

LONGITUDINAL DISPERSION IN
NON-UNIFORM POROUS MEDIA

by

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ABSTRACT

Title of Thesis:

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This thesis is submitted to the Department of Civil Engineering on the 22nd of May, 1964 in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

Thesis outline

When two miscible fluids of approximately the same densities and viscosities flow through a porous medium, in the process of one displacing the other, a mixing phenomenon takes place. Such mixing is the combined effect of molecular diffusion and convective dispersion. Molecular diffusion has been found to be generally negligible compared to convective dispersion and is therefore often neglected in the analysis. Convective dispersion which then is the dominant mode of mixing has been an important subject of study. The dispersion results from the fact that in a porous medium not all flow channels are of the same length, so that fluid elements taking different paths to the same point do not require the same time, and consequently causes longitudinal mixing.

In recent years dispersion phenomena have been studied quite extensively by Civil, Chemical, and Petroleum Engineers. Sea water intrusion into coastal aquifers and salinity and water logging problems may be mainly identified with the interests of Civil Engineers in such studies. Efficient reactor design has engaged Chemical Engineers and new economic oil extraction processes have attracted the imagination of Petroleum Engineers to study dispersion phenomena.

Until recently such a study was mainly aimed at fundamental research but knowledge of the past decade has opened avenues for technology to coordinate and utilize such basic research in actual physical situations.

The aim of the present work is to evaluate certain useful correlations of dispersion parameters for non-uniform sands in terms of measurable field quantities.

From both a statistical approach and gross behavior of fluid flow, one-dimensional dispersion is described by the convective dispersion equation of the form

$$\frac{\partial S}{\partial t} + u_s \frac{\partial S}{\partial x} = D \frac{\partial^2 S}{\partial x^2}$$

where D is the coefficient of dispersion which essentially depends upon the media characteristics.

The dispersion coefficient, in the region where molecular diffusion is negligible, is often expressed as

$$D = \alpha u_s^n$$

Different values of n have been reported by various investigators, but so far there was nothing in literature which gave quantitative or even qualitative trends on the variation of the exponent. It has now been established that n is a strong function of the uniformity coefficient and decreases toward unity with increasing non-uniformity.

It has also been shown in the present investigation that the intrinsic permeability is a more representative characteristic length parameter than a geometric mean of the grain size distribution. It is proposed to use this parameter in preference to d_{50} as a non-dimensionalizing length characteristic in correlating dispersion coefficients.

Finally, it has been concluded that dispersion is appropriately represented by a function of the form

$$D = f(u_s, k, U.C.)$$

Such functional relationship also helped to present dispersion data in a useful correlation for fluids of

different viscosities. A correlation of the form

$$D/\nu = \Phi R \frac{n}{k}$$

has indicated that even a tenfold change in viscosity does not produce significant change from that of water. This is generally in agreement with the findings of some of the other investigators.

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LIST OF NOTATIONS

- d_{50} = fifty percent size of media, ft.
- D = dispersion coefficient, ft^2/sec .
- D_m = effective molecular diffusivity in porous media, ft^2/sec .
- g = acceleration of gravity.
- h = piezometric head, ft.
- k = intrinsic permeability, ft^2 .
- K = Darcy's permeability.
- n = dimensionless constant related to pore geometry.
- q_s = $iu_s + iv_s + kw_s$.
- S = concentration of tracer substance at any point in porous media = $S_s + \overset{\circ}{S}$.
- S_0 = initial concentration of tracer substance (constant).
- S_s = concentration of tracer substance at a point in porous media due to local average seepage velocity.
- $\overset{\circ}{S}$ = excess or deficit of tracer concentration at a point S_s due to local spatial velocity variations.
- t_{50} = time at which $S/S_0 = .5$ past the probe, sec.
- U = superficial velocity $u = \frac{\text{volume of flow}}{\text{total area of column}}$, ft/sec .
- u, v, w = velocity components in x, y, z coordinate directions at a point in porous media = $u_s + \overset{\circ}{u}$, etc., ft/sec .

LIST OF NOTATIONS (continued)

- u_s, v_s, w_s = average seepage velocity component, ft/sec.
- $\hat{u}, \hat{v}, \hat{w}$ = spatial variations from average seepage velocity components, ft/sec.
- x, y, z = three coordinate directions, ft.
- β = dimensionless constant related to pore geometry.
- γ = specific weight of the liquid, lbs/ft³.
- η = dimensionless parameter = $D/u_s x$.
- θ = porosity of the porous media.
- μ = dynamic viscosity of liquid, lb-sec/ft².
- ν = kinematic viscosity, ft²/sec.
- ξ = dimensionless parameter = $u_s t/x$.
- ρ = density of the liquid, lb-sec²/ft⁴.
- U.C. = uniformity coefficient. = $\frac{d_{60}}{d_{10}}$
- α = dispersion constant in the relation $D = \alpha u_s^n$.
- Φ = dimensionless coefficient.
- Ψ = dimensionless coefficient.
- R_k = permeability Reynolds number = $(u_s \sqrt{k})/\nu$.
- R_{d50} = particle Reynolds number = $(u_s d_{50})/\nu$.
- P_k = permeability Peclet number = $(u_s \sqrt{k})/D$.
- P = particle Peclet number = $(u_s d_{50})/D$.

INTRODUCTION

The Dispersion Problem in Porous Media

Mixing processes in porous media have in recent years acquired particular importance with Civil, Chemical, and Petroleum Engineers. Much attention has been given by researchers in these fields to explore theoretical and experimental devices in establishing generalizations which will help put such knowledge on a sound basis. Until the present time, however, only moderate success has been achieved in this direction. The main reason for the inadequacies as may exist arise from the basic fact that a porous medium is a complicated assemblage of particles and pore sizes. To define precise boundary conditions for any problem becomes an insurmountable obstacle. In the absence of a rigorous mathematical treatment, therefore, recourse has been made to simplified models and experimental studies. The first of these models was the Capillary Model, then came the Kozenys Model, the Drag Theory, and the Network Models. These models have been somewhat useful in explaining the permeability of the porous medium but they have not been able to explain the experimentally observed dispersion of a fluid in a porous medium, except in the case of the Network Model.

In recent years two approaches have met with successful results. One is statistical and the other is based on considerations of gross behavior of the fluid flow. The latter is fully described in the section dealing with theoretical considerations.

Statistical model on the other hand is another ingenious tool which provides a means to circumvent the difficulty of the complicated internal pore structure. The concept is not new and was first applied to the kinetic theory of gases where the problem is similar inasmuch as it is impossible to follow the path of each molecule contained in a gas under consideration. The statistical mathematics that is used was devised by Gibbs and then developed by Einstein who used it to explain the Brownian Motion.

The statistical model of porous media based on Einstein's Theory of Brownian Motion is usually referred to as the completely disordered model and is shown to work well⁽¹¹⁾. It leads to a diffusivity equation for the probability function $X(x,t)$ with some limiting expressions for the diffusivity constant, D . Chemical Engineers have arrived at the same result by assuming completely random distribution of residence times in their study of packed tower operations Dankwerts, 1953⁽¹²⁾, Wilhelm, 1953⁽¹³⁾, Baum, 1953⁽¹⁴⁾. As pointed

out by Scheidegger⁽¹¹⁾, the two models are somewhat different but because of the basic statistical assumption one arrives at the same diffusivity equation, $\frac{\partial c}{\partial \theta} = D \frac{\partial^2 c}{\partial x^2}$ $\theta = \text{time}$, $c = \text{mean concentration}$.

