TRANSIENT RESPONSES IN MULTIPLE PLATE ABSORPTION COLUMNS

by

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Thesis Supervisor

Accepted by: ....
Head of Department

97 Bay State Road
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May 20, 1960

Professor Philip Franklin
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Franklin:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "Transient Responses in Multiple Plate Absorption Columns".

Sincerely yours,

Signature redacted

William C. Morris

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I. SUMMARY

In order to gain some understanding of the unsteady-state response of multiple plate absorption towers, this thesis uses a Reeves Electronic Analogue Computer to duplicate the changes in plate liquid concentration of an absorber whose vapor feed concentration has experienced a step change. Assumptions used in formulating the tower model include: theoretical plates, no concentration gradient across the plates, constant inert liquid and vapor flow rates, negligible vapor holdup, and a linear equilibrium relationship.

It was found that after an input step the liquid concentration on each plate approached a new steady-state value along a path which could be closely approximated by a time lag (or jump) followed by an exponential rise.

The time lag was different on each plate of a tower, being largest (and positive) on the top plate and smallest (negative) on the bottom plate. The lag was independent of the size of the input disturbance.

The exponential was independent of both the plate position and the size of the input step. It is suggested that the first exponent in the analytical solution of Lapidus and Amundson (5) can be used to approximate the predominant exponent $s_1 = \frac{-VK}{h}\left[\frac{L}{VK} + 1 - 2\sqrt{\frac{L}{VK}}\cos\frac{\pi}{N+1}\right]$

II. INTRODUCTION

Traditionally, chemical engineers have focused their attention on the steady-state characteristics of chemical processes. As the sophistication of steady-state techniques has grown, however, increasing attention has been given to the time dependent aspects of process response.

Basically, there are two general types of unsteady-state phenomena: the response of a system to a cyclicly changing input, termed frequency response, and the response of a system to a non-repetitive input disturbance, termed transient response. Both of these general response characteristics are of importance in chemical process equipment, but of the two, transient response is both easier to grasp conceptually, and more valuable from an elementary standpoint.

A particular case of interest from the transient point of view is the multiple plate fractionating column. In such a column a binary feed is introduced somewhere between the top and bottom plate, the volatile component taken out the top, and the non-volatile component off the bottom plate. Normally heat is added to the bottom plate, or still, and some fraction of the top vapor is condensed and returned as liquid reflux to the top plate. Obviously, the reflux ratio, and the rate at which heat is added will considerably affect the purity of the product streams. Steady-state methods enable one to calculate the composition of the product streams, given fixed values of feed composition, reflux ratio, heat supplied, equilibrium data, and other tower variables. If such a tower is once characterized, and a discontinuous change (step change) is made in a parameter such as feed composition, then the same steady-state technique will furnish the new equilibrium values of product concentration. It will furnish nothing, however, about the time necessary for the output to equilibrate, nor will it define the path taken by the output in reaching its new equilibrium values.

The usefulness of being able to define the transient response for control of a column such as this is pointed out by Gilliland (9). It is often desired to control the reflux ratio of such a tower by means of a servomechanism coupled to a sensing device which monitors the composition of the material in the tower. The object is to control the reflux ratio in such a manner as to isolate the top product from disturbances in operation parameters such as feed composition. Unfortunately, however, sensing devices able to withstand the rigors of plant operation are not capable of detecting the minute swings in purity which occur at the top of such a tower. For this reason the device is usually placed further down in the tower where concentration swings are proportionally larger. Thus a lag occurs between the time a disturbance reaches the sensing device, and the time it reaches the top of the tower. In this case, the sensing device can operate on the reflux only through some functional lag operator which depends on the column's input-output transient response as well as its input-sensing device transient response.

Work in this field has been going on for some time. At M.I.T., Smith and Polk (12), Jordan (4), O'Donnel (8), and Davis (2) have all examined the transient response of a fractionating column with this control problem in mind. In every case they developed mathematical models of a column and solved for the transient solution on a digital computor.. Davis was even able to duplicate the reflux control problem, and hold product composition constant while supplying a step feed change. Jackson and Pigford (3) characterized the transient responses of columns with both very large and very small reboilers. Their work was also done on a digital computor. Rose, Johnson, and Williams (10,11) examined both column control and general column transients on digital and analogue computors. Almost all of this previous work assumed non-linear equilibrium relationships in construction of the models. While specific solutions to the cases studied were obtained, attempts to correlate the data for general applicability were not so successful. In addition, the effect of changes in the operating and design parameters were complicated and difficult to understand.

In an attempt to clarify the existing results, this project reverts to the examination of a simpler- but related- case, the case of a linear equilibrium relationship and half of a fractionating tower. These conditions correspond to a multiple plate absorber.

Marshall and Pigford (6) first derived the solution to a set of linear absorber equations. Lapidus and Amundsen (5) extended this treatment to cases with arbitrary end conditions. Their result is stated in the form of a set of sums and integrals whose solution is involved for many cases. Acrivos and Amundson (1) examined absorbers with non-linear equilibrium relationships on an analogue computor. No correlations were presented in any of this work to establish the effect of parameters on the transients.

This project also examines the transient response of an absorption tower by means of an analogue computor. Linear equilibrium relationships are used and an attempt made to recognise the effect of the various design and operating variables on the transient times.

A multiple plate absorber consists of a tower filled with plates which contain either small holes or bubble caps to allow the upward passage of vapor. The top side of the plates is covered with liquid absorbant which flows across the plate and down an overflow to the plate below. This liquid absorbant is introduced at the top of the tower and allowed to mix with the rising vapor until it emerges from the bottom of the unit enriched in acquired solute. The vapor feed with a large solute concentration is introduced at the bottom of the tower, and passed upward through the plates until it emerges from the top stripped to some fraction of its initial solute concentration.

Examining an arbitary absorber tower such as is shown in Figure 1, and assuming theoretical plates, no concentration gradient on the plates, and no mass transfer other than solute between the streams, it is possible to relate the c change in any plate concentration to the material flows to that plate by the expression:

$$Vy_{n+1} + Lx_{n-1} - Vy_n - Lx_n = h\frac{\partial x_n}{\partial \theta} - H\frac{\partial y_n}{\partial \theta} \qquad (1)$$

Where: V = The solute free vapor rate, moles/time

L = The solute free liquid rate, moles/time

x = moles solute/mole solute free liquid

y = moles solute/mole solute free vapor

h = liquid holdup per plate

H = vapor holdup per plate

Assuming a negligible vapor holdup (a reasonable condition in the case of an absorber), and a linear equilibrium expression, Equation (1) reduces to:

$$\frac{VK}{h}x_{n+1} + \frac{L}{h}x_{n-1} - \left(\frac{L+VK}{h}\right)x_n = \frac{\partial x_n}{\partial \theta} \qquad (2)$$

where K is defined by the equilibrium relationship:

$$y_n = Kx_n \qquad (3)$$

Equation (2) is the general form of N equations corresponding to the N plates of any given absorber.

In this work, three sets of such equations are solved for stepped feed composition on the Reeves Analogue Computer located at M.I.T. The effects of the parameters V, L, and Δy, and the design parameter N on a transient resulting from stepped input concentration are analysed.

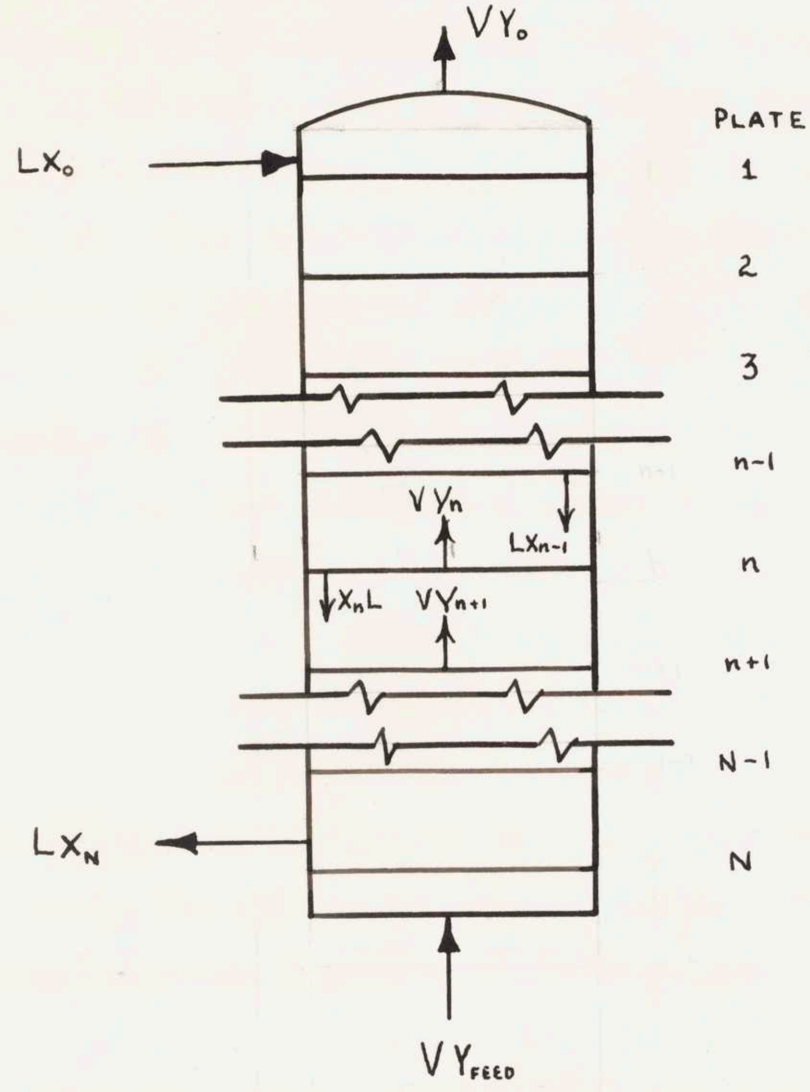


FIGURE 1

Detail of Absorber

III. PROCEDURE

A. Conditions Studied

In orded to evaluate the effects of the parameters L, V, Δy and N on absorber response three absorber models were studied.

The first model, used in Series 2, had five plates, a vapor rate of 25 moles/hour, a liquid rate of 5 moles/hour, and a liquid plate holdup of 1 mole/plate. In this, as well as the subsequent series, a negligible amount of vapor holdup, and an equilibrium constant of 0.20 was assumed. The second model, used in Series 3, had six plates. Otherwise the assumed parameters were identical with the model in Series 2. The third model, Series 4, had six plates, and a liquid rate of 6 moles/hour. In all other respects it was identical with the absorber in Series 2. Series 1, which used the same model as Series 2, gave results which were discarded from the analysis because of unsophisticated computer programing techniques. Table I is a summary of parameters in the three series.

B. The Reeves Analogue Computer

The Reeves Analogue Computer (REAC) duplicates variables in an equation with voltages and time. A sub-circuit is set up for each plate equation, and linked with appropriate positions in the preceeding and following plate sub-circuits. Initial conditions are fed to the circuit in the form of voltages, and the network is allowed to equilibrate through the flow of currents. A solution to the transient response is obtained by measuring the change in voltage with time at an appropriate place in the circuit.

The primary components of the computer are high gain d.c. amplifiers arranged to perform the functions of integration, summation, and inversion. The Reves Electronic Analogue Computer (REAC) located at M.I.T. contains twenty of these d.c. amplifiers. Seven are used as summers, seven as integrators, and six as inverters. As long as an absorber with linear characteristies is assumed, only summers, integrators and potentiometers are needed to duplicate the problem.

TABLE I

Summary of Values Used in Absorber Models

PARAMETERS	SERIES 2	SERIES 3	SERIES 4
L	5	5	6
V	25	25	25
$\frac{L}{V}$	0.2	0.2	0.24
N	5	6	6
y_F	varied	varied	varied

The circuit components in the REAC are all individually wired to a patch-bay located on the front of the computer. This bay is designed to accept a pre-patch-board on which a combination of computer components can be connected by plug-in leads. The pre-patch-board can be removed from the machine without disturbing the problem patched onto it. Thus, one problem need not be destroyed before another can be run.

Six of the integrators, whose inputs and outputs are wired to the patch bay, have internal circuits which permit initial conditions in the form of constant voltages to be added to their outputs. These initial conditions are set with six potentiometers (or simply pots) on the computer front. Associated with each pot is a switch which permits its output to be of either algebraic sign.

The high gain amplifiers used in both the integrators and summers are designed to operate with their outputs in a range of $\pm$ 100 volts. Driving the element outside this range produces nonlinearity, and neon overload lights are provided to indicate this. All of the amplifiers in the computer give rise to a sign change between their outputs and their inputs.

Any two voltages in the computer can be monitored with the externally mounted Output Table. This table consists of a cylindrical drum on which graph paper is mounted, and a traversing arm (with a pen holder) which moves across the outside of the drum in a line parallel with its axis. Both the rotation of the drum, and the displacement of the arm are controlled with servomechanisms that derive their positioning signals from any desired point in the computer circuit. For a more detailed discussion of computer components and techniques the reader is referred to the REAC Theory and Operation Manual (7).

III. PROCEDURE

C. Programing and Operating the Computer

Scaling the Problem

In order to convert the absorber model from the moles/mole -hour domain to the voltage-second domain of the computer, two changes of variables must be made. To relate x with voltage (e) the factor C is chosen so that

$$e_n = Cx_n \quad . \tag{4}$$

To speed up the problem solution, and to stay within the dynamic range of the computer a factor is used in the time scale.

$$t = a\theta \tag{5}$$

where t = computer time (in seconds)

θ = physical time (in hours)

a = scale factor

Substituting these two variable changes (4 and 5) into (2) we arrive at a computer equation,

$$\frac{VK}{ha} e_{n+1} + \frac{L}{ha} e_{n-1} - \left(\frac{VK+L}{ha}\right) e_n = \frac{\partial e_n}{\partial t} \tag{6}$$

which corresponds to the physical problem.

Since C does not appear in Equation (6), the scaler factor relating x and e does not affect solution time, and may be choosen with regard only for full utilization of the voltage range in the machine. As most electrical devices the computer is most accurate when opperated near its maximum voltage limit. The time factor a, however, directly influences the solution time by operating on the constants applied to each integrator output. In all the runs of this thesis the following values were used for a and C:

$a = 20$ $\qquad$ $C = 300$

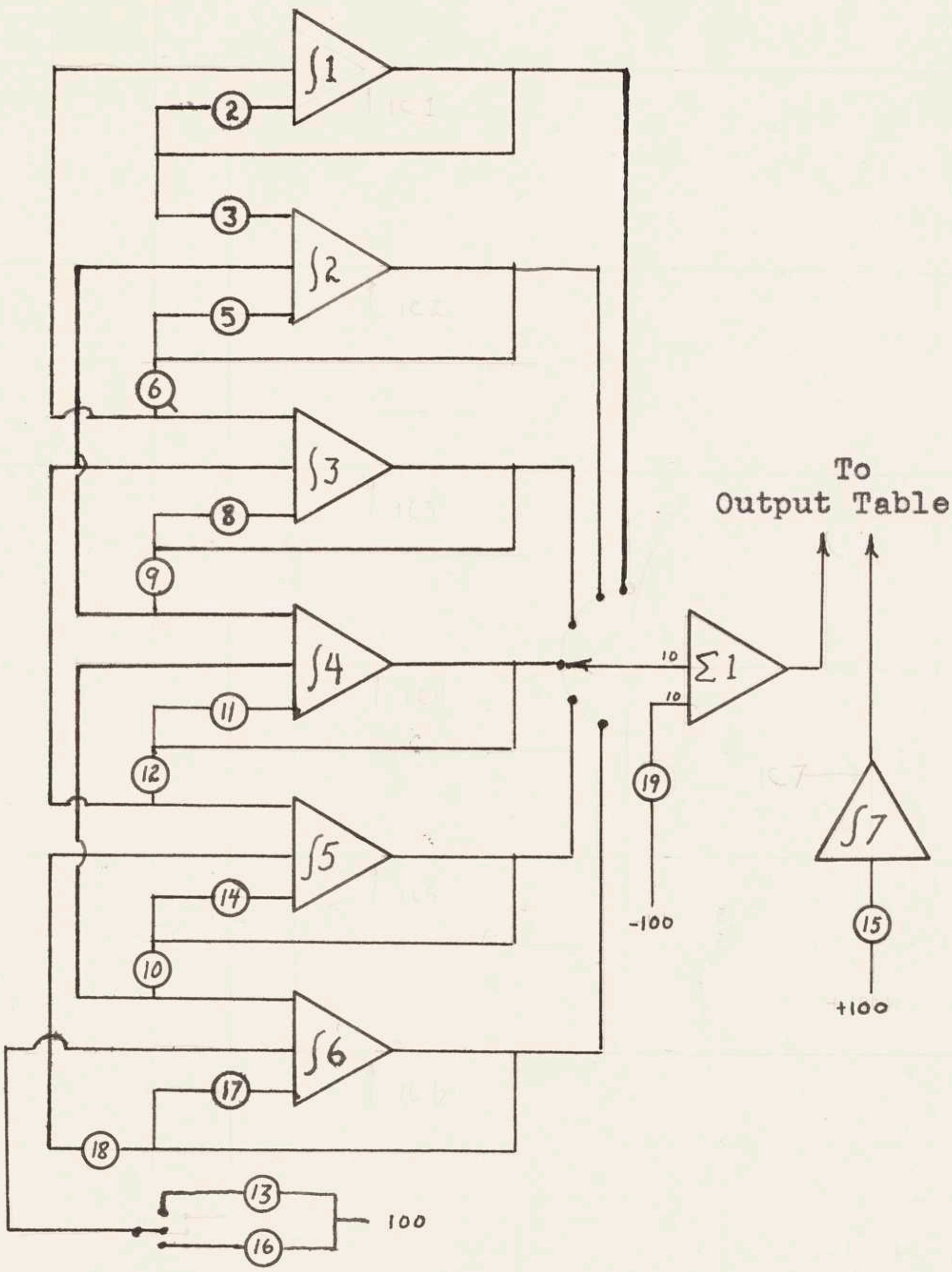
C. Programing and Operating the Computer

Patching In the Problem

Figure 2 shows the complete block diagram of Series 3 on the computer. Because the solute concentration in the liquid feed is zero, the only voltage input to the network corresponds to the feed vapor solute concentration entering plate (integrator) number 5. The initial condition circuits simply boost each plate to near steady-state value and shorten the time necessary for equilibration of the computer before the step is introduced. Because of the sign change in the amplifiers, consecutive integrators alternate in the algebraic sign of their outputs.

