(2.*sqrt(tim))
fet=fet-(2.852e-9)
fet=24.*fet
if(fet.lt.eta) prec=0.0
if(fet.ge.eta.and.sk.ge.sini) eta=fet
601 sk1=sk+((Dt/(un*Zr))*(-eta-yga))
if(sk1.lt.0.0) sk1=0.0
eta=eta*597./24.
if(sk1.eq.0.0) eta=0.0
go to 12
c FINITE DIFFERENCE EQUATION FOR EVALUATING SOIL MOISTURE
c DURING A RAINY PERIOD.
14 l=0
yt=1.0
HP=0.193*0.99*Ta
```

```
 cv=1.
sk1=sk+((Dt/(un*Zr))*(4.632-yg-ys-((sk-so)*ami*(c3+c2))))
sk1=sk1-(Ep*Dt/(un*Zr*597.))
if(sk1.gt.1.0) sk1=1.0
c AIR TEMPERATURE
12 arg=t-15.
arg=arg*3.14/12.
Ta=25.8+(7.8*cos(arg))
c WIND SPEED
arg1=t-18.5
arg1=arg1*3.14/12.

Ua=360.+(180.*cos(arg1))
Ri=392400./(Ua**2.)
Ri=Ri*(Ta-t1k)/(ta+546.+t1k)
if(Ri.lt.0.0.and.Ri.ge.-0.0014) ch=0.0028
if(Ri.le.-0.0014.and.Ri.ge.-0.0054) ch=0.0029
if(Ri.le.-0.0054.and.Ri.ge.-0.0105) ch=0.0030
if(Ri.le.-0.0105.and.Ri.ge.-0.0205) ch=0.0032
if(Ri.le.-0.0205.and.Ri.ge.-0.0402) ch=0.0035
if(Ri.le.-0.0402.and.Ri.ge.-0.0793) ch=0.0039
if(Ri.le.-0.0793.and.Ri.ge.-0.1569) ch=0.0044
if(Ri.le.-0.1569.and.Ri.ge.-0.3119) ch=0.0052
if(Ri.le.-0.3119) ch=0.0058
if(Ri.ge.0.2) ch=0.0
if(Ri.ge.0.0.and.Ri.lt.0.2) ch=0.00277*((1.-(Ri/0.2))**2.)
c EVAPORATION RATE
es=6.11+(0.6102*t1k)
Ep=0.00119*0.622*ch*Ua*3600.*(es-9.67)/1013.25
Ep=Ep*597.
if(l.eq.0) EL=Ep
if(eta.lt.Ep) th1=0.0
if(eta.ge.Ep) th1=un*sk
if(l.eq.0) th1=un*sk
if(sic.eq.1) th1=un*sk
if(th1.eq.0.0) eps=0.9
if(th1.gt.0.0) eps=0.95
if(eta.ge.Ep.and.l.eq.1) EL=Ep
if(eta.lt.Ep.and.l.eq.1) EL=eta
if(sic.eq.1.) EL=Ep
c ALBEDO
th11=2.*th1
if(th11.gt.un) A=aw

if(th11.le.un) A=ad+((aw-ad)*th11/un)

c SHORT WAVE RADIATION

agu=t-12.
agu=agu*3.14/12.
sna=(sin(0.375)*sin(0.611))+(cos(0.375)*cos(0.611)*cos(agu))
if(sna.le.0.0) go to 210
a1=0.128-(0.054*(alog10(1./sna)))

a1n=a1*2.5/sna
if(a1n.ge.88.) a1n=88.
if(a1n.le.-88.) a1n=-88.
ae=exp(-a1n)
vc=1.-(0.65*(cv**2.))
wb=120.*sna*ae*vc
```

```
if(sna.gt.0.0) si=wb
210 if(sna.le.0.0) si=0.0

c DOWN LONGWAVE RADIATION
dli=(9.37e-6)*(0.826e-10)*60.
dli=dli*((Ta+273.)**6.)
 cvv=1.+(0.17*cv**2.)
dli=dli*cvv

c BACK LONG-WAVE RADIATION
uli=(t1k+273.)**4.
uli=uli*(0.826e-10)*60.*eps
c SENSIBLE HEAT TRANSFER
H=-0.00119*0.2399*ch*Ua*3600.*(Ta-t1k)
c FORCE-RESTORE METHOD FOR ESTIMATING
c SURFACE TEMPERATURE.
t1k=t1k+273.
t2k=t2k+273.
G=-uli-EL-H+(HP*yt)+((1.-A)*si)+(eps*dli)
G=G-(0.99*sic*(t1k-273)*Ep/597.)
rs=((1.-un)*2.0e6)+(un*sini*4.2e6)+((un-(un*sini))*1.25e3)
rs=rs/4.2e6
if(soil.eq.1) aml=1.3e-3
if(soil.eq.2) aml=4.e-3
ssk=aml/rs
d2=20.*ssk*86400.
d2=sqrt(d2)
t2k=t2k+(0.25*G/(rs*d2))
pr1=((1.-un)*2.e6)+(th1*4.2e6)+((un-th1)*1.25e3)
pr1=pr1/4.2e6
ht=th1
if(soil.eq.2) go to 200
if(ht.ge.0.4) alm=3.75e-3
if(ht.le.0.4.and.ht.ge.0.3) alm=3.5e-3
if(ht.le.0.3.and.ht.ge.0.2) alm=3.0e-3
if(ht.le.0.2.and.ht.ge.0.1) alm=2.65e-3
if(ht.le.0.1.and.ht.ge.0.05) alm=1.5e-3
if(ht.le.0.05) alm=0.5e-3
go to 300
200 if(ht.ge.0.3) alm=8.e-3
if(ht.le.0.3.and.ht.ge.0.2) alm=7.1e-3
if(ht.le.0.2.and.ht.ge.0.1) alm=6.4e-3
if(ht.le.0.1.and.ht.ge.0.05) alm=5.e-3
if(ht.le.0.05) alm=2.e-3
 300 clam=0.3*sqrt(alm*pr1)
 clam=clam+(0.7*sqrt(aml*rs))
 c11=2.*(sqrt(3.14/86400))/clam
 c22=2.*3.14/24.
t1k=t1k+(0.25*c11*G)-(0.25*c22*(t1k-t2k))
t1k=t1k-273.
t2k=t2k-273.
 sk=sk1
EL=EL*24.
write(6,120) EL,t1k,l,t2k,G,sk,Ri
120 format(f10.4,4x,f10.4,4x,i1,4x,f10.4,4x,f10.4,4x,f10.4,4x,f10.6)
tc=tc+0.25
 t=t+0.25
if(t.eq.24.) t=0.0
if(l.eq.1) go to 13
go to 15
end
```