We conclude, therefore, that the dispersion process is expressed by the same convective dispersion equation by both approaches which contains a dispersion constant, D . The solution to the diffusivity equation is in the form of an error function which also contains the dispersion constant, D .

It is convenient to distinguish the mixing that occurs in the direction of mean flow - Longitudinal Dispersion - and that perpendicular to the mean flow - Lateral Dispersion. Many investigators have shown that longitudinal mixing is much greater than lateral mixing ^(4,7) and therefore is more important. The present investigation is concerned exclusively with longitudinal dispersion.

Not too many efforts have been made in the past to determine all the factors that determine the Dispersion Coefficient. A modest beginning has, however, been made in the field and reasonable knowledge both from theoretical and experimental^(1,2,3,4,5,9) investigations have led to some broad generalizations, at least in the laminar flow region

to establish its functional dependence upon mean seepage velocity and the physical characteristics of the porous medium. The present work is also an effort to know more about the influence of different factors on D.

Statement of the Problem

As is always the case, basic studies in the field of Engineering, as distinct from pure science, often result from a need to organize scientific knowledge in a form readily applicable to the requirements of advancing Engineering Technology. Such knowledge is then applied to actual physical situations.

This is by and large the reason which aroused interest in the present study.

Harleman and Rumer⁽³⁾ through a semi-empirical approach derived two very useful relations that could readily be applied to actual field conditions. They compared their theoretical considerations with reasonably extensive experimental investigations of Harleman and Mehlhorn⁽²⁾ and it turned out that a unique correlation between the ratio of Dispersion Coefficient and Kinematic Viscosity to Permeability and Particle Size Reynolds Numbers resulted. The usefulness of this work was the determination of a method of predicting

Dispersion Coefficient from permeability, which is a parameter that can be determined in the field.

The experimental work of Harleman and Mehlhorn⁽²⁾ against which these correlations were tested, essentially dealt with uniform spheres and sands. Therefore, the effect of non-uniformity was not studied. Physical problems in the field generally involve materials that are not uniform and therefore the study of the effect of non-uniform grain size distributions was essential to any intelligent appraisal of their relations. Thus this forms the first aspect of the present study.

To further broaden the scope, the effect of viscosity has also to be included. This would help the Chemical and Petroleum Engineers in their reactor design and oil recovery processes, respectively.

Thus two effects are investigated: (1) non-uniform grain size distribution, and (2) viscosity, and their influence on the relationship

$$D/v = \Psi R_{d50}^n \quad \text{and} \quad D/v = \Phi R_k^n$$

is established.

The intrinsic permeability is believed to be a much more representative parameter of the complex porous medium

than a particular grain size and an attempt is made to see if any general correlations between the intrinsic permeability and other dispersion parameters can be obtained for a non-uniform porous medium.

Physical Situations where Dispersion is Important

There are mainly three fields in which the dispersion process in porous media is important. These are Civil Engineering, Chemical Engineering, and Petroleum Engineering.

Sea water intrusion in coastal aquifers has become a continued menace in areas where unplanned development of groundwater resources has progressed to such an extent that gradient of flow has been reversed. Such adverse gradients tend to move sea water inland and, as has been found, it disperses in a manner such that salt is carried well ahead of the actual bulk of the flow. Since concentrations of a few ppm can render a supply unfit for human consumption, this problem has attracted the attention of more and more researchers in Civil Engineering in an effort to prevent aquifer pollution. Tidal effects and periodic pumping increase dispersion and over a period of time contaminated water fills a volume several times the amount of actual salt water that travels inland.

Lately the idea of storage and disposal of radioactive

waste water has also promoted dispersion studies so that reasonable predictions can be made of the extent of contamination caused by such recharging.

Reclamation of saline and waterlogged lands is another important field in Civil Engineering, where knowledge of dispersion is necessary to any quantitative predictions with regard to quality, quantity, and time required for leaching away salinity concentrations to desired levels. Such knowledge would probably be a major tool for the reclamation Engineers who are combating this dual menace in West Pakistan.

Chemical Engineers on the other hand have been interested in displacement processes from the point of view of mixing effects upon reactor efficiencies. Wilhelm and others⁽¹⁶⁾ have specifically studied mixing in fixed beds reactors. But since high reactor efficiencies often require turbulent motion, most of these studies have laid emphasis on mixing in turbulent or almost turbulent flow regimes.

Petroleum Engineers, have, however, worked quite extensively on miscible displacements in laminar flow in porous media. Their imagination has been captured by new economic processes of oil recovery by miscible displacements.

It had been realized that capillary forces are essentially responsible for retaining residual oil behind a

displacing flood front. Such capillary forces result from interfacial tension between in-place oil and the displacing fluid. Therefore if the abrupt interface between the displaced and displacing fluid was removed so would be the capillary forces. This led to miscible displacements by using various solvents and it is now asserted that 100% oil recovery in the swept area is realized⁽¹⁷⁾. Since solvents are expensive, Petroleum Engineers are continually working to understand more about dispersion so that economy in reduced solvent mixing is achieved.

LITERATURE REVIEW

There has been much work done on dispersion phenomena in porous media both through analytical and experimental models and therefore a reasonable amount of literature is available. Concerning the present study, however, there is nothing sufficient from which to establish generalizations for non-uniformity and viscosity effects upon dispersion. However, a brief and salient literature review, pertinent to the present study, is presented here.

Harleman and Rumer⁽¹⁾ investigated sea water intrusion and dispersion into coastal aquifers. They studied both analytical and experimental phases of the problem in a confined isotropic and homogenous porous medium. First part of the work dealt predominantly with lateral and longitudinal dispersion for one-dimensional flow. The second phase of the study was concerned with steady-state fresh-salt water interface position of the saline wedge and its dispersion without tidal effects. The last part dealt with the additional dispersion due to tidal oscillations. Pertinent to this work was the first phase of the investigation for which they investigated longitudinal and lateral dispersion coefficients as functions of the pore geometry and the seepage velocity. Also for isotropic medium the ratio of the longitudinal coefficient to the lateral coefficient of dispersion

was experimentally determined to be a function of the pore geometry and the seepage velocity. For glass beads in R range 1, they obtained a relation, $D = .027 u_s^{1.10}$, where $D =$ Dispersion Coefficient in $\text{cm}^2/\text{sec.}$, $u_s =$ seepage velocity in $\text{cm}/\text{sec.}$

In another study, Harleman, Rumer and Mehlhorn⁽²⁾ experimentally investigated the effects of certain important characteristics of the porous medium on one-dimensional longitudinal dispersion. Grain size, shape and, to some extent, the uniformity of the grain size distribution of spheres and sands were studied. They obtained good correlation between α_1 and grain size, $D = \alpha_1 u_s^n$, $D =$ Long.Disp. Coeff. for uniform sands and spheres. Also, α_1 and the intrinsic permeability which are both functions of the medium were seen to be correlated by a relation, $\alpha_1 = \text{const. } \sqrt{k}$. $k =$ intrinsic permeability. The constant was larger for sands than for spheres.