A step change in vapor feed concentration is accomplished by switching rapidly from Pot 13, which provides the initial y_F, to Pot 16 which is set at the new value of y_F. A double-throw function switch is provided on the computer to facilitate such a change. The inputs of Pot 13 and Pot 16 are connected to a 100 volt supply on the patch board.
The pots operating on each integrator's output correspond to the constants $\frac{VK}{ha}$, $\frac{L}{ha}$, and $\frac{VK+L}{ha}$ in Equation (6).
In Series 3 only two pots per plate were needed because $\frac{VK}{ha}$ was numerically equal to $\frac{L}{ha}$.

Each integrator's output has a potential corresponding to the liquid solute concentration on that plate. These outputs are fed consecutively to Summer 1 which is used to amplify the voltage by a factor of ten before sending it to the arm of the Output Table. The high gain of Summer 1 which is necessary for accuracy in the results requires the



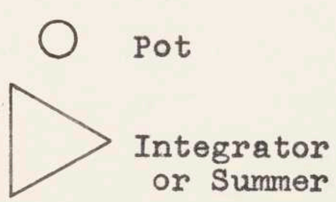


FIGURE 2

Computer Diagram
Series 3

output to be greater than ±100 volts if the plate voltage is larger than ±10 volts. In order to circumvent this problem (i.e. to amplify increments in plate voltage and not its D.C. value) a compensating voltage of opposite polarity is added to the input of Summer 1. Thus, the displacements registered from zero on the Output Table have no absolute significance and are valuable for indicating changes only.

To provide a time scale for the drum rotation (x-axis) of the Output Table a one volt signal is supplied to Integrator 6 by Pot 15. This integrator thus generates a ramp of one volt per second which rotates the Output Table's drum at a constant velocity. Because the Output Table reads from -100 to +100 volts, the initial condition pot on Integrator 6 is used to position the drum at its -100 volt extreme at the beginning of a run.

Making a Run

Each of the three series of runs made for this study used a different absorber model. Individual runs in each series differed only in the magnitude of the step change in their vapor feed concentration.

The first step in making a run was determining the physical parameters of the absorber model. Having these, the machine parameters and constants could be determined from Equations (4) and (5). Once the machine constants were known, the problem was patched into the computer (see Figure 2), and the pots and initial conditions set at their proper values.

Following this, graph paper and pen were placed on the Output Table which had its y-axis servo connected through a summer to one of the integrator outputs and its x-axis servo to a ramp producing integrator. The Output Table axies were then positioned with Pot 19 and the initial condition pot on Integrator 6 in order to prevent the servos from receiving more than ±100 volts during the run.

After placing the function switch in contact with Pot 13, the operating switch, which closes the integrator input connections was closed and the run started. The Output Table pen plotted the selected plate voltage as a function of time. Normally initial conditions were not set with complete accuracy, and several seconds had to be allowed for the machine to equilibrate before introducing the step in y_F. Once this equilibrating had occurred, the function switch was used to connect Pot 16 with the circuit. The effect of the step in the monitored integrator (as well as the preceeding equilibration changes) was graphed on the Output Table. In order to complete a run with data from each integrator, this procedure had to be repeated once with the Output Table connected to each plate. No significant error was introduced by this technique (see DISCUSSION OF RESULTS, <u>C. Accuracy of Results</u>).

The five or six curves which corresponded to each plate in the tower were drawn by the Output Table on a single sheet of graph paper. Figure 3 is an example of one such data sheet. It should be pointed out that the alternating directions of change following the step are the result of the polarity reversal in each consecutive integrator output.

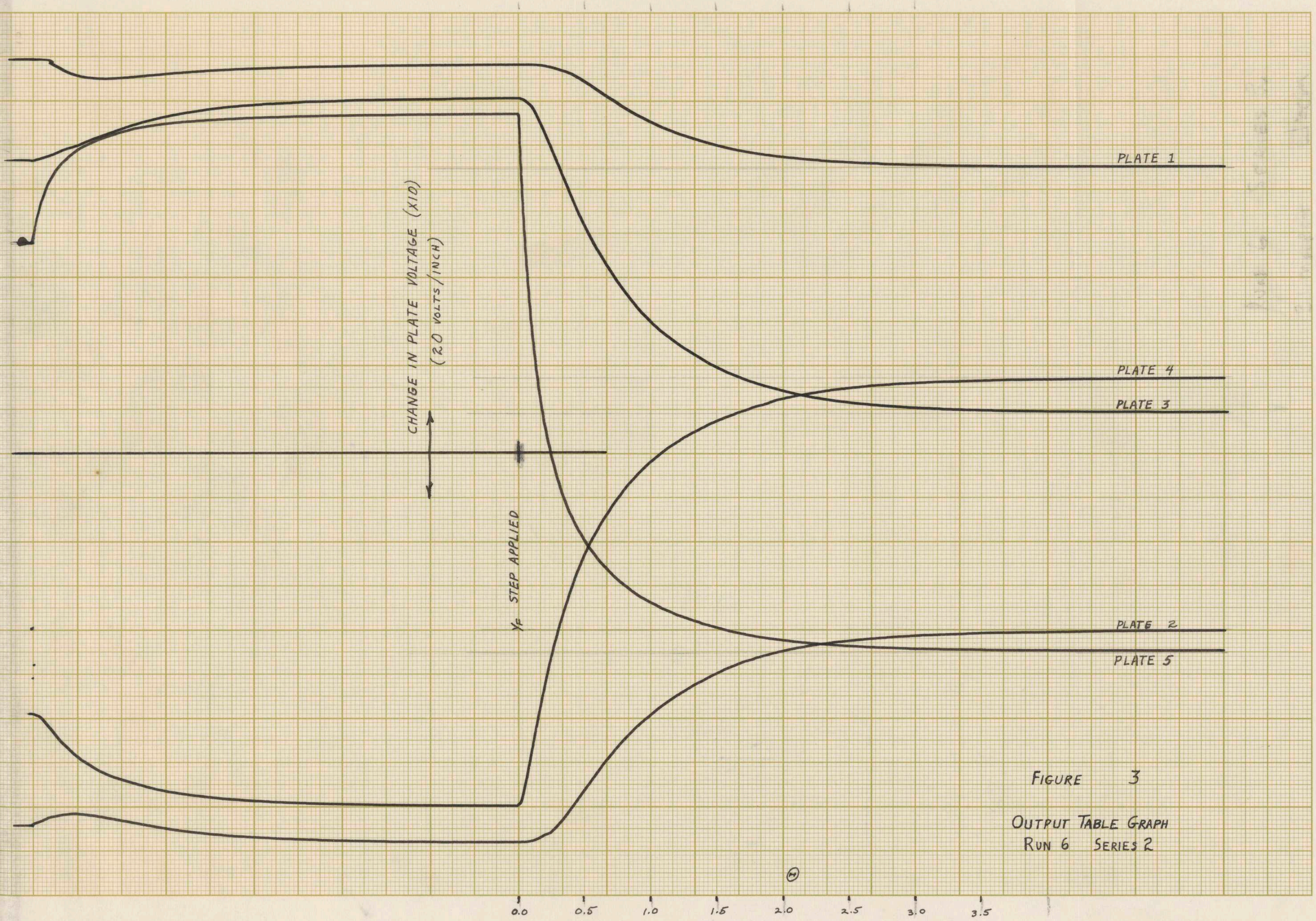


FIGURE 3

OUTPUT TABLE GRAPH
RUN 6 SERIES 2

IV. RESULTS

The data taken from the computer on graphs such as Figure 3 was reduced to numerical form by measuring the amount of change in x_n after each three squares on the time axis (about 10 seconds on the computer or 0.5 hours in the tower).

The curves for each plate seemed to rise or fall exponentially from their initial to their final equilibrium values. In order to examine any exponential tendency in the response, the curves were replotted on semi-logarithmic graph paper.

Because a rising exponential cannot be depicted as a straight line on semi-log paper, the values measured from the raw data were related to the final equilibrium value rather than the initial equilibrium value, i.e. $x_n(\infty)-x_n(\theta)$ was measured rather than $x_n(\theta)-x_n(0)$. In addition, these values were normalized to facilitate interplate comparison. The final result was taken to be the quantity E_D, where

$$E_D = \frac{x_n(\infty)-x_n(\theta)}{x_n(\infty)-x_n(0)} \qquad (7)$$

The curves of log E_D -vs- θ are reproduced in Figures 4 to 20. From these figures it can be seen that each plate in a tower may be approximated by a time lag or jump, where the exponential goes to one at some time other than zero, followed by a single exponential change. It appears from Figures 4 to 20 that all plates in any given tower have the same time constant for their predominant exponential (i.e.

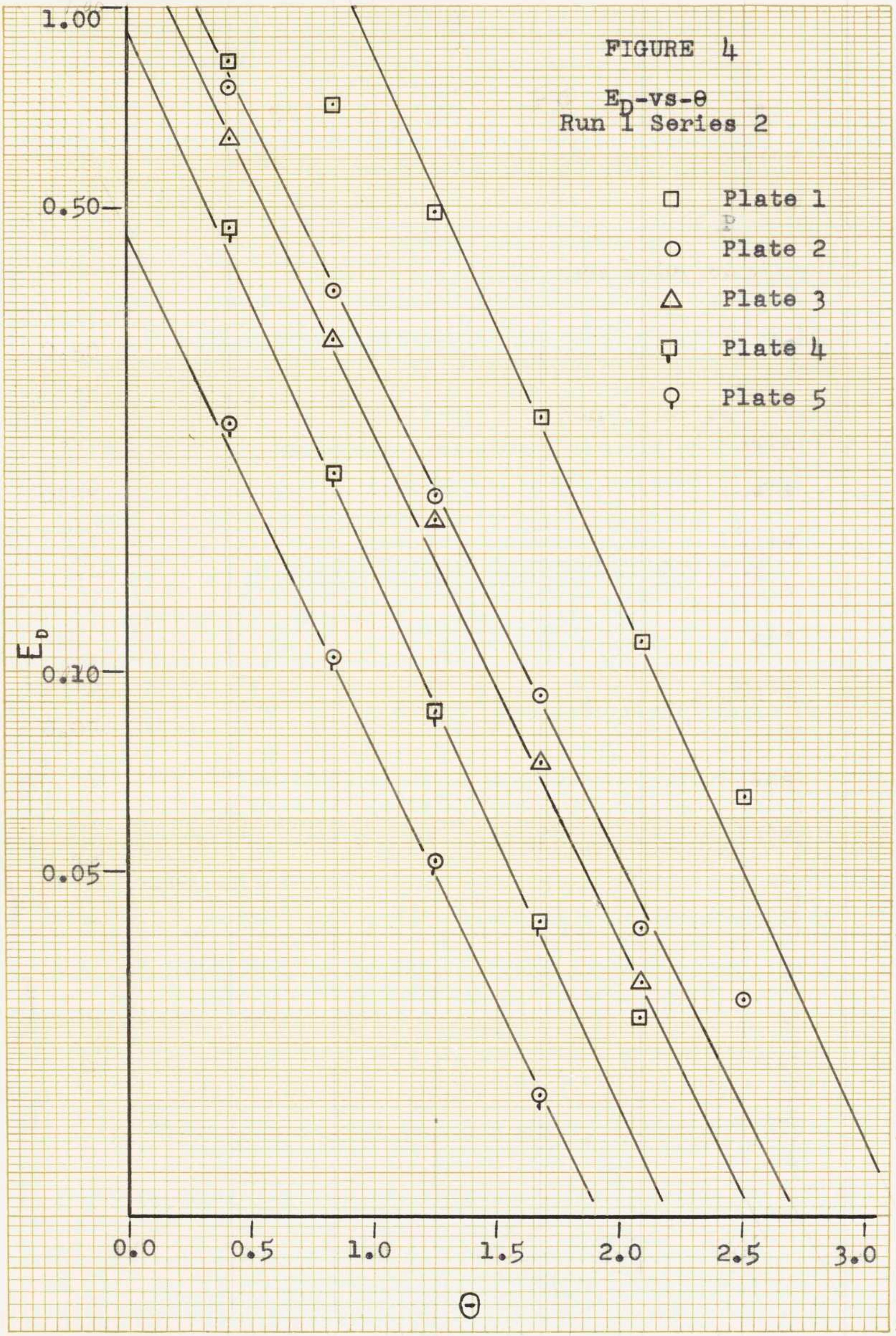


FIGURE 4
E_D-vs-θ
Run 1 Series 2
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Θ

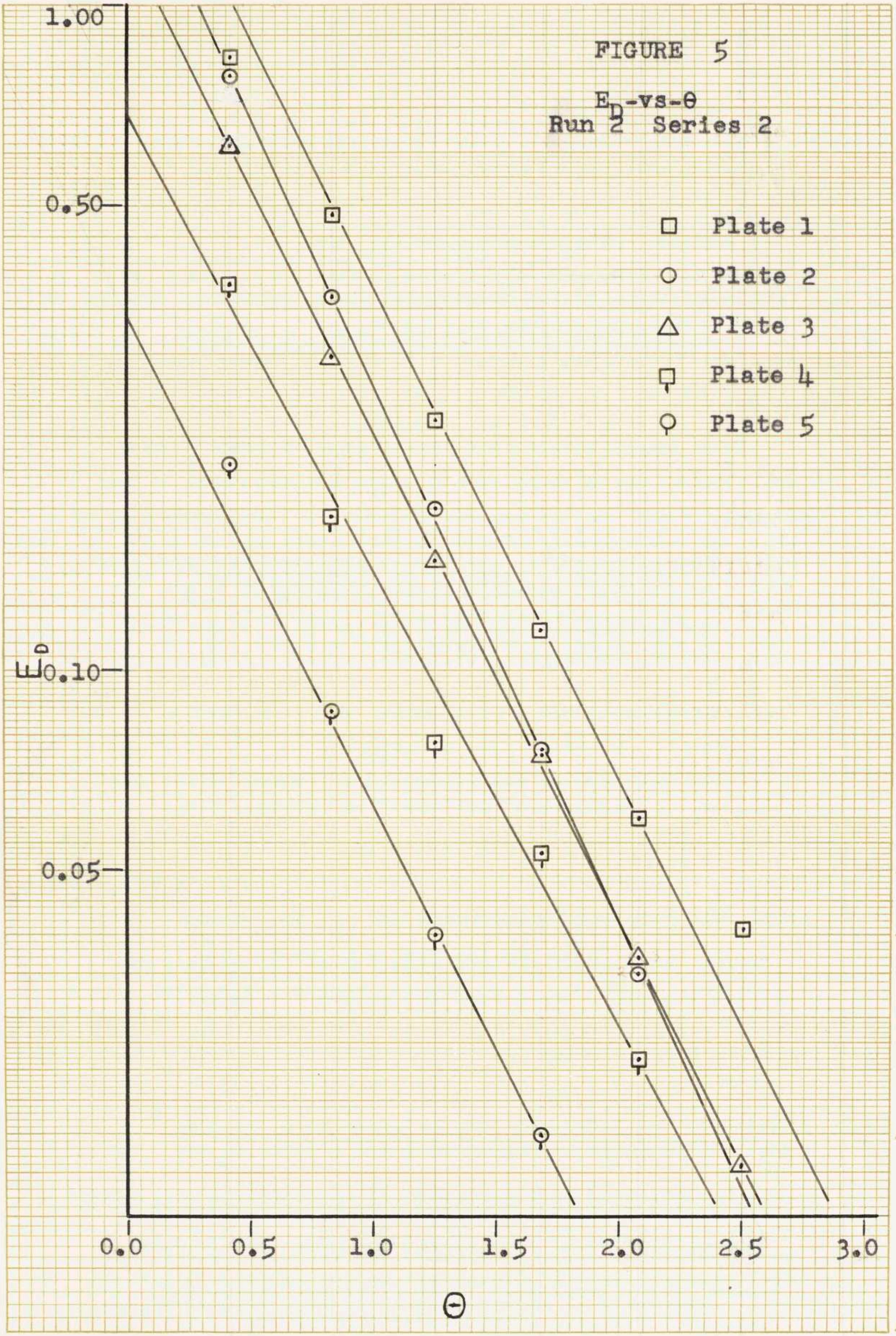


FIGURE 5
E_D-vs-θ
Run 2 Series 2
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Θ