In yet another work by Harleman, Mehlhorn, and Rumer⁽³⁾ results based on their earlier investigations⁽²⁾ were correlated. For various sizes of uniform spheres and uniform sands, it was found that Longitudinal Dispersion Coefficient when correlated with Reynolds number, based on both mean size and the intrinsic permeability, was independent of the particle size, (Range R d_{50} $0.01 < 10$), but the dispersion was

greater in sands, probably because of the angular shape and slight non-uniformity. The principle involved is simple yet this is very useful work because of its ready applicability in the field. The permeability Reynolds number vs. Dispersion correlation is a valuable tool for field Engineers, since fairly accurate field permeability measurements can be made. One of the main reasons for interest by the author in the present work was to try and generalize if possible the correlations achieved in this paper⁽³⁾ for non-uniform grain size distributions.

In their latest publication, Harleman and Rumer⁽⁴⁾ have investigated longitudinal and lateral dispersion coefficients in steady uniform laminar flow through an isotropic porous medium. They established that the ratio of the longitudinal dispersion coefficient to the lateral dispersion coefficient depends upon the absolute magnitude of the seepage velocity and is independent of the distance. The ratio is represented as

$$D_1/D_2 = \lambda R^{1/2}$$

where λ is a dimensionless coefficient which they maintain, presumably depends on particle shape for uniform media and particle size distribution for non-uniform media.

Some effects of non-uniformity upon dispersion are

found in the work of Raimondi, Gardner, and Petrick⁽⁵⁾.

Generally they have considered equations of motion both for linear and radial flow in porous media, mixing they maintain is essentially controlled by two parameters, one in the molecular diffusion in fluids and the other is a constant that depends upon the porous medium, and is independent of the fluid and rate of flow.

The effective dispersion coefficient for random packing of glass spheres over the entire viscous region was described by an equation, $D_e = D + \alpha u_s dp$, $dp = av$. sphere diameter, u_s = mean fluid velocity, D = coefficient of molecular diffusion, and α is a constant, independent of the fluid properties and which has dimensions of length. It can be expressed as

$$\alpha = \sigma^2 dp$$

where σ^2 is a dimensionless constant which varies with the standard deviation of the distribution of velocity components in the direction of flow. As such it would also be a function of the grain size distribution. dp is the mean particle size. σ^2 for uniform spheres was found to be .68. This value increased as the grain size distribution increased. While keeping permeability constant, it was shown that α ($\alpha = \sigma^2 dp$) increased with increasing non-uniformity. Also by keeping particle size distribution constant it was shown

that α increases almost linearly with increasing permeability.

In an attempt to study the effects of entrapped gases on hydraulic characteristics of porous media, Orlob and Radhakrishna⁽⁶⁾ also investigated the effect of grain size distribution on the slope of breakthrough curves. They showed that with increasing non-uniformity the slope of the breakthrough curve increased. They also determined that a 10% increase in air content of media voids is capable of producing a 15% reduction in effective porosity, a 35% decrease in permeability and about 50% reduction in hydraulic dispersion.

Crane and Garner⁽⁷⁾ in their work on lateral dispersion performed some experiments on longitudinal dispersion and observed that at sufficiently low flow rates longitudinal and transverse dispersion are equal and are essentially controlled by molecular diffusion of the fluid and the formation factor of the porous medium. At high flows, however, longitudinal and lateral dispersions are independent of the molecular diffusion and are directly proportional to the velocity of flow and independent of the fluid properties. Longitudinal dispersion, they maintain, may be up to 50 times the lateral dispersion. This is in agreement with Harleman and Rumer's work⁽⁴⁾.

Essentially lateral dispersion with some longitudinal

dispersion was studied by Blackwell⁽⁸⁾ in packed columns for laminar flow in unconsolidated sands and he also concluded that mixing at small flow rates both in lateral and longitudinal directions is governed by molecular diffusion. At higher flow rates, however, convective mixing is the dominant process. Blackwell also studied the viscous effects by changing the viscosity from 0.9 to 22.6 centipoises. A unique relationship between $(dp u_s)/D$ and K_t/D was obtained, ($K_t = \text{lat. disp. coeff.}$, $D = \text{molecular diff. coeff.}$, $dp = \text{mean particle size}$, $u_s = \text{seepage velocity}$), for any one particular medium. Increasing fineness of particle range gave increasing values of K_t/D for the same $(dp u_s)/D$. ($u_s = \text{seepage velocity}$).

Ebach and White⁽⁹⁾ carried out extensive investigations of longitudinal mixing in packed columns. They obtained almost a linear correlation between dispersion coefficient and the mean particle diameter and velocity. Uniform spheres, raschig rings, intalox saddles and berl saddles were used as bed material. They employed pulse function method for low velocity runs and a frequency response technique for higher velocity runs. Viscous influence was also investigated by them for 1 mm spheres in a packed column for a flow rate corresponding to a Reynolds number of 0.25 for water. In this region a change of

viscosity from 0.95 to 27 CP did not influence the axial dispersion coefficient.

Henico, Jacques, and Vermeulen⁽¹⁰⁾ in their extensive work on extraction-tower design have investigated various aspects of axial dispersion coefficients by a step input method over a wide range of Reynolds numbers (3-2000). Various bed materials were used. Constant values of Peclet numbers are found in the turbulent and the laminar regions which are differentiated by a fairly sharp transition region. They also observed that viscosity had large effects upon axial dispersion at high Reynolds numbers ($R > 20-25$). Peclet numbers remained the same for all concentrations of glycerol solutions (ν varied from 1 to 5-6 centistokes) at the same Reynolds number (ν for water = 1 centistoke).

Axial Peclet number was shown to vary inversely with bed porosity and no effect of packing arrangement was observed.

THEORETICAL CONSIDERATIONS

General Analytical Concepts

The present chapter mainly deals with two analytical concepts. One is the development of the convective dispersion equation from conservation of dispersing mass considerations; the other is the derivation of the semi-empirical relations

$$1. \quad D/v = \Psi R_{d50}^n$$

$$2. \quad D/v = \Phi R_k^n$$

which were earlier developed by Harleman and Rumer⁽³⁾. It might again be mentioned that the present experimental investigation is a further effort to establish the validity of these relations and determine the effect of non-uniformity of grain size upon the various coefficients, so that they are more readily applicable to field conditions.

For the purpose of developing the convective dispersion equation from considerations of conservation of mass, laminar flow within Darcy's range is assumed. In addition, the fluid is considered incompressible and the medium completely saturated.

Linear Law

The linear law or the Darcy's Law needs no elaborate introduction. It is well known and is important in many

fields of Engineering, yet is amazingly simple. It was discovered by Darcy* in his extensive experiments on filters. He found that velocity is linearly related to the hydraulic gradient in a particular region of flow through porous media. The Darcy Equation is expressed as

$$\frac{Q}{A} = u = k \frac{dh}{ds} \quad (1)$$

where $K = k\gamma/\mu$ and is called the Darcy coefficient of permeability. It has dimensions of velocity.

$$k = C d_{50}^2 \quad (1a)$$

(C = const. of the porous media; d_{50} = mean particle size).

This law is not universal and breaks down even in laminar flow when the inertial forces become of the same order as viscous forces. This breakdown point is conveniently expressed by a dimensionless parameter, $(u_s d_{50})/\nu = R$, where u_s = seepage velocity = u/θ ; d_{50} = geometric mean of the grain size distribution; and ν is the kinematic viscosity. It has been shown that at about $R = 5$, laminar flow still exists but Darcy's Law no longer holds exactly.

Dispersion and Diffusion

It is well to distinguish between dispersion and

* French hydrologist.

diffusion so that in the development of the convective dispersion equation, when we neglect molecular diffusion, the significance of such an assumption will be clear. Dispersion and diffusion are synonymous terms and indeed mean mixing, but the mode through which mixing takes place in the two processes is remotely different.

Diffusion has been known for a long time and is recognized as a mixing phenomenon caused by molecular activity between two miscible fluids in contact. Though the magnitude of such mixing largely depends upon diffusing fluids and the gradients of concentrations, it is very small compared to dispersion in the Reynolds number range that is generally important in Civil and Petroleum Engineering.

The awareness to a mixing other than molecular diffusion became apparent when investigators in different fields found that the amount of mixing between two miscible fluids in the process of one displacing the other, in the mean direction of flow, through a porous medium, was by far in excess of that which could be accounted for by molecular diffusion. Thus originated the idea that in miscible displacements through porous media not only is molecular diffusion effective but a more important mechanical mixing process is very important.

Such mixing is the result of the condition that not

all flow channels leading to the same point are of the same length, so that the fluid elements taking different paths to the same point in a porous medium do not require the same time and consequently create longitudinal mixing. The name "dispersion" for such mixing was first given by Scheidegger⁽¹⁹⁾.

Diffusion and dispersion are therefore caused by entirely different processes and, as Scheidegger put it⁽²⁰⁾, the fact that both show phenomenologically in a similar fashion is more or less accidental.

Conservation of Mass

Consider the macroscopic fluid element with dimensions dx, dy, dz in a pore space (Figure 1).

The flux of the dispersing mass (substance) into the element in the x-direction is $(\rho s u) dy, dz$ (neglecting molecular diffusion compared to the mechanical mixing of dispersion due to intertwining of flow paths), where

$$\rho = \text{density of the liquid} = \frac{\text{mass of the sol.}}{\text{volume of the sol.}} = \frac{M}{L^3}$$

$$s = \text{concentration of the substance} = \frac{\text{mass of the substance}}{\text{mass of the solution}}$$

$$\rho s = \frac{\text{mass of the substance}}{\text{volume of the solution}} = \frac{M}{L^3}$$

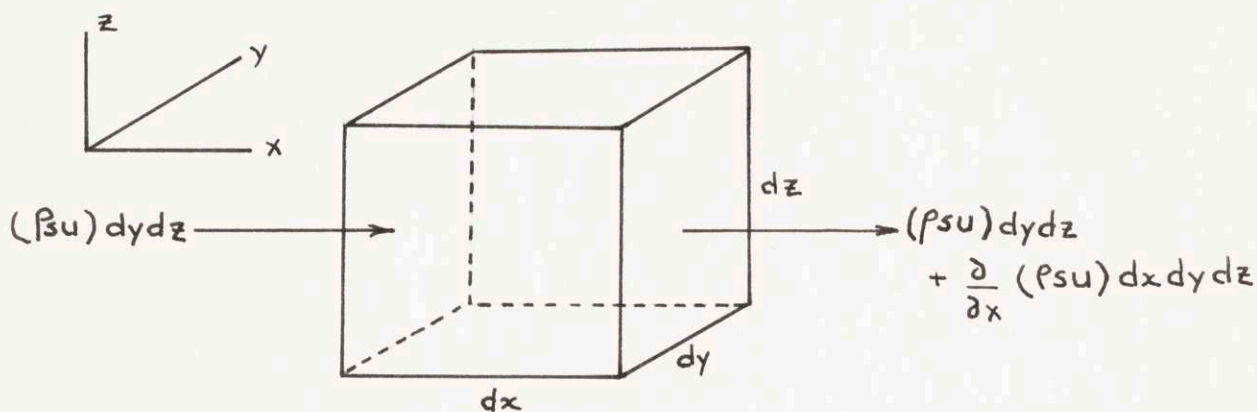


Figure 1.

The outgoing flux from the element in the x-direction is

$$(\rho su) dydz + \frac{\partial}{\partial x} (\rho su) dx dy dz$$

Therefore the change of the mass flow in the x-direction is

$$-\frac{\partial}{\partial x} (\rho su) dx dy dz$$

Obtaining similar expressions for the other two coordinate directions and the sum of the net changes equated to the time rate of change of the substance within the element, one has, after dividing by the elemental volume

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho S u) + \frac{\partial}{\partial y}(\rho S v) + \frac{\partial}{\partial z}(\rho S w) = 0 \quad (2)$$

Expanding the triple derivative, equation (2) becomes

$$\frac{D}{Dt}(\rho S) + \rho S \operatorname{div} q = 0 \quad (3)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

and

$$\operatorname{div} q = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Since the fluid considered is incompressible, the volumetric dilation is zero, i.e. $\operatorname{div} q = 0$, and equation (3) becomes

$$\frac{D}{Dt}(\rho S) = 0 = \rho \frac{DS}{Dt} + S \frac{D\rho}{Dk} \quad (4)$$

To satisfy the continuity equation for the liquid, we have

$$\frac{D\rho}{Dt} = \rho \operatorname{div} q = 0 \quad (5)$$

Since $\operatorname{div} q = 0$, $D\rho/Dt = 0$, and equation (4) becomes

$$\frac{DS}{Dt} = 0 = \frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} + w \frac{\partial S}{\partial z} \quad (6)$$

Defining the fluid particle velocities as

$$\begin{aligned} u &= u_s + \hat{u} \\ v &= v_s + \hat{v} \\ w &= w_s + \hat{w} \end{aligned} \quad (7)$$

where u, v, w are the actual fluid velocities at a point; u_s, v_s, w_s are the average seepage velocities; and $\hat{u}, \hat{v}, \hat{w}$ are the spatial variations of velocities within the pore system from the average seepage velocities. For an isotropic, homogeneous porous medium, the average seepage velocities are defined as

$$u_s = \frac{k\gamma}{\mu\theta} \frac{dh}{dx}, \quad v_s = \frac{k\gamma}{\mu\theta} \frac{dh}{dy}, \quad w_s = \frac{k\gamma}{\mu\theta} \frac{dh}{dz} \quad (8)$$

Tortuosity of the flow paths and no-slip condition at the flow boundaries gives rise to spatial variations. In a similar way the substance concentration can also be defined as

$$S = S_s + \hat{S} \quad (9)$$

where S represents the actual concentration at a point, S_s would be concentration at a point only if the convective velocities (u_s, v_s, w_s) defined the transport of the dispersing mass (slug flow), and \hat{S} represents a spatial variation from S_s due to spatial velocity variations.