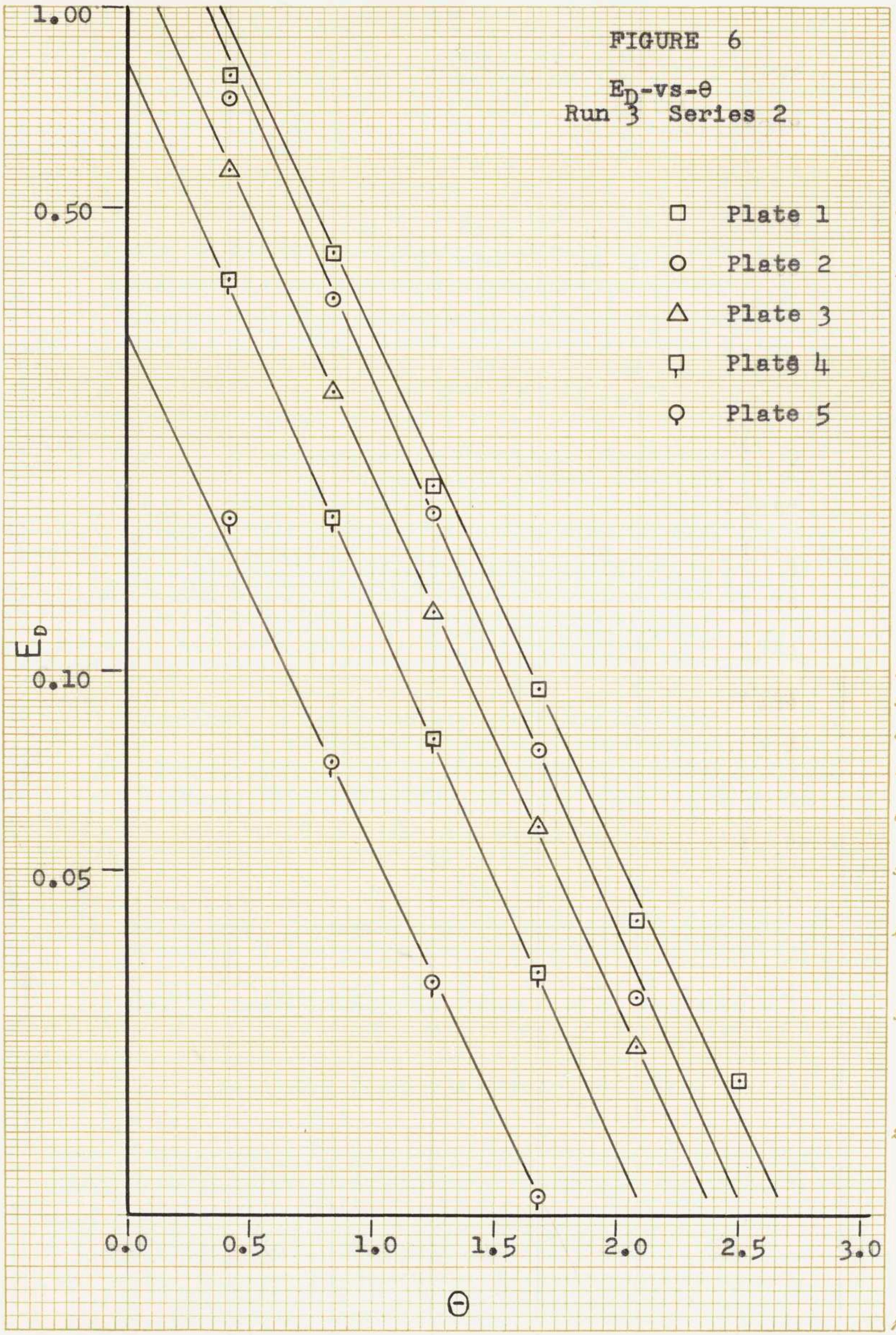


FIGURE 6
E_D-vs-θ
Run 3 Series 2
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Θ

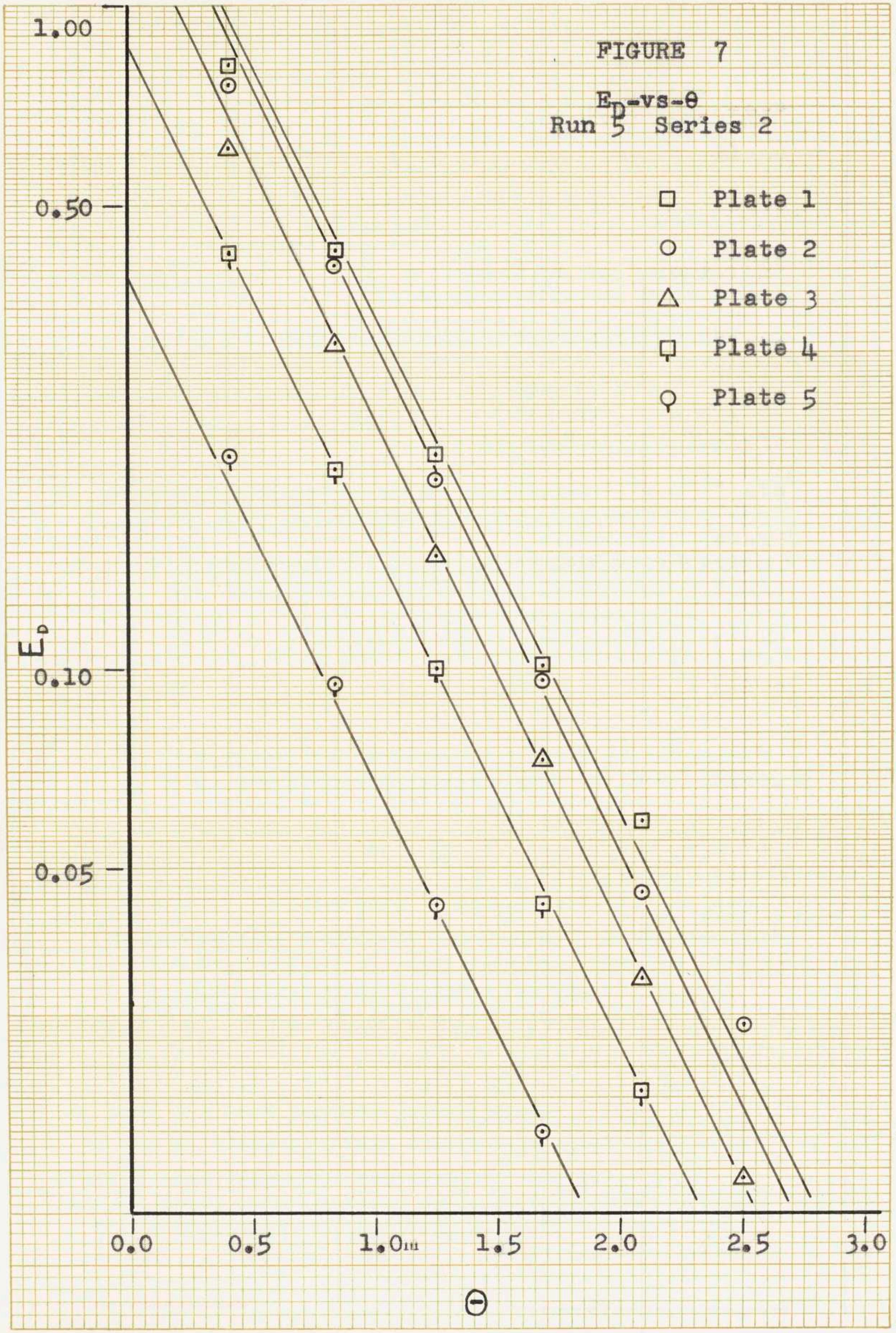


FIGURE 7
E_D-vs-θ
Run 5 Series 2
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Θ

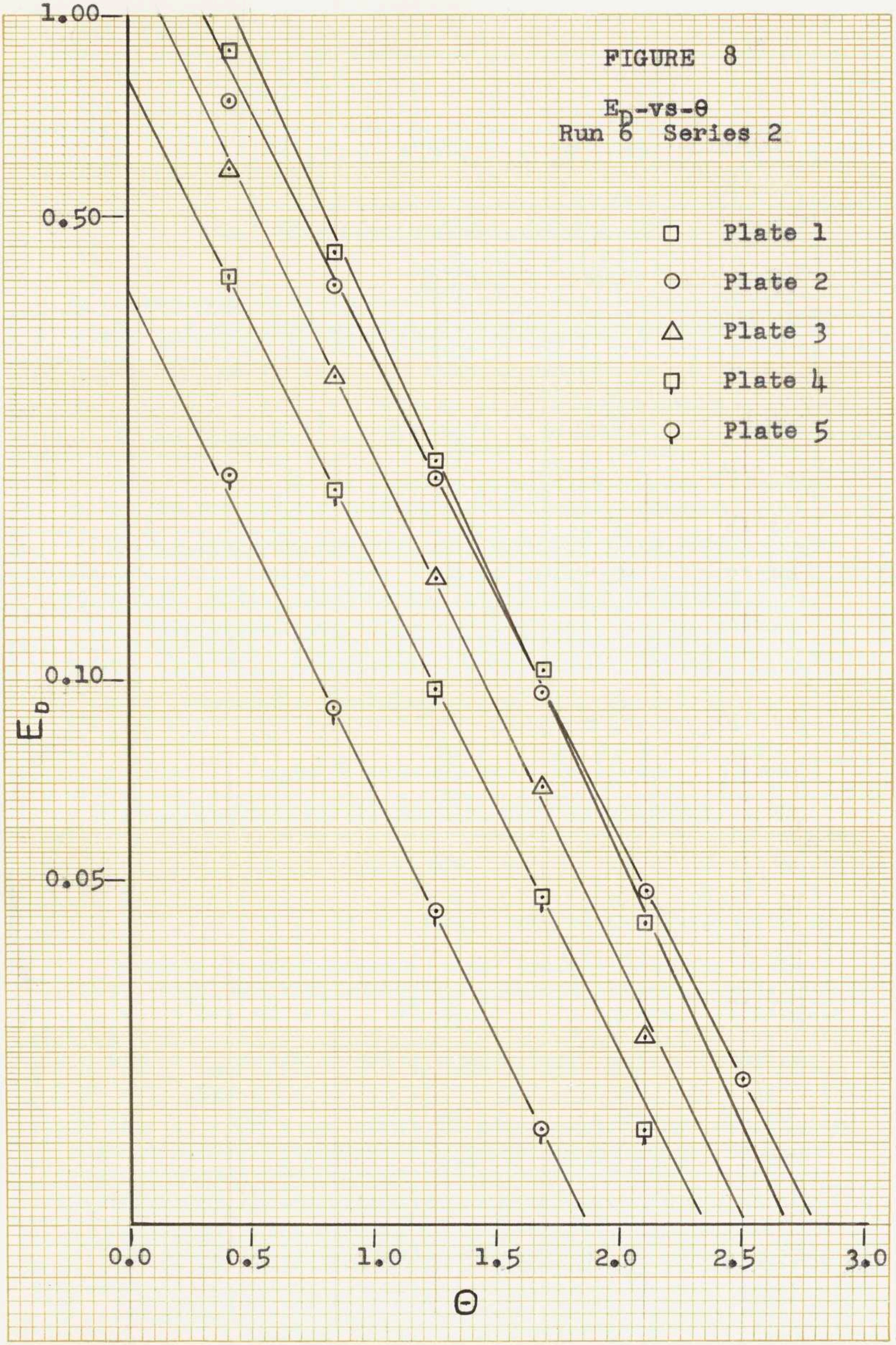


FIGURE 8
E_D-vs-θ
Run 6 Series 2
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Θ

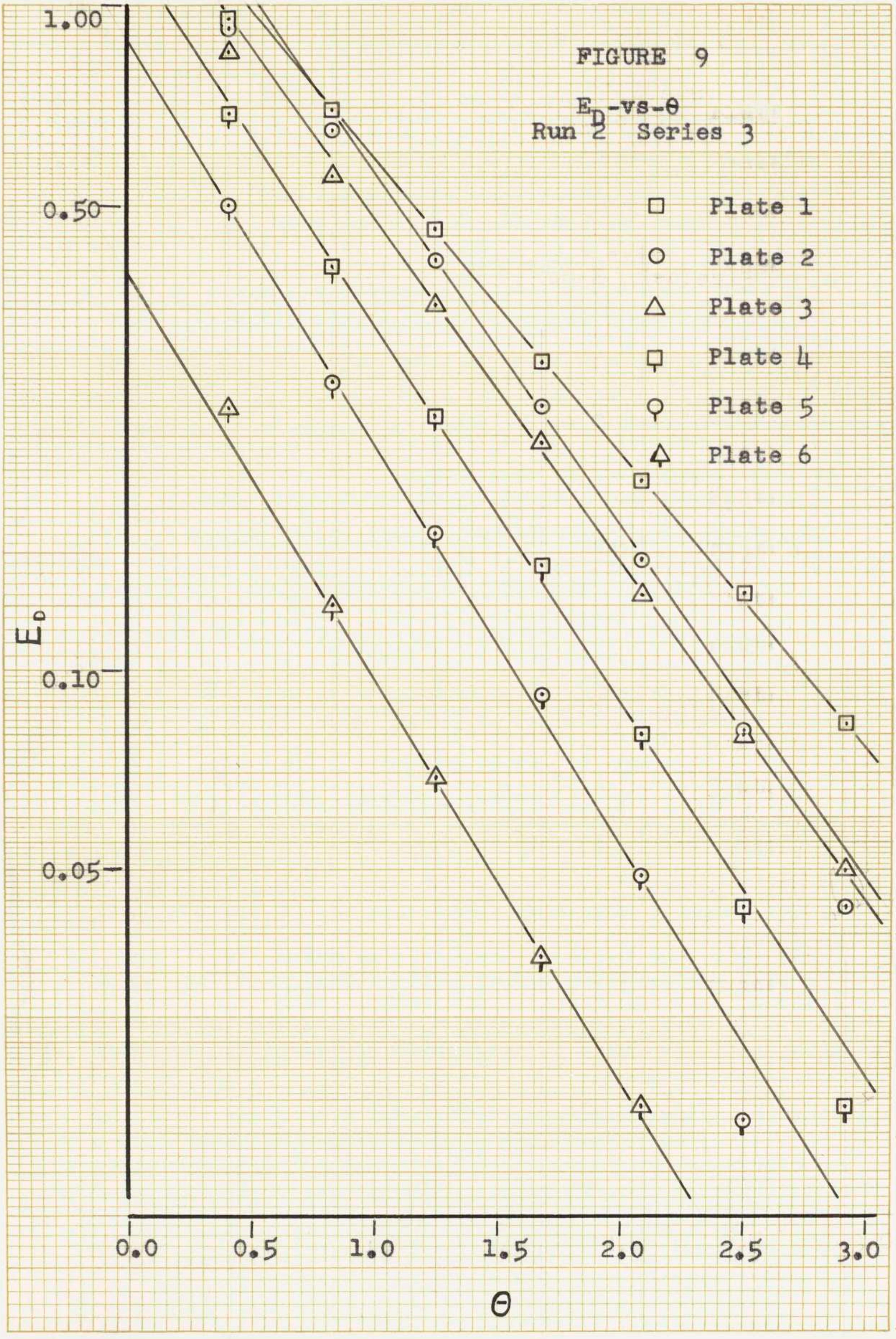


FIGURE 9
E_D-vs-θ
Run 2 Series 3
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

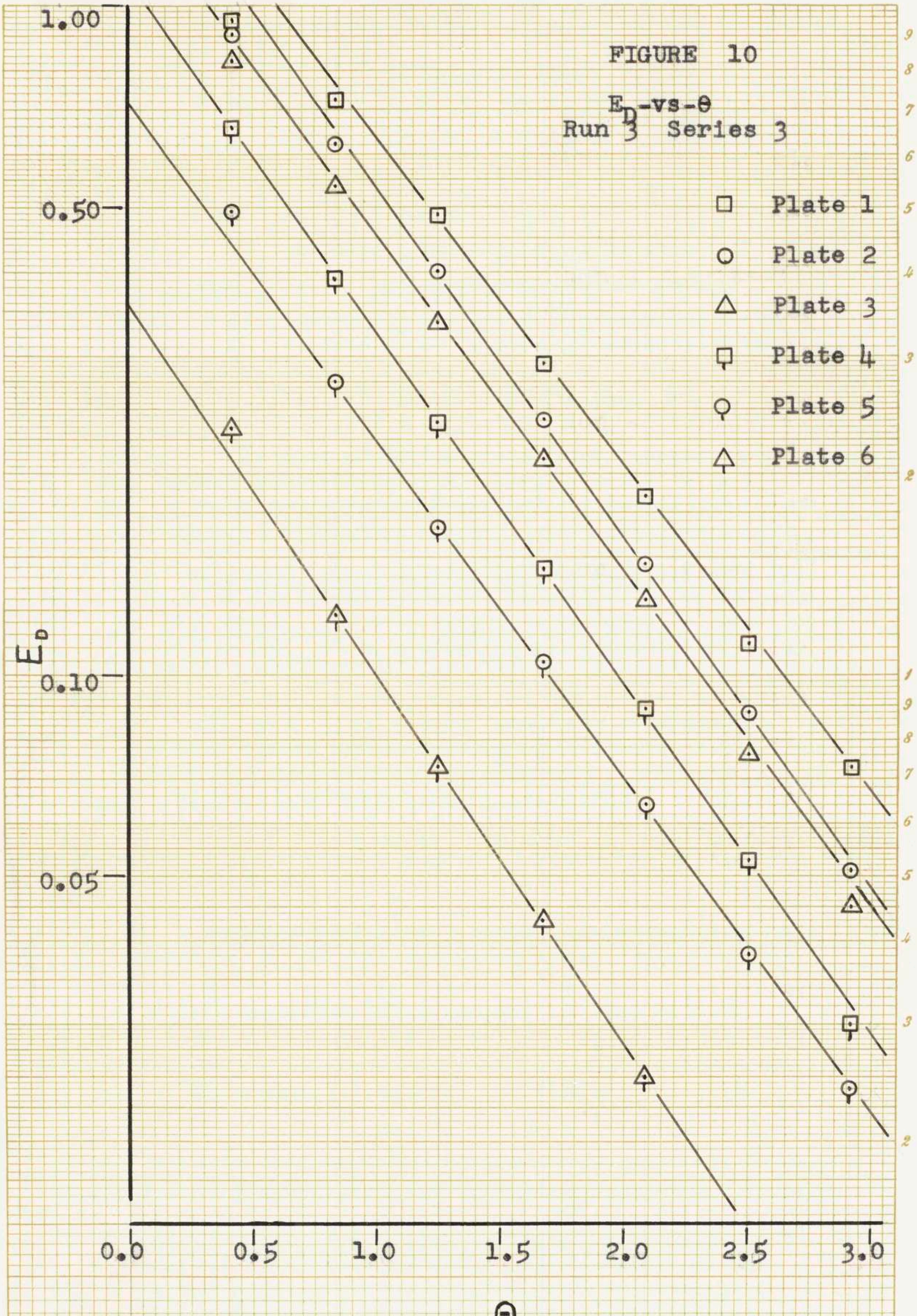


FIGURE 10
E_D-vs-θ
Run 3 Series 3
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Θ

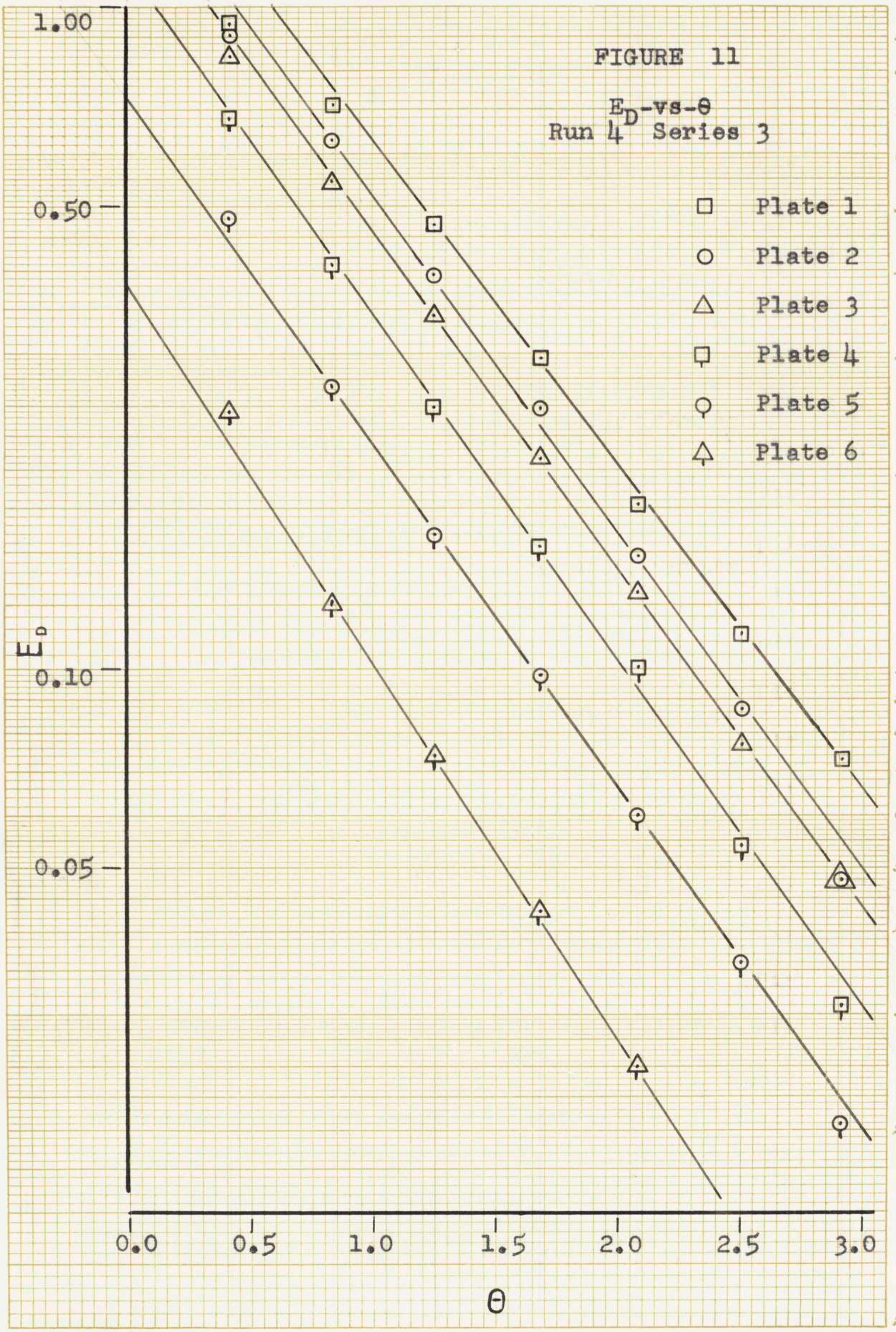


FIGURE 11
E_D-vs-θ
Run 4 Series 3
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

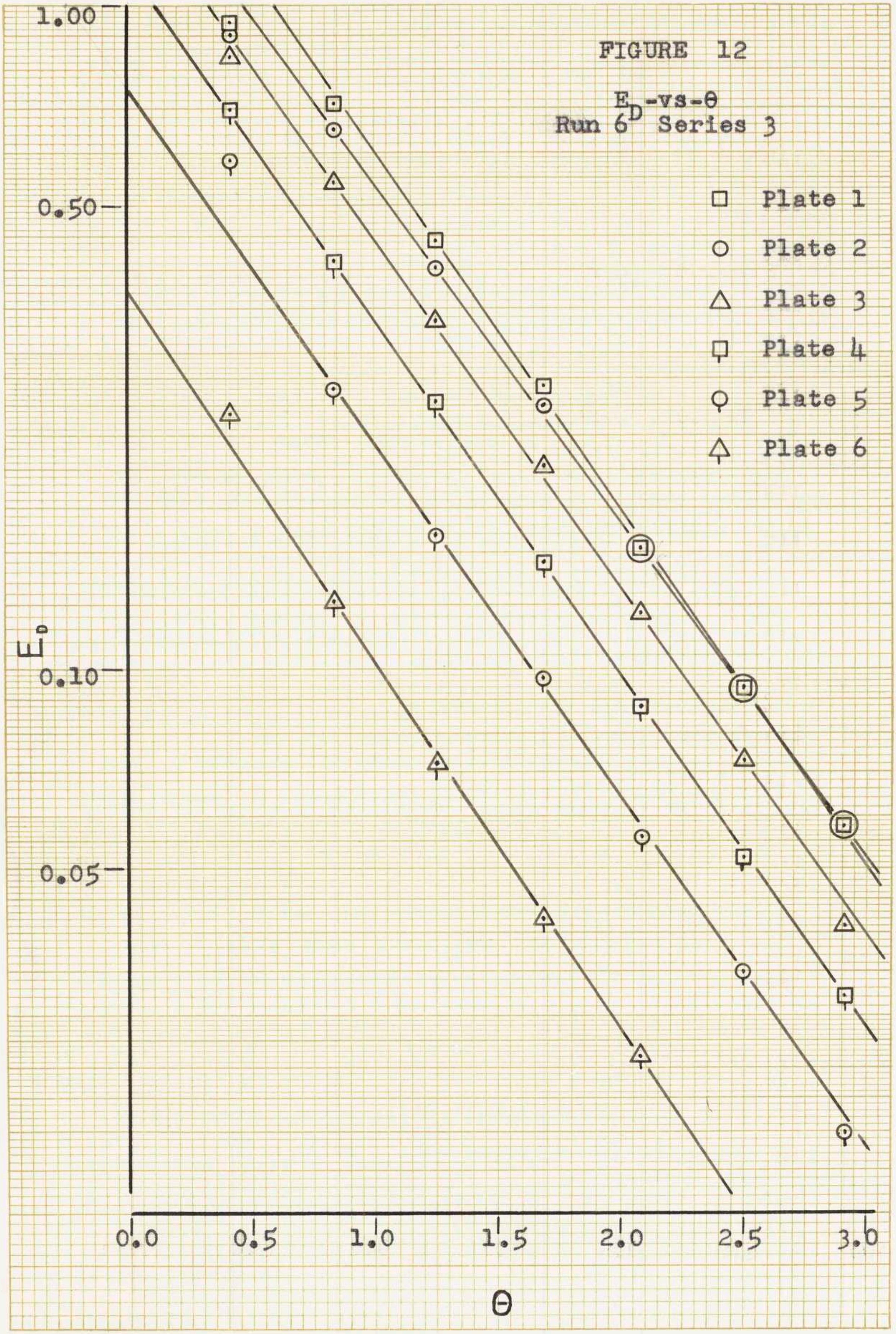


FIGURE 12
E_D-vs-θ
Run 6 Series 3
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

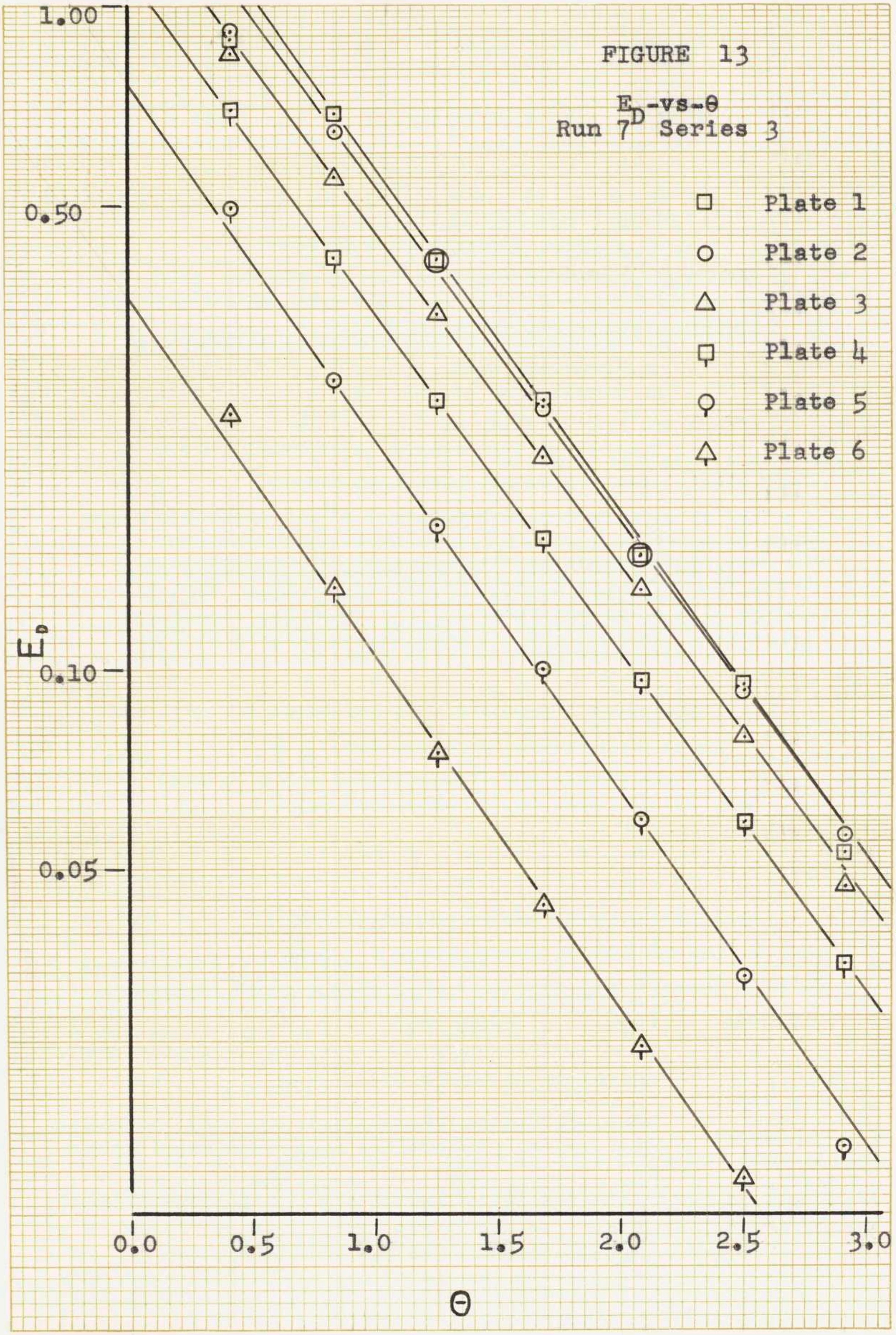


FIGURE 13
E_D-vs-θ
Run 7 Series 3
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

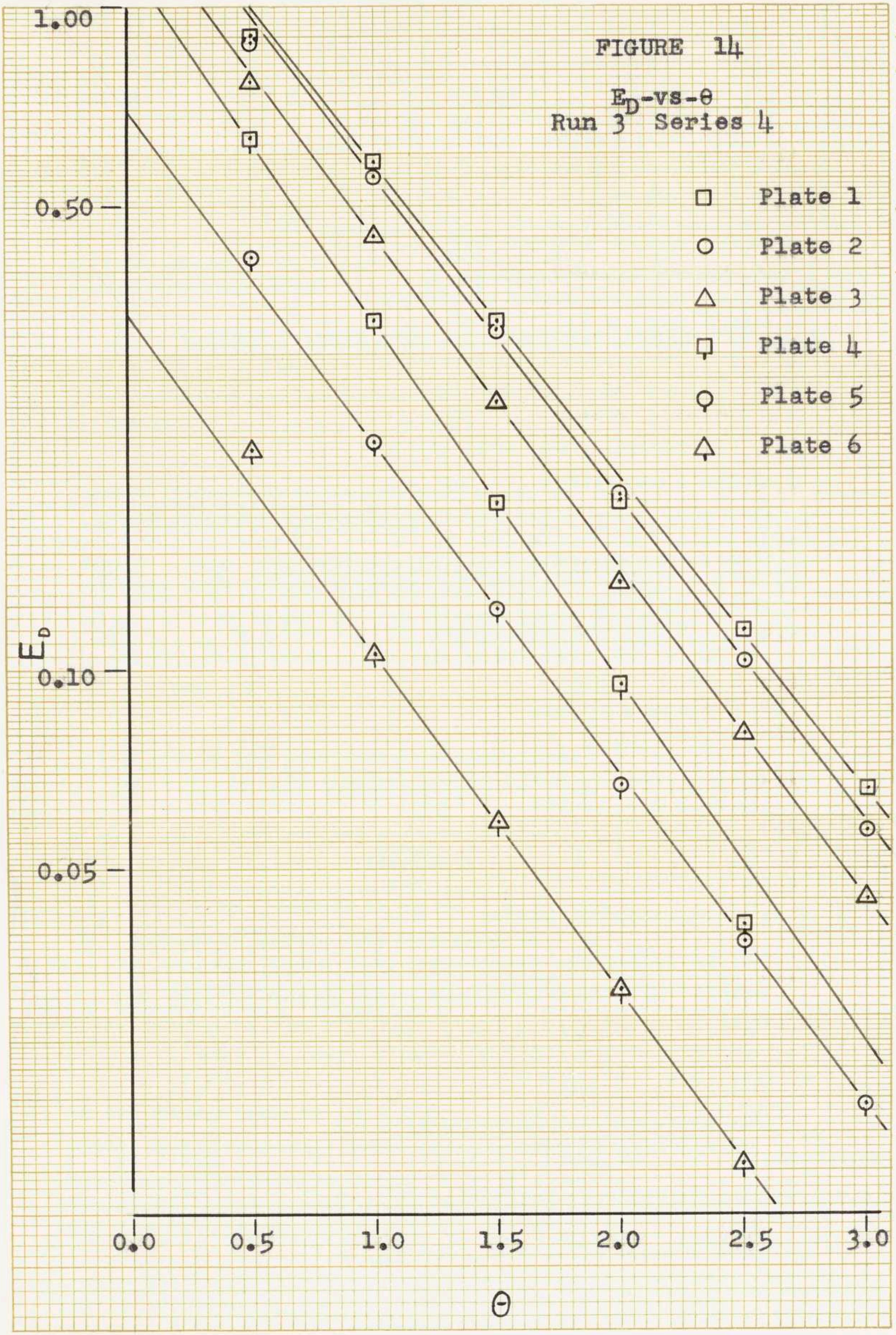


FIGURE 14
E_D-vs-θ
Run 3 Series 4
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