Substituting equations (7) and (9) into equation (6) we obtain

$$\frac{\partial}{\partial t}(S+\hat{S}) + (u_s+\hat{u})\frac{\partial}{\partial x}(S+\hat{S}) \quad (10)$$

$$+ (v_s+\hat{v})\frac{\partial}{\partial y}(S+\hat{S}) + (w_s+\hat{w})\frac{\partial}{\partial z}(S+\hat{S}) = 0$$

Assuming isotropic and homogeneous medium in which the

variation of the pore geometry is random, equation (10) can be reduced after expanding the indicated products and averaging spatially through the pore system to a form given by

$$\begin{aligned} \overline{\frac{\partial S_s}{\partial t}} + \overline{u_s \frac{\partial S_s}{\partial x}} + \overline{\dot{u} \frac{\partial \dot{S}}{\partial x}} + \overline{v_s \frac{\partial S_s}{\partial y}} \\ + \overline{\dot{v} \frac{\partial \dot{S}}{\partial y}} + \overline{w_s \frac{\partial S_s}{\partial z}} + \overline{\dot{w} \frac{\partial \dot{S}}{\partial z}} = 0 \end{aligned} \quad (11)$$

Adding the zero term $\dot{S} \operatorname{div} \dot{q}$ to equation (11), combining double derivatives and rewriting the equation, we have

$$\begin{aligned} \overline{\frac{\partial S_s}{\partial t}} + \overline{u_s \frac{\partial S_s}{\partial x}} + \overline{v_s \frac{\partial S_s}{\partial y}} + \overline{w_s \frac{\partial S_s}{\partial z}} \\ + \frac{\partial}{\partial x} \overline{(\dot{S} \dot{u})} + \frac{\partial}{\partial y} \overline{(\dot{S} \dot{v})} + \frac{\partial}{\partial z} \overline{(\dot{S} \dot{w})} = 0 \end{aligned} \quad (12)$$

Considering the physical model for the process of dispersion similar to that of turbulent diffusion, nine dispersion coefficients can then be defined such that

$$\begin{aligned} \overline{\dot{u} \dot{S}} &= - D_{xx} \overline{\frac{\partial S_s}{\partial x}} + D_{xy} \overline{\frac{\partial S_s}{\partial y}} + D_{xz} \overline{\frac{\partial S_s}{\partial z}} \\ \overline{\dot{v} \dot{S}} &= - D_{yx} \overline{\frac{\partial S_s}{\partial x}} + D_{yy} \overline{\frac{\partial S_s}{\partial y}} + D_{yz} \overline{\frac{\partial S_s}{\partial z}} \\ \overline{\dot{w} \dot{S}} &= - D_{zx} \overline{\frac{\partial S_s}{\partial x}} + D_{zy} \overline{\frac{\partial S_s}{\partial y}} + D_{zz} \overline{\frac{\partial S_s}{\partial z}} \end{aligned} \quad (13)$$

Now substituting equation (13) into equation (12) and dropping the averaging bar notations and scripts on S, leads to the general convective-dispersion equation.

$$\begin{aligned} \frac{DS}{Dt} = & \frac{\partial}{\partial x} D_{xx} \frac{\partial S}{\partial x} + D_{xy} \frac{\partial S}{\partial y} + D_{xz} \frac{\partial S}{\partial z} + \frac{\partial}{\partial y} D_{yx} \frac{\partial S}{\partial x} + D_{yy} \frac{\partial S}{\partial y} + D_{yz} \frac{\partial S}{\partial z} \\ & + \frac{\partial}{\partial z} D_{zx} \frac{\partial S}{\partial x} + D_{zy} \frac{\partial S}{\partial y} + D_{zz} \frac{\partial S}{\partial z} \end{aligned} \quad (14)$$

For the one-dimensional case, equation (14) reduces to

$$\frac{\partial S}{\partial t} + u_s \frac{\partial S}{\partial x} = D_{xx} \frac{\partial^2 S}{\partial z^2} \quad (15)$$

It has been postulated by Scheidegger⁽²¹⁾ that the longitudinal dispersion coefficient is related to the pore system geometry and the seepage velocity in the following form.

$$D_{xx} = \alpha_1 u_s \quad (16)$$

where α_1 depends upon the geometry of the pore system. It has also been found^(2,5,9) that the coefficient α_1 is proportional to the mean grain size, i.e.

$$\alpha_1 = \beta d_{50} \quad (17)$$

where β depends upon shape and size distribution of particles.

Therefore

$$D_{xx} = \beta d_{50} u_s \quad (18)$$

dividing both sides with the kinematic viscosity, we obtain

$$\frac{D_{xx}}{\nu} = \beta R_{d_{50}} \quad (19)$$

Experiments by others (1,2,4,9,...) including the present investigation indicate a relationship

$$\frac{D_{xx}}{\nu} = \psi R_{d_{50}}^n \quad (20)$$

as a better representation of all previously reported data. The term ψ is not necessarily equal to β . Also equation (1a) can be rewritten as

$$d_{50} = \frac{\sqrt{k}}{\sqrt{c}} \quad (21)$$

Substituting this in equation (18) we have

$$D_{xx} = \frac{\beta}{\sqrt{c}} \sqrt{k} u_s \quad (22)$$

Dividing both sides by kinematic viscosity again, we obtain

$$\frac{D_{xx}}{\nu} = \frac{\beta}{\sqrt{c}} R_k \quad (23)$$

in which β/\sqrt{c} is presumably a function of particle shape and size distribution.

By the same reason a more general relation would be

$$\frac{D_{xx}}{\nu} = \phi R_k^n \quad (24)$$

in which ϕ may not be necessarily equal to β/\sqrt{c} .

The initial and boundary conditions for the present investigation are illustrated below. The solution to equation (15) for these conditions is the following

$$\frac{S}{S_0} = \frac{1}{2} \operatorname{erfc} \frac{z - u_s t}{2 \sqrt{D_{xx} t}} + \frac{1}{2} \exp \frac{u_s x}{D_{xx}} \operatorname{erfc} \frac{x + u_s t}{2 \sqrt{D_{xx} t}} \quad (25)$$

$$S(0, t) = S_0, \quad t > 0$$

$$S(x, 0) = 0, \quad x > 0$$

$$S(\infty, t) = 0, \quad t > 0$$

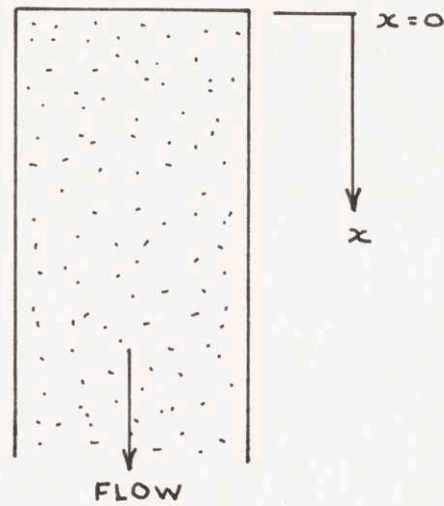


Figure 2.

Illustration of Initial and Boundary Conditions

It has been shown that⁽²³⁾ the second term in equation (16) can be neglected when $D_{xx}/u_s x \approx .0075$. This is true in the present case and equation (16) can be written in a dimensionless form, as

$$\frac{S}{S_0} = \frac{1}{2} \operatorname{erfc} \frac{1 - \xi}{2 \sqrt{\xi \eta}} \quad (26)$$

where

$$\xi = \frac{u_s t}{x} = \frac{t}{t_{50}}$$

and

$$\eta = \frac{D_{xx}}{u_s x}$$

Graphical representation of the solution is shown in the figure below. It has been found that for the solution given the mean point of concentration $S/S_0 = .5$ will move at the average seepage velocity in a saturated medium with no dead end pores.