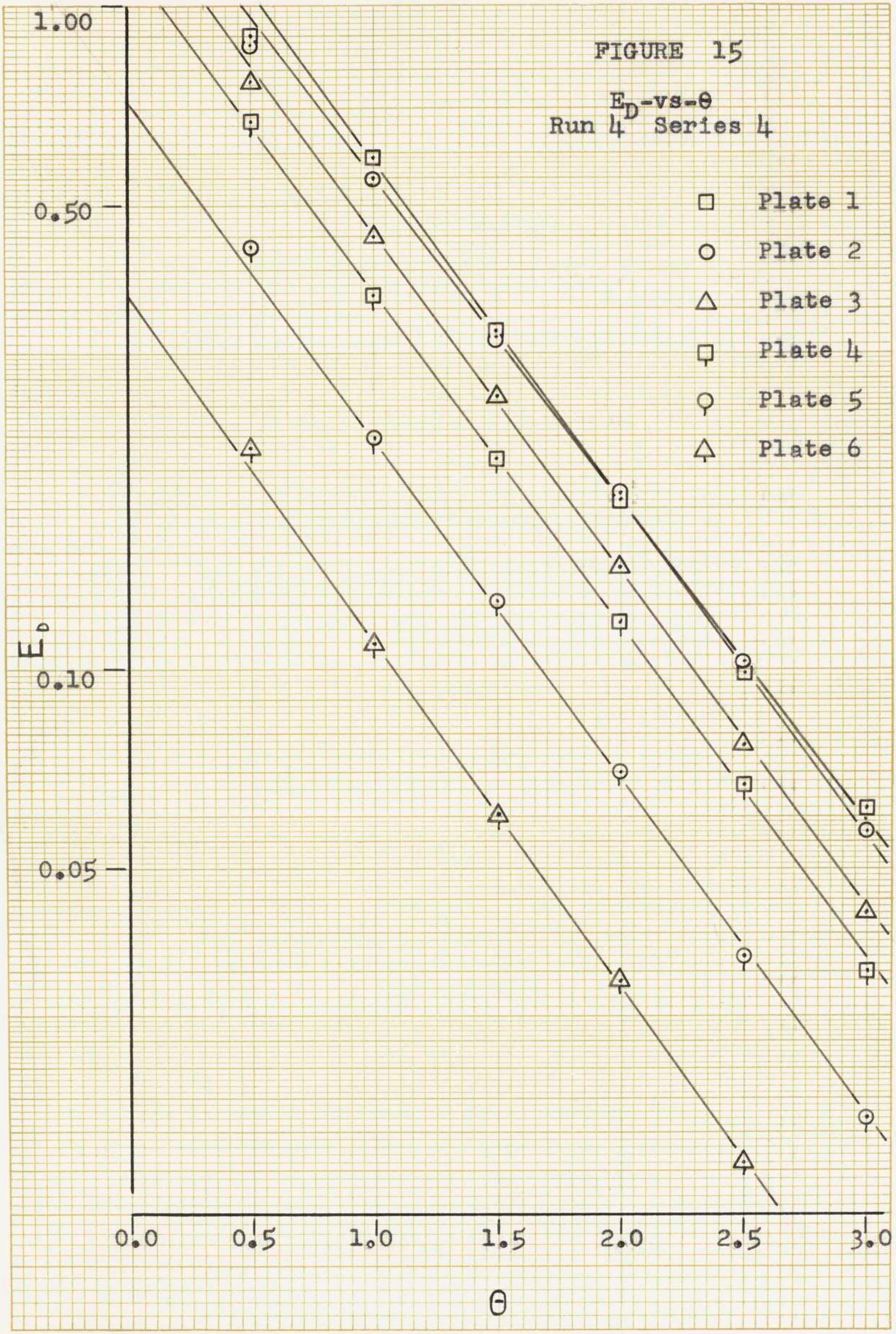


FIGURE 15
E_D-vs-θ
Run 4 Series 4
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

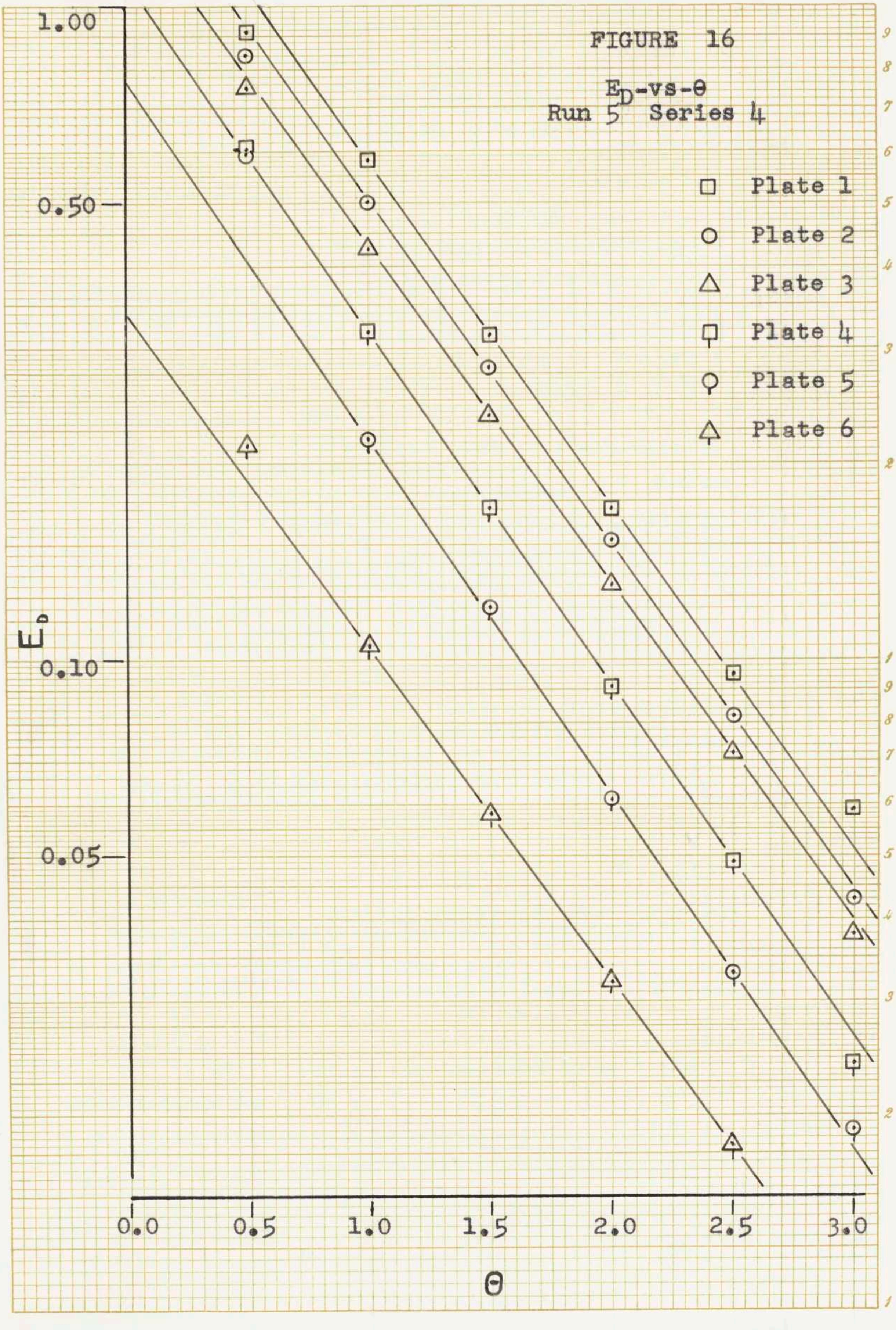


FIGURE 16
E_D-vs-θ
Run 5 Series 4
□ Plate 1
○ Plate 2
△ Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

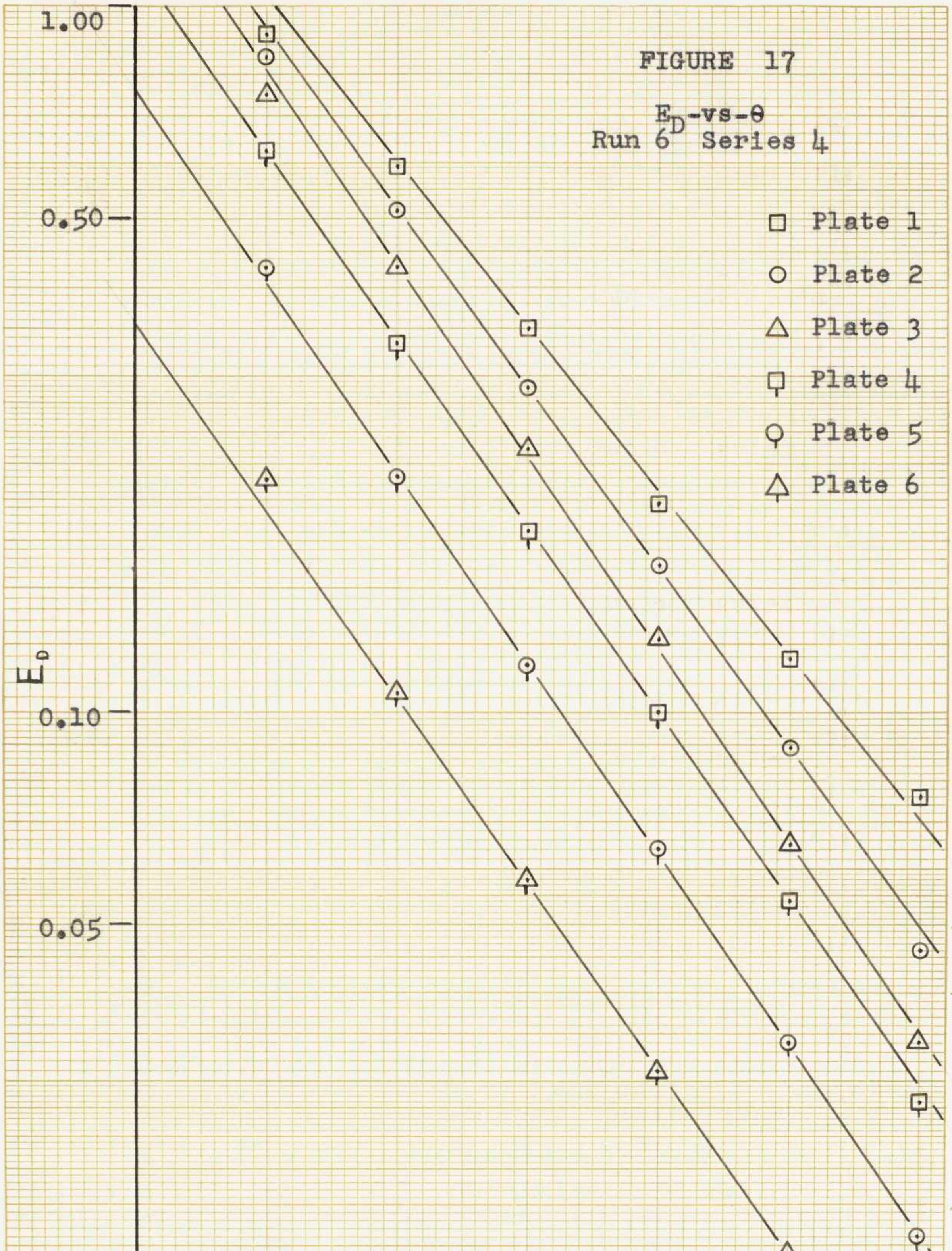


FIGURE 17
E_D-vs-θ
Run 6 Series 4
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

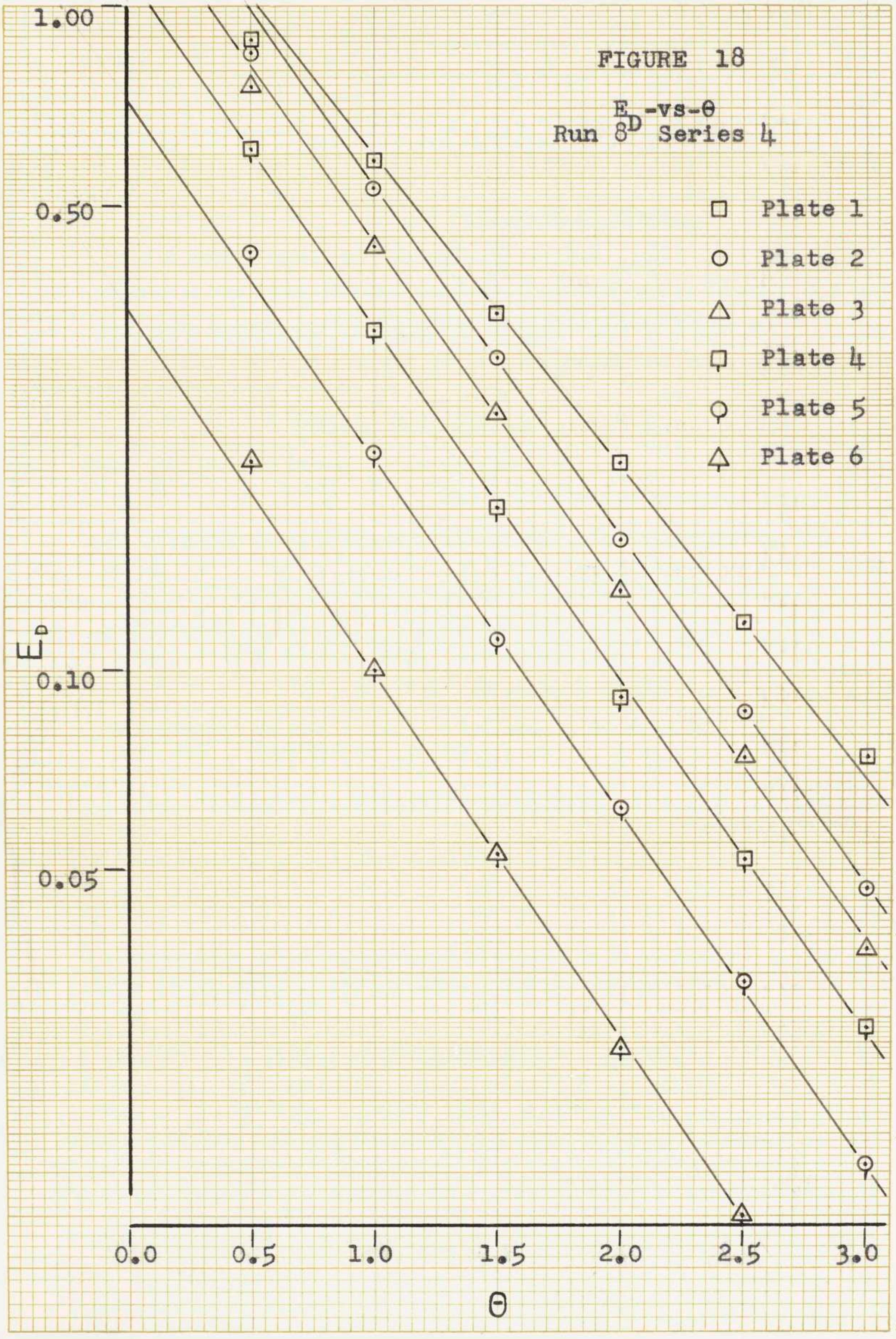


FIGURE 18
E_D-vs-θ
Run 8 Series 4
Plate 1
Plate 2
Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

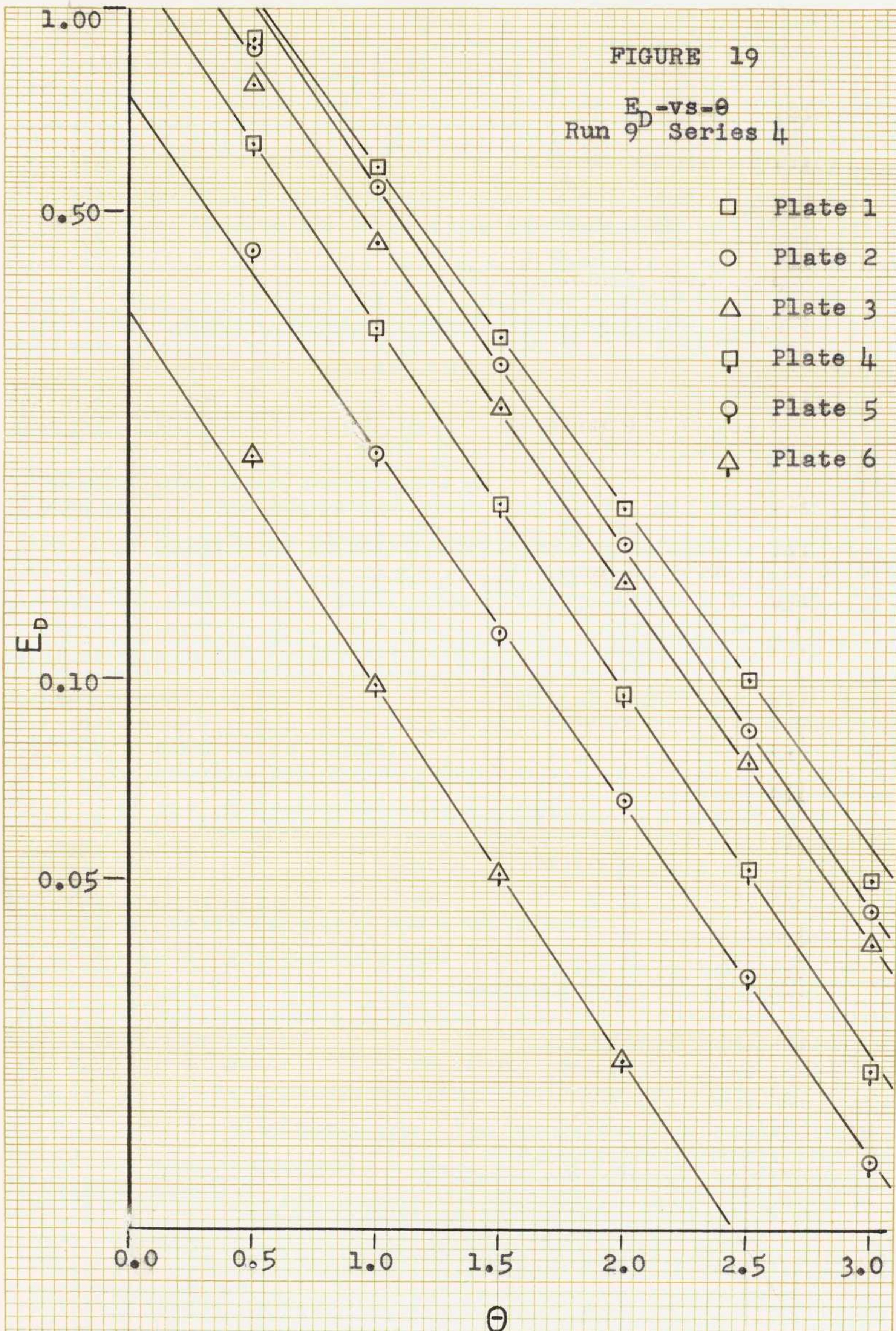


FIGURE 19
E_D-vs-θ
Run 9 Series 4
□ Plate 1
○ Plate 2
△ Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

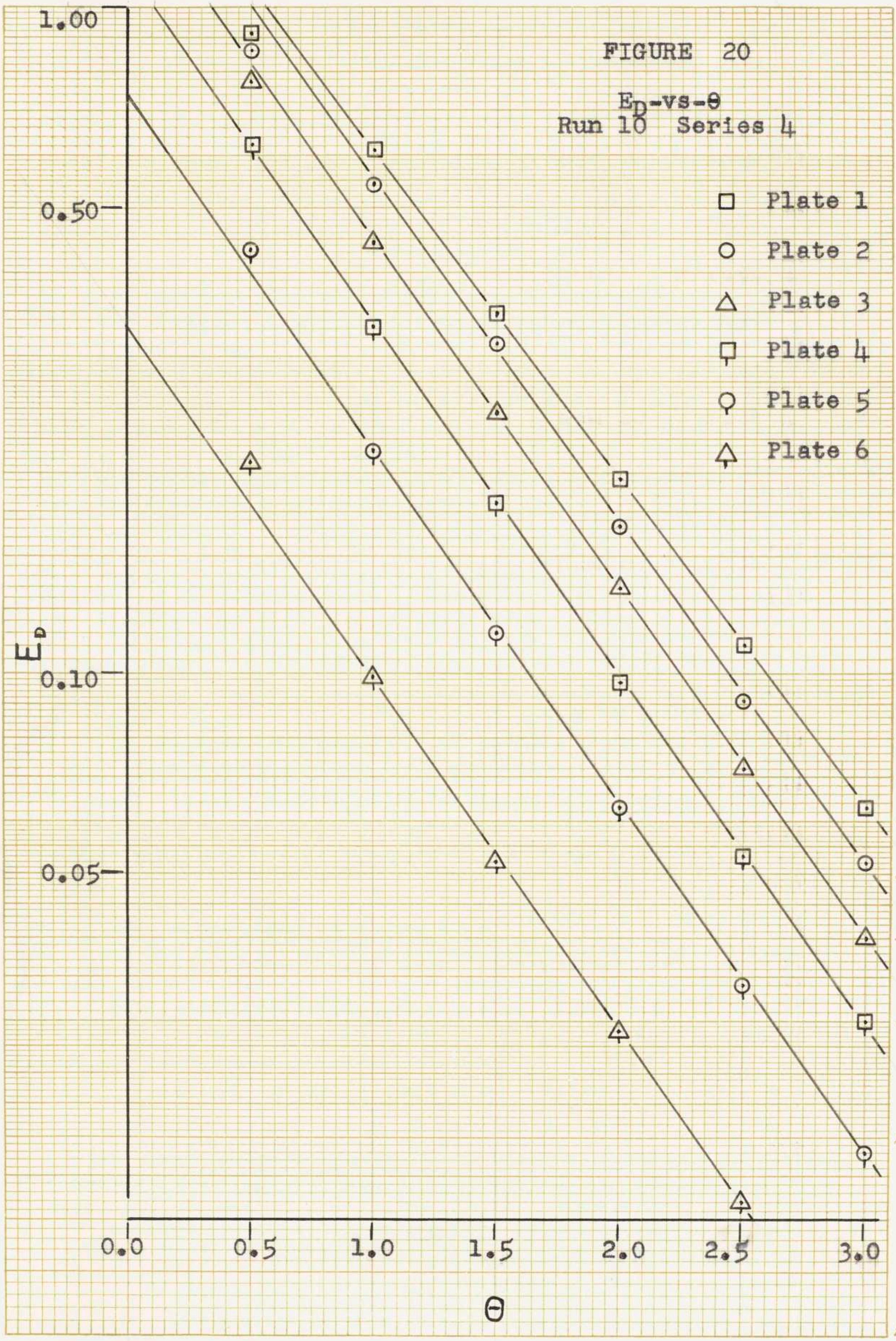


FIGURE 20
E_D-vs-θ
Run 10 Series 4
□ Plate 1
○ Plate 2
△ Plate 3
Plate 4
Plate 5
Plate 6
1.00
0.50
0.10
0.05
E_D
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ

the slopes of the straight lines on the log E_D plots are identical. Each plate, however, has a different time lag (D) in this simplified approximation.

For every plate the time constant (τ) which is the time required for the exponential to fall to $\frac{1}{e}$(or 0.368) of its initial value, and the lag or jump (D) were measured from Figures 4 to 20. Figures 21, 22, and 23 show the distribution of τ for each plate in each run of the three series. On any plate of the absorber, the time constants are randomly distributed for different step changes. Furthermore, the averages of τ for all plates in a series appear to be nearly identical and randomly distributed, i.e. the exponents are the same throughout the absorber regardless of the size or direction of changes in the vapor input concentration.

Figures 24, 25, and 26 show the lag times of each plate for every run in the three series. From these graphs it appears that the lag time of any plate although unique is not changed by differences in the feed change.

Because of the distributions, the average of all the time constants in each series, and the average of the lags for each plate in all runs of each series was taken as a best estimate of the true values of these characteristics.

FIGURE 21

Distribution of τ
Series 2

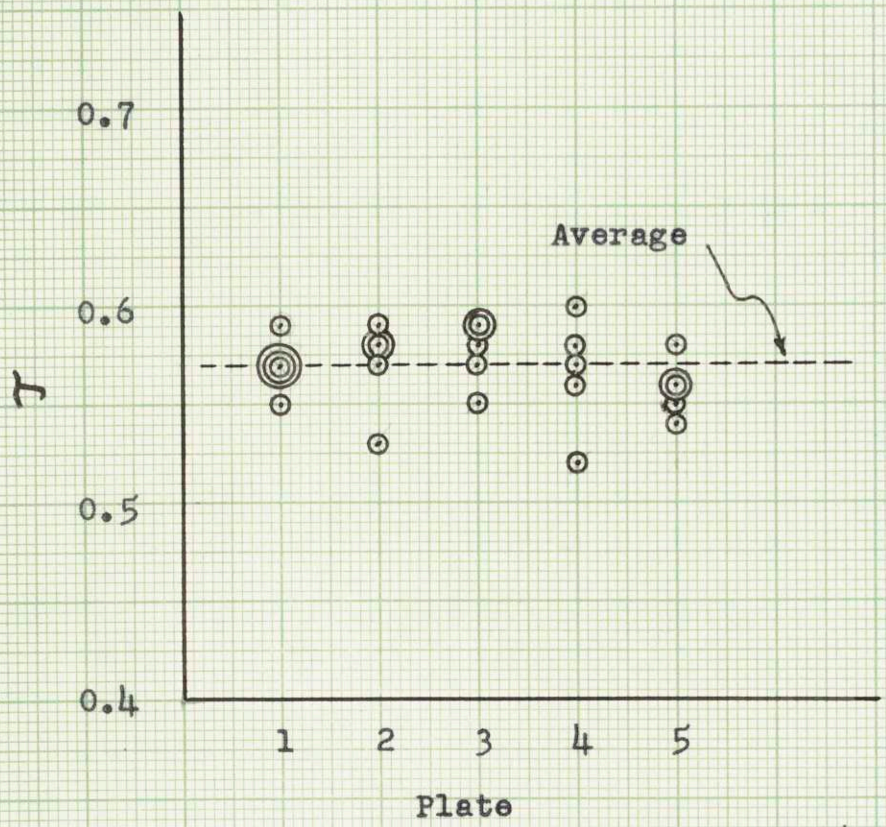
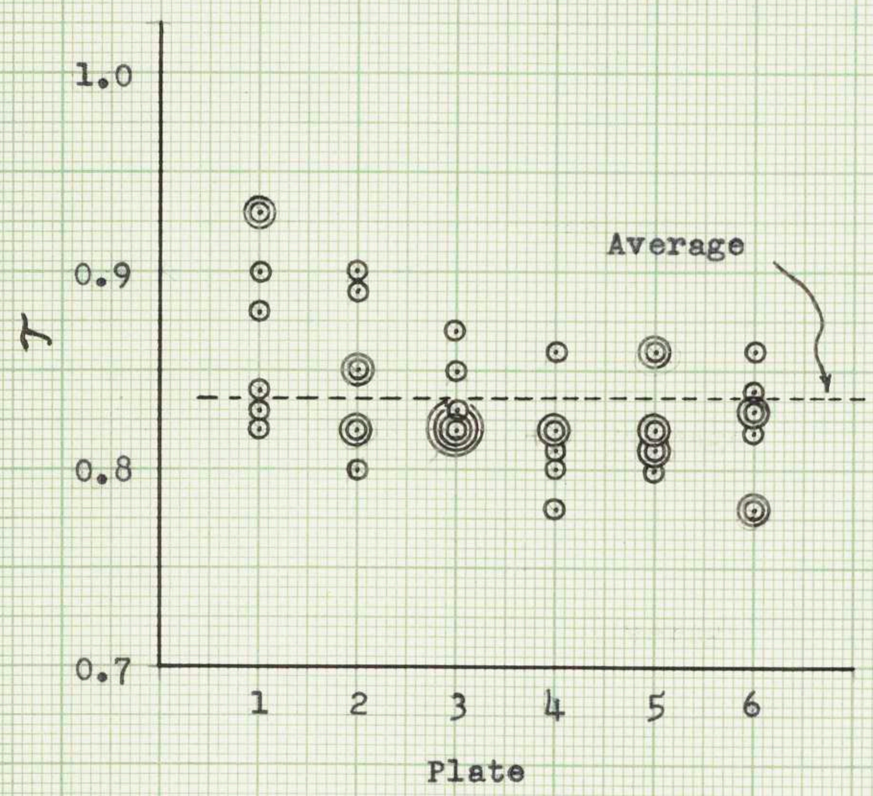