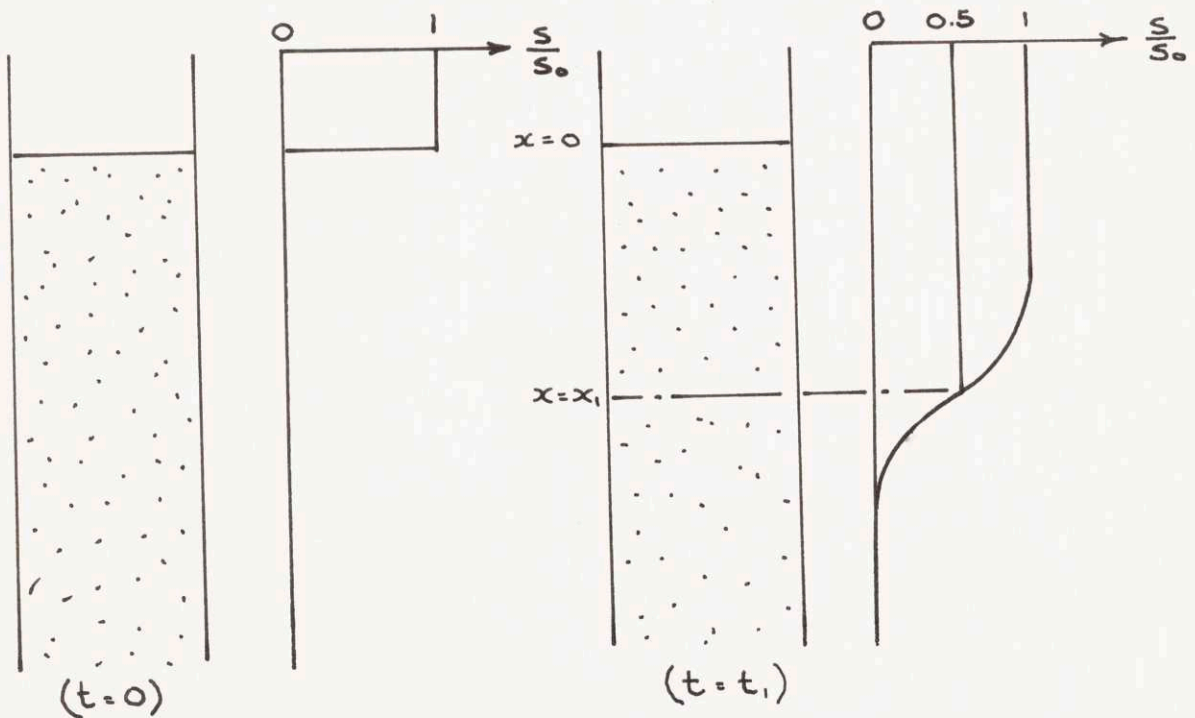


Figure 3.

Graphical Representation of the Solution

EXPERIMENTAL EQUIPMENT AND PROCEDURE

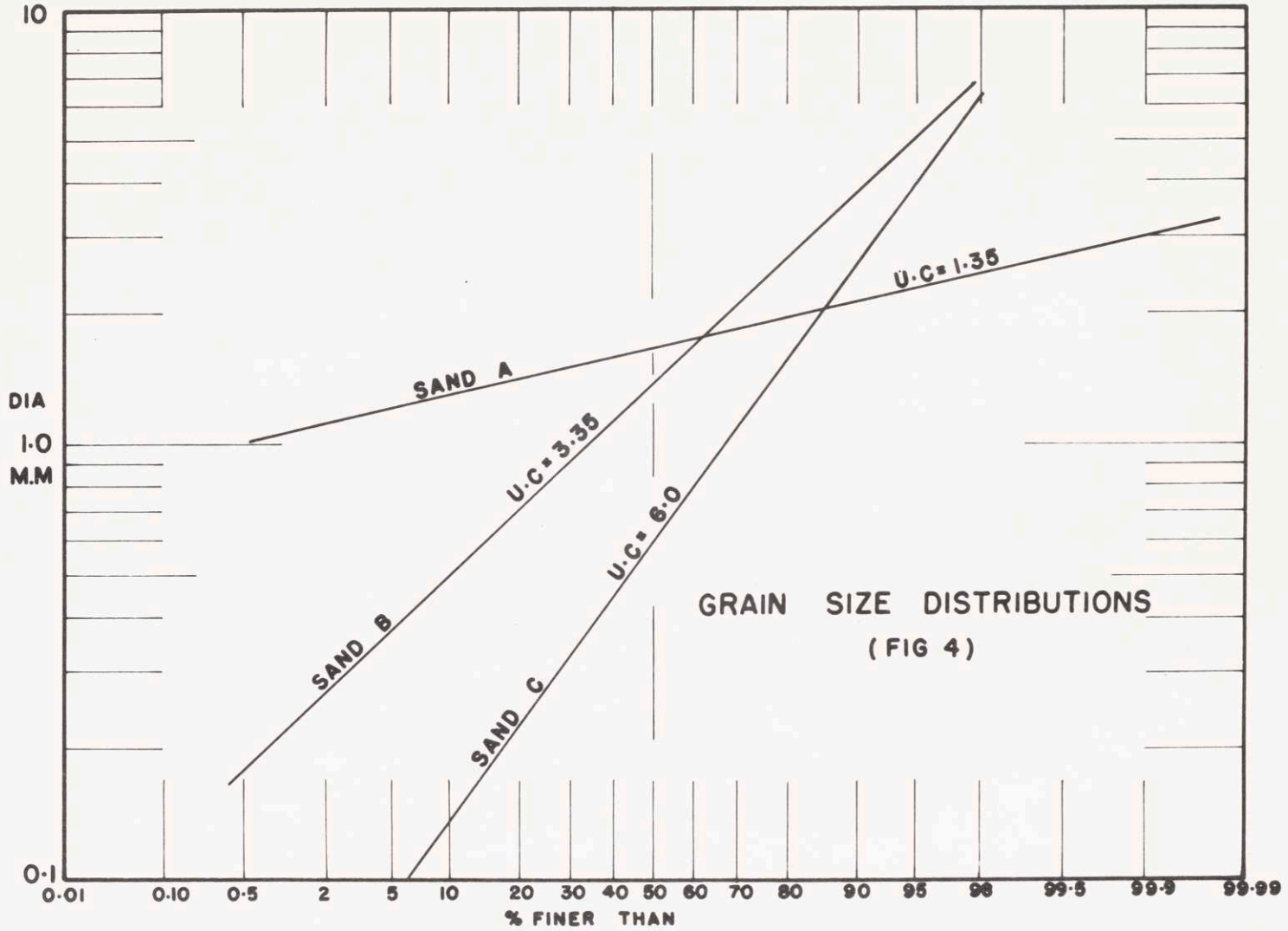
Media Characteristics and Definitions

Three different media with progressively increasing uniformity coefficients were investigated. These were designed mixes, perfectly normally distributed except sand A which was not mixed but used as obtained. It appeared from sieve analysis that (Figure 4) it was fairly normally distributed though not very non-uniform. For the purposes of the present experiments it was sufficiently accurate to assume a continuous distribution of its grain sizes.

As for sands B and C, they were composed of fractions sieved from three different sands and then mixed in continuous distribution of sizes so that within 5 and 95% all consecutive fractions of U.S. standard series were present; as such they could truly be called continuously distributed mixtures.

The range of sizes for sand A (Figure 4) varied from 1.5% finer than 1 mm to 97.9% finer than 2.85 mm. The geometrical mean size was 1.67 mm and a uniformity coefficient equal to 1.35.