FIGURE 22

Distribution of τ
Series 3

FIGURE 23

Distribution of τ
Series 4



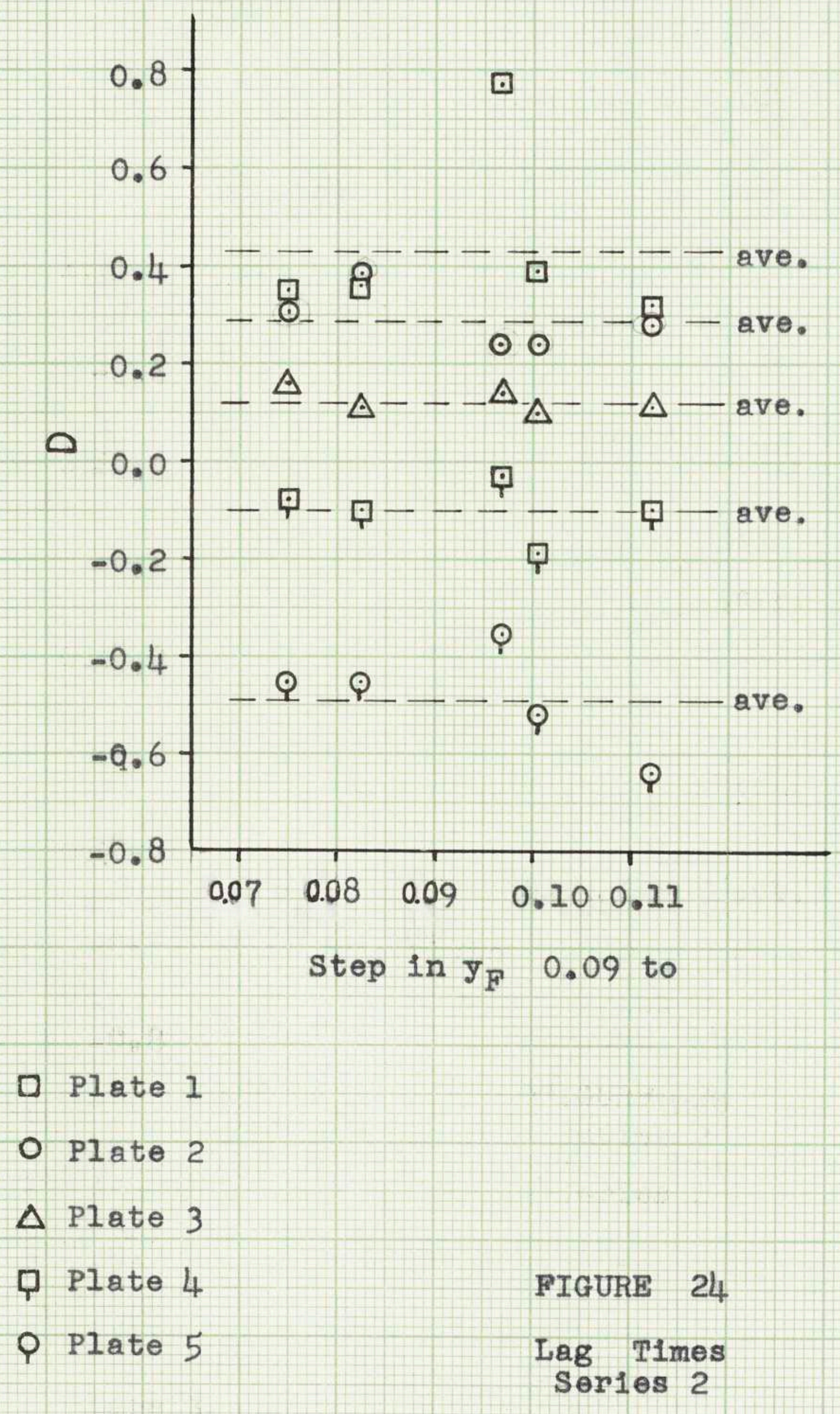


FIGURE 24

Lag Times
Series 2

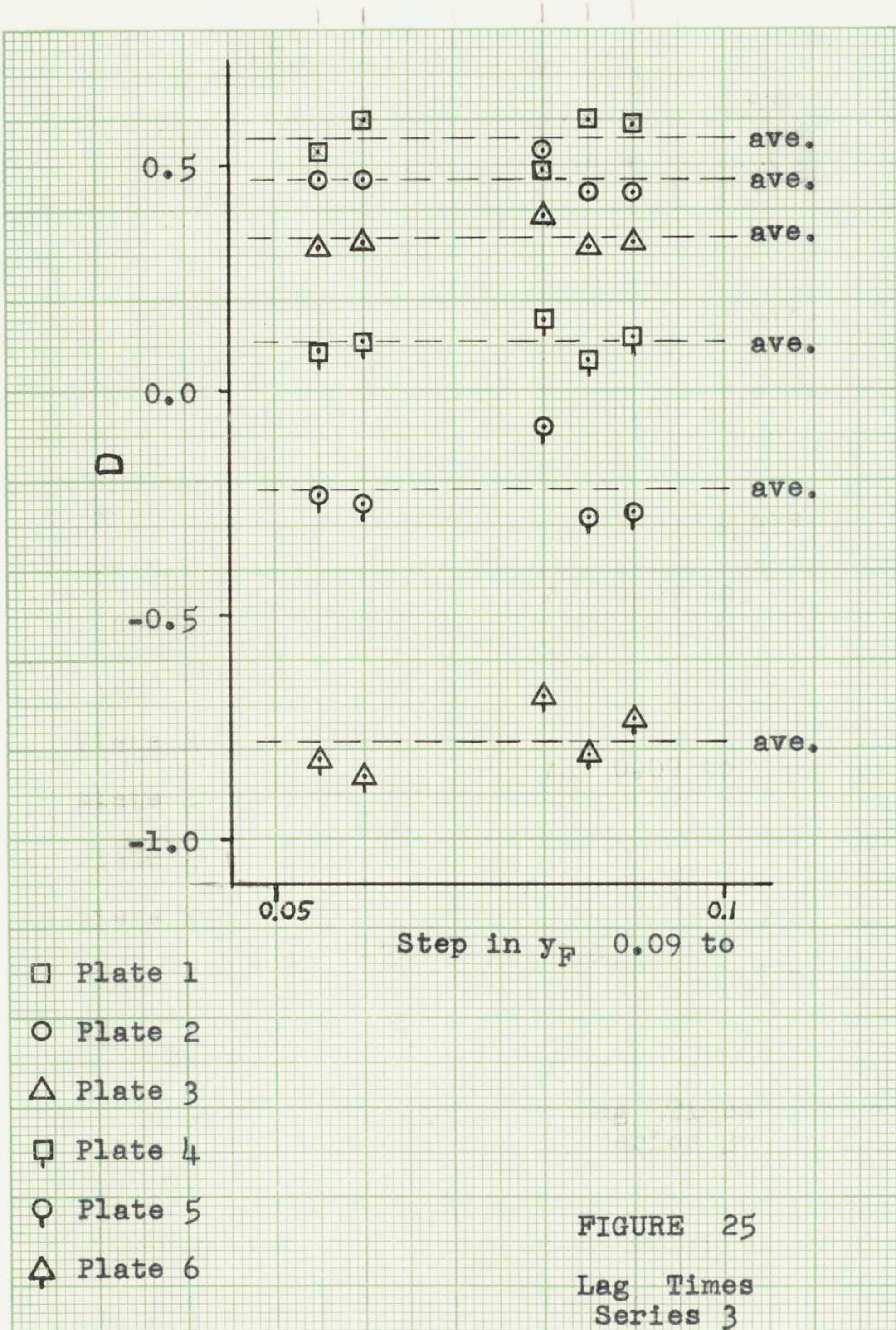


FIGURE 25

Lag Times
Series 3

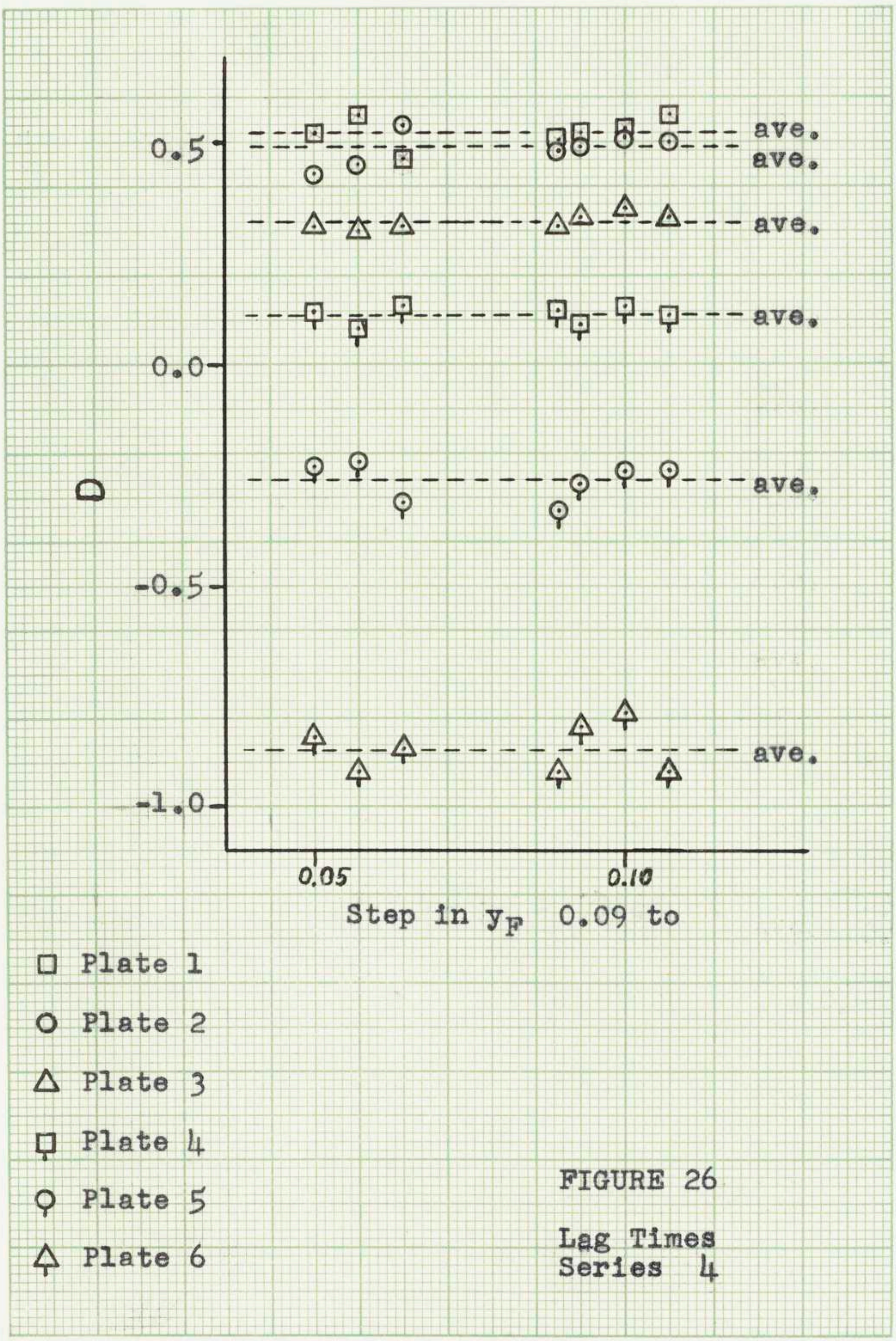


FIGURE 26

Lag Times
Series 4

Table II is a tabular summary of the average experimental lags and time constants.

TABLE II

Summary of Average Values for τ and D

	Series 2	Series 3	Series 4
τ	0.57	0.83	0.84
D_1^*	0.43	0.56	0.52
D_2	0.29	0.47	0.49
D_3	0.12	0.34	0.32
D_4	-0.10	-0.11	-0.11
D_5	-0.49	-0.22	-0.26
D_6	----	-0.78	-0.87

* Subscripts used to denote plate number

V. DISCUSSION OF RESULTS

A. Factors Affecting The Transient Response

As was explained above the time dependent response of each plate in these absorbers may be approximated with a time lag of either sign followed by an exponential decay. It was also pointed out that τ and D were independent of the size of the step in vapor feed concentration (see Figures 21 to 26), and that τ was the same for any plate on an absorber. Referring to Table II it is possible to make tentative conclusions about the effects of L, V, Δy_F, and N on τ or D. Table III lists these qualitative effects.

TABLE III

Effect of Parameters On τ and D

The Effect of Increasing	On τ	On D
N	increase	increase overall spread
y_F	no effect	no effect
$\frac{L}{V}$	Apparently no effect	apparently no effect

It should be pointed out that the value of $\frac{L}{V}$ in Series 4 was only slightly different from that used in Series 2 and 3, so it is difficult to appraise the effect of this change on τ and D.

V. DISCUSSION OF RESULTS

B. Comparison Of Results With The Analytical Solution

The solution to a linear absorber's equations (2) has been developed by Lapidus and Amundson (5). Their solution is given as:

$$X_n(\theta) = \frac{-2}{N+1} \sum_{k=1}^{N} (-1)^k \sin \frac{\pi k(N-n+1)}{N+1} \sum_{j=1}^{n} A_j (\sqrt{c})^{n-j} \sin \frac{\pi k j}{N+1}\, e^{s_k \theta}$$

$$- \frac{2}{N+1} \sum_{k=1}^{N} (-1)^k \sin \frac{\pi k n}{N+1} \sum_{j=n+1}^{N} A_j (\sqrt{c})^{n-j} \sin \frac{\pi k (N-j+1)}{N+1}\, e^{s_k \theta}$$

$$- \frac{2(\sqrt{c})^{n-N}}{d(N+1)} \int_0^{\theta} \left[\frac{g(\theta-b)}{K}\right] \left[\sum_{1}^{N} (-1)^k \sin \frac{\pi n k}{N+1} \sin \frac{\pi k}{N+1}\, e^{s_k b} \right] db$$

$$- \frac{2(\sqrt{c})^{n+1}}{d(N+1)} \int_0^{\theta} f(\theta-b) \left[\sum_{1}^{N} (-1)^k \sin \frac{\pi k}{N+1} \sin \frac{\pi k(N-n+1)}{N+1}\, e^{s_k b} \right] db \qquad (8)$$

where:

$$s_k = \frac{-1}{d} \left[c+1-2\sqrt{c} \cos \frac{\pi k}{N+1} \right] \quad k=1,2,\cdots,N \qquad (9)$$

$$c = \frac{L}{VK}$$

$$d = \frac{h}{VK}$$

$$y_n = K x_n$$

$$A_j = x_j(0)$$

$$f(\theta) = x_0(\theta)$$

$$g(\theta) = y_F(\theta)$$

When $f(\theta)=0$ and $g(\theta)=$ Constant, as is the case in this thesis, Equation 8 reduces to:

$$X_n(\theta) = \frac{-2}{N+1} \sum_{k=1}^{N} (-1)^k \sin \frac{\pi k(N-n+1)}{N+1} \sum_{j=1}^{n} A_j (\sqrt{c})^{n-j} \sin \frac{\pi k j}{N+1} e^{s_k \theta}$$

$$- \frac{+2}{N+1} \sum_{k=1}^{N} (-1)^k \sin \frac{\pi k n}{N+1} \sum_{j=n+1}^{N} A_j (\sqrt{c})^{n-j} \sin \frac{\pi k(N-j+1)}{N+1} e^{s_k \theta}$$

$$- \frac{2(\sqrt{c})^{n-N}}{d(N+1)} \left(\frac{Y_F(\theta>0)}{K}\right) \sum_{1}^{N} (-1)^k \sin \frac{\pi n k}{N+1} \sin \frac{\pi k}{N+1} \left(\frac{e^{s_k \theta}-1}{s_k}\right) \qquad (10)$$

When the series are expanded, this expression is a sum of N exponentials whose time constants are determined by s_k;

$$\tau = \frac{-1}{s_k} \qquad (11)$$

The experimental results indicated that one exponential in the solution was predominant. In order to determine which s_k corresponded to the main exponential, the experimental time constants were compared with the set of s_k's for each run. Table IV lists the results of this comparison.

From Table IV it can be seen that the experimental time constants correspond most closely to the theoretical value of τ for k=1. Unfortunately, the diversity in conditions studied was not great enough to provide a more exact evaluation of this correspondence.

It is valuable, however, to examine the nature of Equation 10 as though k=1 were the only term in the expansions. This assumption is reasonably valid in that the small exponents require all other terms to approach zero rapidly.

TABLE IV

Comparison of Experimental Time Constant With Values of s_k

k	Series 2 $\tau = 0.569$ $\frac{-1}{S_K}$	Series 3 $\tau = 0.832$ $\frac{-1}{S_K}$	Series 4 $\tau = 0.836$ $\frac{-1}{S_K}$
1	0.746	1.000	0.910
2	0.200	0.270	0.260
3	0.100	0.130	0.120
4	0.067	0.082	0.074
5	0.054	0.061	0.056
6	-----	0.058	0.053

If the only allowable value of k in the exponents is one, Equation 10 becomes:

$$X_n(\theta) = \left[\frac{2}{N+1} \sin\frac{\pi(N-n+1)}{N+1} \sum_{j=1}^{n} A_j(\sqrt{c})^{n-j} \sin\frac{\pi j}{N+1}\right] e^{s_1\theta}$$

$$+ \frac{2}{N+1}\left[\sin\frac{\pi n}{N+1} \sum_{j=1+n}^{N} A_j(\sqrt{c})^{n-j} \sin\frac{\pi(N-j+1)}{N+1}\right] e^{s_1\theta}$$

$$- \frac{2(\sqrt{c})^{n-N}}{d(N+1)}\left(\frac{Y_F(\theta>0)}{K}\right)\left[\sin\frac{\pi n}{N+1} \sin\frac{\pi}{N+1}\right]\frac{e^{s_1\theta}}{-s_1}$$

$$+ \frac{2(\sqrt{c})^{n-N}}{d(N+1)}\left(\frac{Y_F(\theta>0)}{K}\right) \sum_{1}^{N} (-1)^k \sin\frac{\pi n k}{N+1} \sin\frac{\pi k}{N+1} \left(\frac{1}{+s_k}\right) \qquad (11)$$

Where all of the k's must be evaluated in the non-exponential integration constant. When $\theta = 0$ this expression becomes:

$$X_n(0) = \left[\frac{2}{N+1} \sin\frac{\pi(N-n+1)}{N+1} \sum_{j=1}^{n} A_j(\sqrt{c})^{n-j} \sin\frac{\pi j}{N+1}\right] + \left[\frac{2}{N+1} \sin\frac{\pi n}{N+1} \sum_{j=n+1}^{N} A_j(\sqrt{c})^{n-j} \sin\frac{\pi(N-j+1)}{N+1}\right]$$

$$-\frac{2(\sqrt{c})^{n-N}}{d(N+1)}\left(\frac{Y_F(\theta>0)}{K}\right)\left[\sin\frac{\pi n}{N+1} \sin\frac{\pi}{N+1}\right]\frac{1}{-s_1} + \frac{2(\sqrt{c})^{n-N}}{d(N+1)}\left[\frac{Y_F(\theta>0)}{K}\right]\sum_{1}^{N}(-1)^k \sin\frac{\pi n k}{N+1} \sin\frac{\pi k}{N+1}\left(\frac{1}{s_K}\right) \qquad (12)$$

As θ approaches infinity the value of x_n from Equation 11 becomes:

$$x_n(\infty) = \frac{2(\sqrt{c})^{n-N}}{d(N+1)} \left[\frac{y_F(\theta>0)}{K}\right] \sum_{1}^{N} (-1)^k \sin\frac{\pi n k}{N+1} \sin\frac{\pi k}{N+1} \left(\frac{1}{s_k}\right) \qquad (13)$$

The path taken by x_n between these two extremes is determined by the exponential $e^{s_1\theta}$. Thus it appears that the predominate exponential in the step transient has an exponent which corresponds to Equation 9 for $k = 1$,

$$s_1 = \frac{-VK}{h}\left[\frac{L}{VK} + 1 - 2\sqrt{\frac{L}{VK}} \cos\frac{\pi}{N+1}\right] \qquad (14)$$

Since Equation 11 (and Equation 10) reduce to Equation 13 as θ goes to infinity, it would be expected that it gives the final equilibrium value of x_n. Evaluations of Equation 13 for selected plates in Series 3 are compared with final steady-state values in Table V.

TABLE V

Comparison Between $x_n(\infty)$ From Steady-state Analysis and Equation 13

(Series 3, Run 2)

Plate	$x_n(\infty)$ Steady-State	$x_n(\infty)$ Equation 13
1	0.056	0.055
3	0.173	0.167
6	0.345	0.342

It appears that the initial values of x_n given by Equation 12 are not the initial steady-state values. Table VI compares the values of $x_n(0)$ given by steady-state analysis and Equation 12.

TABLE VI

Comparison Between $x_n(0)$ From
Steady-State Analysis and Equation 11

(Series 3, Run 2)

Plate	$x_n(0)$ Steady-State	$x_n(0)$ Equation 11
1	0.05	0.0421
3	0.15	0.1386
6	0.30	0.3289

The differences presented in Table VI may be explained as part of the lag producing mechanism. The value of Equation 12 for the first plate is smaller than the steady-state value. It can be assumed that the other exponentials counteract this main term (summing to zero) until enough time has elapsed to make it equal to the initial steady-state value. At this point it becomes the predominate term and provides a single exponential path to the final value.

V. DISCUSSION OF RESULTS

C. Accuracy of Results

The accuracy obtainable in the final values of τ and D for each plate depends primarily on three factors, static accuracy of the computer, dynamic accuracy of the computer, and graphical accuracy in measuring the computer data.

Static accuracy in the computer can be divided into two significant catigories, individual component accuracy and overall power supply accuracy. As in most electronic equipment precision of components in the REAC cannot be expected to be much less than about 1%, but with carefull compensation of the pots and integrators by a vacuum tube volt meter and with computer operation over its full voltage range, accuracy can be maximized. In addition, loading effects in the pots can be compensated from a correction chart (7) or by analyzing their load circuit. The effect of component static accuracy is to change the equilibrium (and transient) voltage on a particular integrator output. Since the equilibrium value of the integrator voltage is not read out of the computer, it does not affect the results, and since each plate transient voltage is normalized (from zero to one), its absolute value tends to wash out of E_D. The deviation of any plate from its actual value is less than about five percent. Inaccuracy in the power supply voltage was large, ranging over long periods about ±8 volts from its nominal 100 volt value. This change, however, displaces all the output voltages equally, and since the static values do not appear in the results, except insofar as they influence dynamic response, an overall static error washes out completely. The time axis was accurately set with a volt meter.

Dynamic limitations in the computing elements were negligible in this thesis. The only elements with narrow bandwidths were integrators, and the runs were made with ample provision for limitations in this type of element.

The most critical error in deriving the results was the accuracy with which the Output Table graphs (see Figure 4) could be read. Two factors affected this accuracy: the precision obtained in alligning the graph paper on the drum, and accuracy with which the transient displacement could be read from the graph. The allignment of the graph was accomplished with an accuracy of ±0.3 small squares per three hours (values from the graph were tabulated in small square units). Since $x_n(\infty)-x_n(0)$, expressed in small squares, varied between 10 for plate one and about 150 for plates five and six, this cumulative error ranged from ±3% to ±0.3%. The error was largest for plates one and two.