The synthetic mixtures of sand B (Figure 4) had a continuous distribution of sizes varying from 1.6% finer than .25 mm to 91.5% finer than 4 mm. Such a distribution



Grain Size Distributions of Sands (on Log Probability Paper)

gave a geometric mean size of 4.51×10^{-3} ft. and a uniformity coefficient $(d_{60})/(d_{10}) = 3.35$. This is quite a highly non-uniform mixture.

Sand C (Figure 4) was again designed as a synthetic mixture, continuously and normally distributed between the range of 5% finer than .088 mm to 90% finer than 4 mm. Such a distribution gave a highly non-uniform mixture, perfectly normally distributed with a geometrical mean size of .6 mm (1.968×10^{-3} ft.) and a uniformity coefficient of 6.0. This was the most highly non-uniform mixture that could be obtained from the sands that were available. It's not common occurrence in nature to find sands more non-uniform than those investigated. Therefore a non-uniformity of such a magnitude was considered sufficient for the interest of the present study.

Reason for generating normally distributed mixture is that natural sands and aquifer material generally are normally distributed about a geometric mean.

The hydraulic properties of packed beds depends on mean size as well as on the spread of sizes. However, research has dealt primarily with mean size with little attention to characteristics of distribution.

The distribution of sizes may be represented graphically by plotting "percent by weight finer than" vs.

"grain size". Such a graph is known as cumulative frequency distribution. There are many mathematical formulations available for fitting frequency distributions, only the normal or Gaussian distribution is most useful because of its common occurrence in natural materials and random processes.

Frequency distribution on an ordinary arithmetic paper plots an S-curve. It is usually difficult to readily determine whether the S-curve plotted through a set of data points is an S-curve representing normal distribution. Thus it is helpful to distort a graph paper so that a cumulative normal distribution appears as a straight line. Such a paper is called arithmetic probability paper. When the spread is large, log probability paper is used. Natural sands plot nearly straight lines on such paper with slight distortions at the ends.

One of the greatest advantages of log probability paper is that one can read off geometric mean size (d_{50}) and geometric standard deviation ($\sigma_g = \frac{d_{84.1}}{d_{50}} = \frac{d_{50}}{d_{14.9}}$) (measure of spread) without going into lengthy calculations. Measure of non-uniformity can also conveniently be determined ($U.C. = \frac{d_{60}}{d_{10}}$ on log probability paper).

A cumulative frequency plot for sands A, B, and C with

other pertinent information is given in Figure 4.

Sands from which different fractions were obtained were very similar in texture (Figures 5,6,7). This delineates shape effects in the experiments.

Packing of Media

Packing of non-uniform material to obtain isotropic and homogeneous media is no easy task. This difficulty was, however, overcome by dividing the mixture into large numbers of random samples by means of a random samples (Figure 8). This assured uniform distribution of sizes in each of the subdivided samples which were in turn poured into the column through a specially constructed long tailed tunnel. The tunnel was continuously moved in all directions inside the column to assure a uniform spreading of the falling material. Such a device also standardized the length of fall of each subdivided sample so that if there was any sorting of sizes during falling it would be the same for subsequent samples. Since the diameter of the tail of the tunnel was not large, there was no appreciable sorting of sizes.

Column

All experiments were carried out in a column of lucite with an internal diameter of 9.78 cms (Figure 9). The base of the column was provided with a porous plate which contained