Displacement from the final equilibrium value could be measured with dividers to an accuracy of about ±0.1 small squares. The percent error caused at any point by this deviation depended on both the value of $x_n(\infty)-x_n(0)$ and the degree to which the transient had diminished this value. Plates one and two show a high percentage error because their initial values were low. Initially, the errors in these plates were on the order of ±1% (since no run with an $x_1(\infty)-x_1(0)$ of less than 10 small squares was included in the analysis), but as these first plates approached their new equilibrium, the error increased exponentially to about ±10% (no value smaller than one small square was considered in the results). Thus, accuracy fell off sharply with time. Since the first plates had low equilibrium changes, and since the last rose rapidly because of its time jump, the middle plates were the least affected by this limitation. It should be pointed out that the graph displacements were amplified by a factor of ten in order to decrease this inaccuracy.

In summary, the most important limitations on accuracy were caused by imperfect graph allignment and lack of accuracy in measuring small increments. In the range of time where the lines determining τ and D were measured, these limitations caused an error of up to about ±10% in the top two plates, about ±4% in the middle plates, and about ±6% in the bottom plates. For large increments in y_F these errors were reduced appreciably.

Error in τ and D caused by these effects corresponded to the same values, but the averages which were derived as best estimates placed limits of ±2% on τ and about ±4% on D.

VI. CONCLUSIONS

From the results of this thesis the following conclusions may be made about the response of an absorber tower to a step change in the vapor feed concentration, when the tower is chacterized by perfect plates, negligible vapor holdup, constant inert liquid and vapor streams, and a linear equilibrium relationship.

1. The resultant change in plate concentration may be closely approximated by a time lag followed by an exponential transient.
2. The exponent characterizing the exponential is the same in every plate, and independent of the size of the feed disturbance.
3. The transient's exponent may be closely approximated by $s_1 = \frac{-VK}{h}\left[\frac{L}{VK} + 1 - 2\sqrt{\frac{L}{VK}}\cos\frac{\pi}{N+1}\right]$
4. The time lag preceeding the exponential is different on every plate, being largest and positive on the top plate, and smallest (negative) on the bottom plate
5. The time lag is independent of the size of the feed disturbance.

VII. RECOMMENDATIONS

As an aid to continuing and expanding the results of this thesis the following recommendations may be made.

1. More data on different absorber modles should be run to check the proposed correlation for the time constant (Equation 14). It should be necessary to run only one or two steps on each model if the steps are large enough to produce good accuracy.
2. Some investigation should be made into the effect of changing the vapor-liquid ratio in a tower such as the one used in this thesis. Such a change (in the liquid rate for example) is the most obvious way to control product purity over changes in the vapor feed concentration.

VIII. APPENDIX

A. Summary of Data and Calculated Values.

E_D Series 2

	PLATE NUMBER					
	1	2	3	4	5	
RUN 1	E_D	E_D	E_D	E_D	E_D	θ
	1.000	1.000	1.000	1.000	1.000	0.
	.833	.760	.637	.468	.237	.417
	.713	.373	.317	.199	.105	.834
	.491	.184	.159	.087	.052	1.251
	.241	.092	.073	.042	.023	1.668
	.111	.041	.034	.020	.010	2.084
	.065	.032	.021	.011	.003	2.501
RUN 2	1.000	1.000	1.000	1.000	1.000	0.
	.859	.738	.620	.383	.204	.417
	.486	.368	.297	.171	.087	.834
	.238	.176	.147	.078	.040	1.251
	.115	.076	.075	.053	.020	1.668
	.060	.035	.037	.026	.009	2.084
	.041	.041	.018	.013	.005	2.501
RUN 3	1.000	1.000	1.000	1.000	1.000	0.
	.789	.734	.569	.390	.170	.417
	.414	.366	.264	.171	.073	.834
	.190	.173	.123	.079	.034	1.251
	.094	.076	.058	.035	.016	1.668
	.042	.032	.027	.014	.009	2.084
	.024	.014	.012	.005	.004	2.501
RUN 4	Output table paper not correctly alligned on this run.					
RUN 5	1.000	1.000	1.000	1.000	1.000	0.
	.815	.761	.614	.428	.210	.417
	.429	.406	.310	.200	.095	.834
	.210	.193	.148	.100	.044	1.251
	.101	.096	.073	.044	.020	1.668
	.059	.046	.034	.023	.010	2.084
		.029	.017	.015		2.501
RUN 6	1.000	1.000	1.000	1.000	1.000	0.
	.833	.749	.589	.409	.204	.417
	.442	.394	.287	.194	.091	.834
	.214	.200	.142	.097	.045	1.251
	.103	.096	.069	.047	.021	1.668
	.043	.048	.029	.021	.010	2.084
		.025	.014	.010		2.501

Lag Time, Series 2

Plate	Step (o.09 to -)				
	0.075	0.083	0.097	0.105	0.112
1	0.35	0.36	0.79	0.39	0.32
2	0.31	0.38	0.24	0.23	0.28
3	0.16	0.11	0.140	0.10	0.11
4	-0.08	-0.10	-0.03	-0.19-	-0.10
5	-0.45	-0.46	-0.37	-0.52	-0.64

Time Constant, Series 2

Plate	Step (0.09 to -)				
	0.075	0.083	0.097	0.105	0.112
1	0.57	0.57	0.59	0.59	0.57
2	0.57	0.58	0.59	0.58	0.53
3	0.57	0.58	0.59	0.59	0.55
4	0.57	0.58	0.56	0.60	0.52
5	0.55	0.58	0.54	0.56	0.56

E_D Series 3

PLATE NUMBER

1	2	3	4	5	6	
E_D	E_D	E_D	E_D	E_D	E_D	Θ
RUN 1 Small changes invalidate this run						
RUN 2						
1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.952	0.923	0.849	0.689	0.500	0.248	0.417
0.696	0.651	0.553	0.405	0.273	0.125	0.834
0.462	0.415	0.354	0.242	0.161	0.069	1.251
0.290	0.250	0.220	0.144	0.092	0.037	1.668
0.193	0.147	0.130	0.080	0.049	0.022	2.084
0.131	0.081	0.080	0.044	0.021	0.012	2.501
0.083	0.044	0.050	0.022	---	---	2.919
RUN 3						
1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.956	0.906	0.826	0.656	0.493	0.234	0.417
0.722	0.625	0.539	0.391	0.276	0.123	0.834
0.478	0.401	0.338	0.239	0.166	0.073	1.251
0.293	0.242	0.210	0.144	0.104	0.043	1.668
0.185	0.147	0.129	0.089	0.064	0.025	2.084
0.112	0.088	0.076	0.053	0.038	0.015	2.501
0.073	0.051	0.045	0.030	0.024	----	2.919
RUN 4						
1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.943	0.906	0.841	0.680	0.482	0.245	0.417
0.713	0.628	0.546	0.408	0.268	0.127	0.834
0.469	0.394	0.342	0.249	0.160	0.074	1.251
0.295	0.248	0.208	0.153	0.098	0.043	1.668
0.178	0.148	0.131	0.101	0.060	0.025	2.084
0.113	0.087	0.077	0.054	0.036	0.014	2.501
0.073	0.048	0.048	0.031	0.021	0.007	2.919
RUN 5 Small changes invalidate this run.						

E_D Series 3

(continued)

PLATE NUMBER

1	2	3	4	5	6	
E_D	E_D	E_D	E_D	E_D	E_D	Θ
RUN 6						
1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.942	0.906	0.837	0.695	0.584	0.243	0.417
0.710	0.650	0.544	0.141	0.265	0.127	0.834
0.442	0.404	0.336	0.253	0.159	0.072	1.251
0.268	0.249	0.202	0.145	0.097	0.042	1.668
0.152	0.152	0.122	0.088	0.056	0.026	2.084
0.094	0.094	0.073	0.052	0.035	0.014	2.501
0.058	0.058	0.041	0.032	0.020	---	2.919
RUN 7						
1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.889	0.916	0.845	0.697	0.498	0.243	0.417
0.688	0.647	0.551	0.419	0.274	0.133	0.834
0.413	0.413	0.342	0.254	0.165	0.075	1.251
0.254	0.249	0.209	0.157	0.100	0.044	1.668
0.148	0.149	0.132	0.096	0.593	0.027	2.084
0.095	0.093	0.079	0.059	0.344	0.017	2.501
0.053	0.056	0.047	0.036	0.019	0.012	2.919
---	---	0.027	0.023	---	---	----

Lag Time, Series 3

Plate	Step (0.07 to -)				
	0.055	0.06	0.08	0.085	0.09
1	0.53	0.60	0.49	0.60	0.59
2	0.47	0.47	0.53	0.44	0.44
3	0.32	0.33	0.39	0.32	0.33
4	0.09	0.11	0.16	0.07	0.12
5	-0.23	-0.25	-0.08	-0.28	-0.27
6	-0.82	-0.86	-0.68	-0.81	-0.73

Time Constant, Series 3

Plate	Step (0.07 to -)				
	0.055	0.06	0.08	0.085	0.09
1	0.82	0.82	0.98	0.87	0.98
2	0.86	0.87	0.82	0.85	0.86
3	0.87	0.83	0.84	0.86	0.86
4	0.80	0.82	0.77	0.83	0.83
5	0.81	0.81	0.72	0.86	0.83
6	0.81	0.81	0.72	0.78	0.77

E_D, Series 4

PLATE NUMBER

1	2	3	4	5	6	
E_D	E_D	E_D	E_D	E_D	E_D	Θ

RUN 1 Small changes invalidate this run.

RUN 2 Small changes invalidate this run.

RUN 3

1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.902	0.882	0.773	0.636	0.420	0.215	0.5
0.582	0.555	0.452	0.338	0.221	0.106	1.0
0.336	0.327	0.254	0.179	0.123	0.059	1.5
0.180	0.103	0.080	0.041	0.039	0.018	2.5
0.066	0.057	0.045	0.015	0.022	0.010	3.0

RUN 4

1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.901	0.876	0.767	0.673	0.435	0.217	0.5
0.590	0.551	0.449	0.368	0.224	0.109	1.0
0.323	0.319	0.259	0.208	0.127	0.060	1.5
0.180	0.184	0.143	0.118	0.070	0.034	2.0
0.099	0.103	0.077	0.067	0.037	0.018	2.5
0.062	0.057	0.043	0.035	0.021	0.011	3.0
---	---	0.021	0.021	0.021	---	3.5

E_D, Series 4
(continued)

PLATE NUMBER

1	2	3	4	5	6	
E_D	E_D	E_D	E_D	E_D	E_D	Θ
RUN 5						
1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.915	0.841	0.748	0.606	0.600	0.213	0.5
0.581	0.502	0.426	0.319	0.218	0.106	1.0
0.315	0.282	0.237	0.171	0.121	0.058	1.5
0.171	0.153	0.131	0.091	0.061	0.032	2.0
0.095	0.082	0.072	0.049	0.033	0.018	2.5
0.059	0.043	0.038	0.024	0.019	0.011	3.0
RUN 6						
1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.914	0.848	0.752	0.623	0.428	0.214	0.5
0.594	0.515	0.429	0.334	0.217	0.107	1.0
0.350	0.288	0.236	0.181	0.117	0.058	1.5
0.198	0.162	0.127	0.100	0.064	0.013	2.0
0.119	0.089	0.065	0.054	0.034	0.017	2.5
0.076	0.046	0.034	0.038	0.018	0.010	3.0
---	---	0.015	---	---	---	3.5
RUN 7	Small changes invalidate this run.					
RUN 8						
1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.890	0.856	0.759	0.613	0.429	0.207	0.5
0.588	0.533	0.436	0.326	0.213	0.100	1.0
0.346	0.296	0.244	0.176	0.112	0.053	1.5
0.206	0.157	0.132	0.091	0.062	0.027	2.0
0.118	0.087	0.074	0.052	0.034	0.015	2.5
0.074	0.047	0.038	0.029	0.018	0.009	3.0

E_D, Series 4

(continued)

PLATE NUMBER

1	2	3	4	5	6	
E_D	E_D	E_D	E_D	E_D	E_D	θ
RUN 9						
1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.895	0.873	0.771	0.629	0.436	0.215	0.5
0.580	0.542	0.448	0.334	0.218	0.098	1.0
0.325	0.297	0.254	0.183	0.118	0.051	1.5
0.180	0.159	0.139	0.095	0.066	0.027	2.0
0.100	0.084	0.075	0.052	0.036	0.014	2.5
0.050	0.045	0.040	0.026	0.019	0.008	3.0
---	---	0.021	0.016	0.010	---	3.5
RUN 10						
1.000	1.000	1.000	1.000	1.000	1.000	0.0
0.912	0.860	0.769	0.612	0.434	0.208	0.5
0.611	0.545	0.446	0.332	0.217	0.099	1.0
0.348	0.312	0.246	0.181	0.115	0.052	1.5
0.196	0.167	0.134	0.097	0.063	0.029	2.0
0.111	0.091	0.072	0.053	0.034	0.016	2.5
0.063	0.052	0.040	0.030	0.019	0.009	3.0
---	---	0.021	0.016	---	---	3.5

Lag Time, Series 7

Step (0.07 to -)	Plate 1	2	3	4	5	6
0.110	0.52	0.43	0.31	0.12	-0.23	-0.84
0.100	0.56	0.45	0.30	0.08	-0.22	0.92
0.090	0.46	0.54	0.31	0.13	-0.31	-0.87
0.085	0.50	0.48	0.31	0.12	-0.33	-0.92
0.050	0.52	0.49	0.33	0.09	-0.27	-0.82
0.040	0.53	0.51	0.35	0.13	-0.24	-0.79
0.030	0.56	0.50	0.33	0.11	-0.24	-0.92

VIII. APPENDIX

B. Location of Original Data

The Output Table graphs for each run in all of the Series are included in the third copy of this thesis. This copy in in the possession of Professor E.R.Gilliland Department of Chemical Engineering, Massachusetts Institute of Technology.

Numerical data and calculations of the original work may be found in the research notebook "Morris-1960" which is located at the Department of Chemical Engineering, Massachusetts Institute of Technology.

VIII. APPENDIX

C. Nomenclature

A_j	$x_n(0)$ on plate $n = j$	moles/mole
C	Scale factor for the independent variable (time) in the computer	volts
D	Lag time	hours
E_D	$\frac{x_n(\infty) - x_n(\theta)}{x_n(\infty) - x_n(0)}$	dimensionless
H	Vapor holdup per plate	moles/plate
K	Equilibrium constant	dimensionless
N	Number of plates in an absorber tower	dimensionless
V	Solute free vapor rate	moles/hour
a	Scale factor for computer time	seconds/hour
b	Integration variable	
c	$\frac{L}{VK}$	
d	$\frac{h}{VK}$	
e_n	Voltage on n'th integrator output	volts
e	2.7183····	
$f(\theta)$	$x_0(\theta)$	
$g(\theta)$	$y_F(\theta)$	
h	Liquid holdup per plate	moles/plate
j	Summation variable	
k	Summation variable	
n	Denotes one of N plates	
s_k	Exponent in Equation (8) see Equation (9)	
t	Computer time	seconds

x	Moles solute per mole solute free liquid	moles/mole
y	Moles solute per mole solute free vapor	moles/mole
Δ	Signifies an increment in the following quantity	
θ	Physical time	hours
τ	Time constant of an exponential	
π	3.1416···	

Subscripts

F	Denotes a property of the vapor feed
0	Denotes a property of the liquid feed

V. APPENDIX

D. Literature Citations

(1) Acrivos, A., and Amundson, N.R., Ind. Eng. Chem., 45, 467, (1953).

(2) Davis, S.H. Jr., "Transients in Continuous Distillation Columns", Sc.D. Thesis, Chem. Eng., M.I.T. (1957).

(3) Jackson, R.F., and Pigford, R.L., Ind. Eng. Chem., 48, 1020, (1956).

(4) Jordan, J.I. Jr., "Transient Conditions in Continuous Distillation Columns", M.S. Thesis, Chem. Eng., M.I.T. (1954).

(5) Lapidus, L., and Amundson, N.R., Ind. Eng. Chem., 42, 1071, (1950).

(6) Marshall, W.R., and Pigford, R.L., "Applications of Differential Equations to Chemical Engineering Problems", Newark, Del. Univ., Delaware Press, (1947).

(7) Massachusetts Institute of Technology, Servovechanisms Laboratory, Department of Electrical Engineering, "E.E. Department Reac Analog Computer Theory and Operation Manual", Report No. 6080-1, (1954).

(8) O'Donnell, J.F., "Distillation Calculations Using a High-Speed Digital Computer", Sc. D. Thesis, Chem. Eng., M.I.T. (1955).

(9) Robinson, C.S., and Gilliland, E.R., "Elements of Fractional Distillation", 4th edition, New York, McGraw-Hill Book Company, Inc., 1950.

(10) Rose, A., Johnson, C.L., and Williams, T.J., Ind. Eng. Chem., 48, 1173, (1956).

(11) Rose, A., and Williams, T.J., Ind. Eng. Chem., 47, 2284, (1955).

(12) Smith, A.T., and Polk, J.G., "Behavior of Continuous Distillation Columns Under Transient Operating Conditions", M.S. Thesis, Chem. Eng., M.I.T. (1953).