Fig. 5 Enlarged Photograph of Grains of Sand A



Fig. 6 Enlarged Photograph of Grains of Sand B

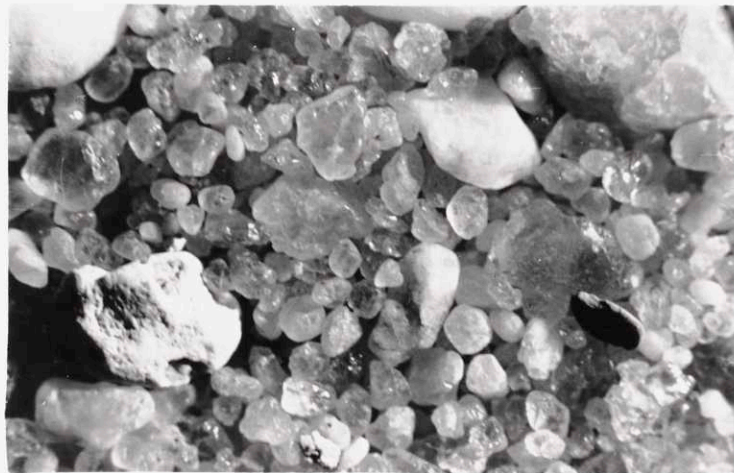
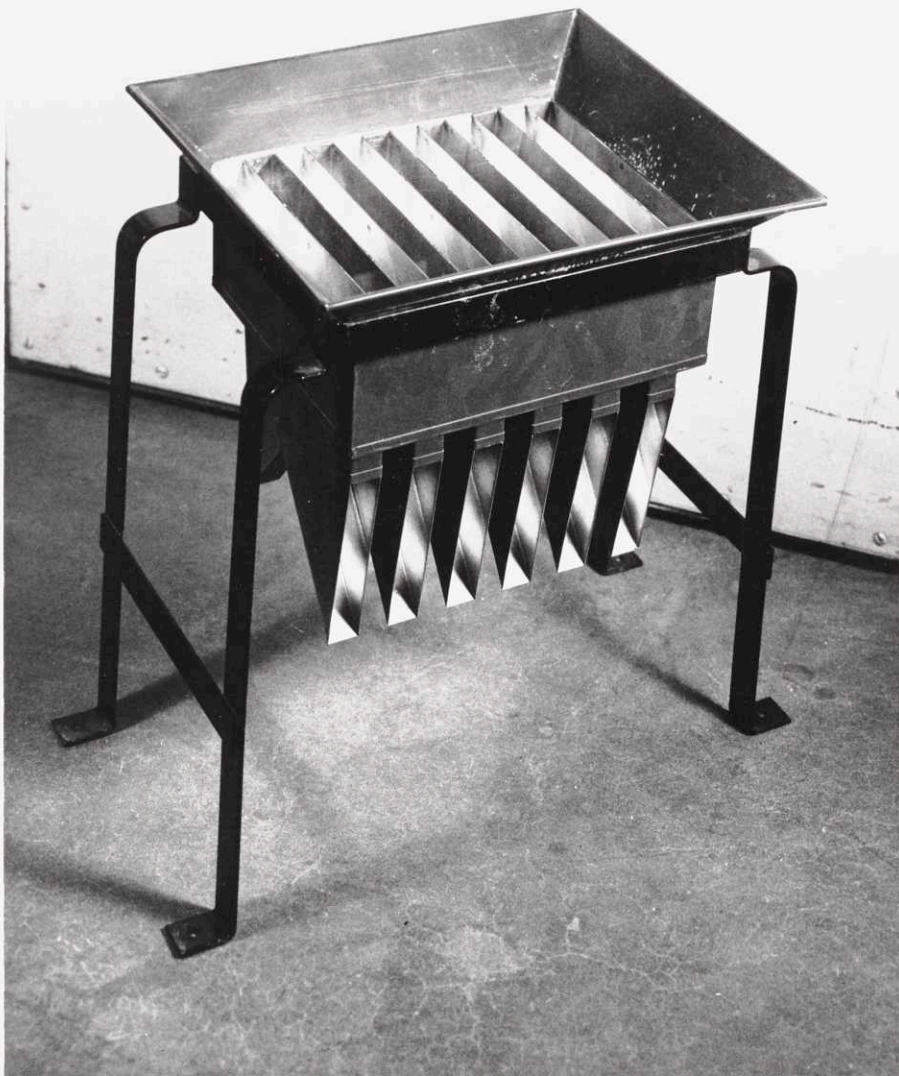
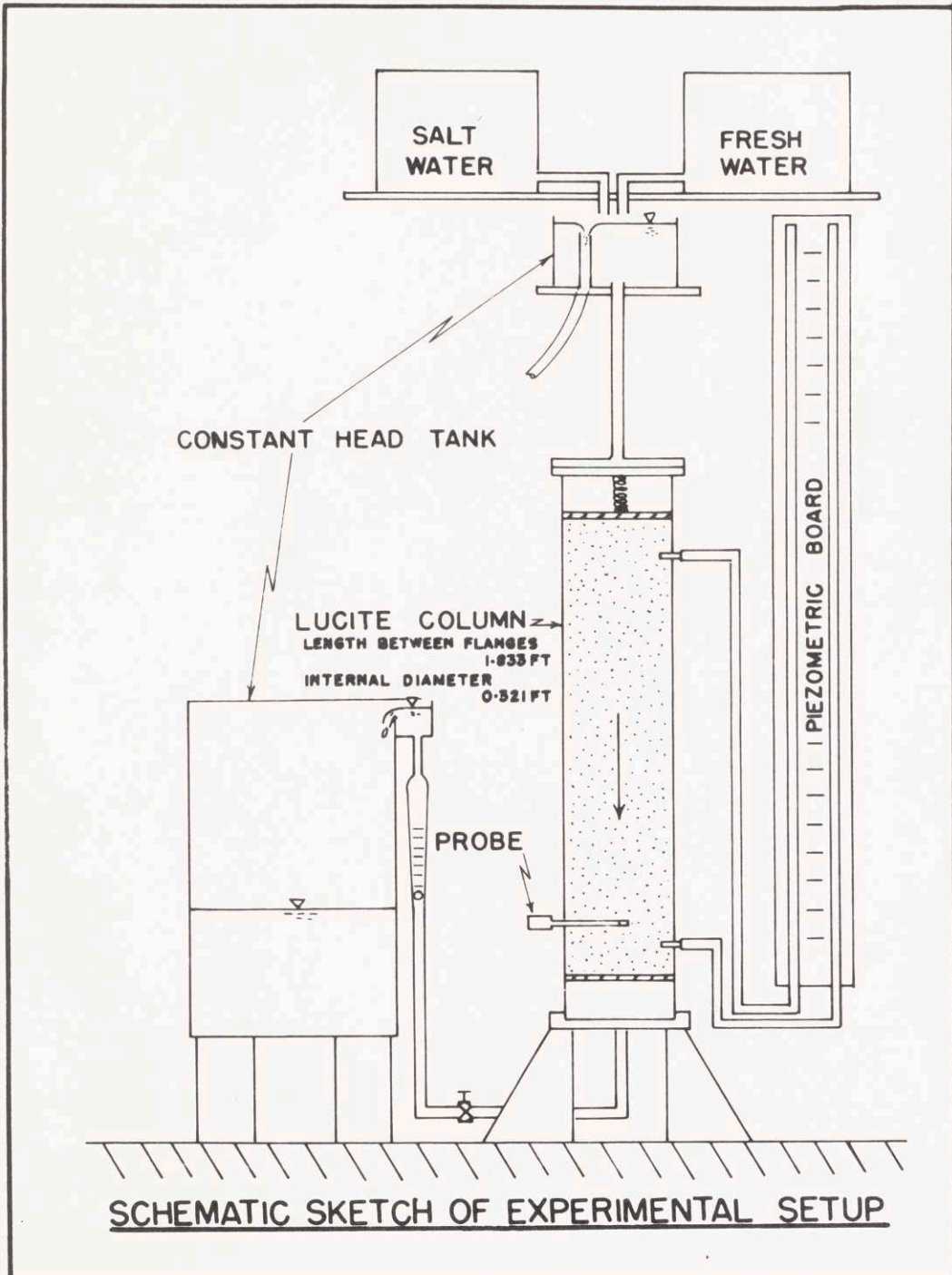


Fig. 7 Enlarged Photograph of Grains of Sand C



SAMPLER

(fig. 8)



(FIG 9)

Schematic Sketch of Apparatus

the material. The top of the media was covered with a 50 mesh screen mounted in a lucite frame which could be moved inside the column. Such an arrangement enabled filling the column to any convenient level.

After achieving a desired packing the frame containing the screen was slid into position to rest on top of the sand and then held in position by means of a compressed spring. The other end of the spring rested against a solid lucite plate provided with a rubber gasket. The neoprene rubber gasket and lucite cover made a leakproof lid for the column. Four half inch openings covered with rubber stoppers provided a means of getting air out of the system before starting the runs. Spring tension eliminated development of void spaces below the screen due to settlement. No settlement was, however, noticed.

Fluids

Three fluids were used during the experiments. For sands A and C, only water which was deaerated by spraying in a vacuum tank was used. However, for sand B, three different fluids -- water ($\nu = 1.09 \times 10^{-7}$ ft²/sec at 68°F); 30% glycerol solution ($\nu = 252 \times 10^{-7}$ ft²/sec at 68°F); and 60% glycerol solution ($\nu = 103 \times 10^{-6}$ ft²/sec at 68°F) were used to investigate the effect of viscosity upon dispersion.

Initial Filling of the Packed Column

This is quite an important step in the experiment because any air entrainment can seriously alter the behavior of the porous medium. This can be seen from Orlob and Radhakrishna's work⁽⁶⁾.

When the media was packed and spring loaded top screen and lid slid into place, with rubber stopper removed, highly deaerated water was very slowly fed from the bottom so that in the process of filling the voids air was continuously being ejected from the top. It was observed that slower filling rates very rapidly removed traces of air in the column. Each initial filling of the column took between 45 to 60 minutes and this gave reasonably good results. Even after the column was saturated, continuous flow of deaerated water was maintained for a reasonably long time. In all cases about 30-40 gallons of deaerated water were passed through the medium before commencing experiments. By this time there was no trace of entrapped air.

After the initial filling and flushing of the column, fluid was then fed from a constant head tank by means of a flexible tubing. The constant head tank was about 2' above the top of the column in the case of the water, and about 8-10' for the case of glycerol solutions.

The constant head tank was fed by two 5 gallon reservoirs, one of which contained tracer solution and the other either water or glycerol. The flow was then collected in another constant tank after passing through a system of valves, one of which was electrically operated. By this means a pre-set valve position was used for each run and enabled to synchronize start of flow with the Sanborn timer.

Measurement of Flow

Three means were at hand for measuring flow. One was by passing the flow through a Rotometer after it left the valves. The second was measuring flow volumetrically by means of a point guage (Figure 9). The third was to directly obtain seepage velocity from the breakthrough curve. In the present experiments seepage velocities as obtained from breakthrough curves have been used. These are the most representative velocities associated with the progressive dispersing front.

Instrumentation and Tracer Solution

About a tenth of a percent by weight of sodium chloride was used as tracer. At such low concentrations the conductance of NaCl is known to vary linearly with concentration⁽²⁾. In literature NaCl solution has been often described to possess

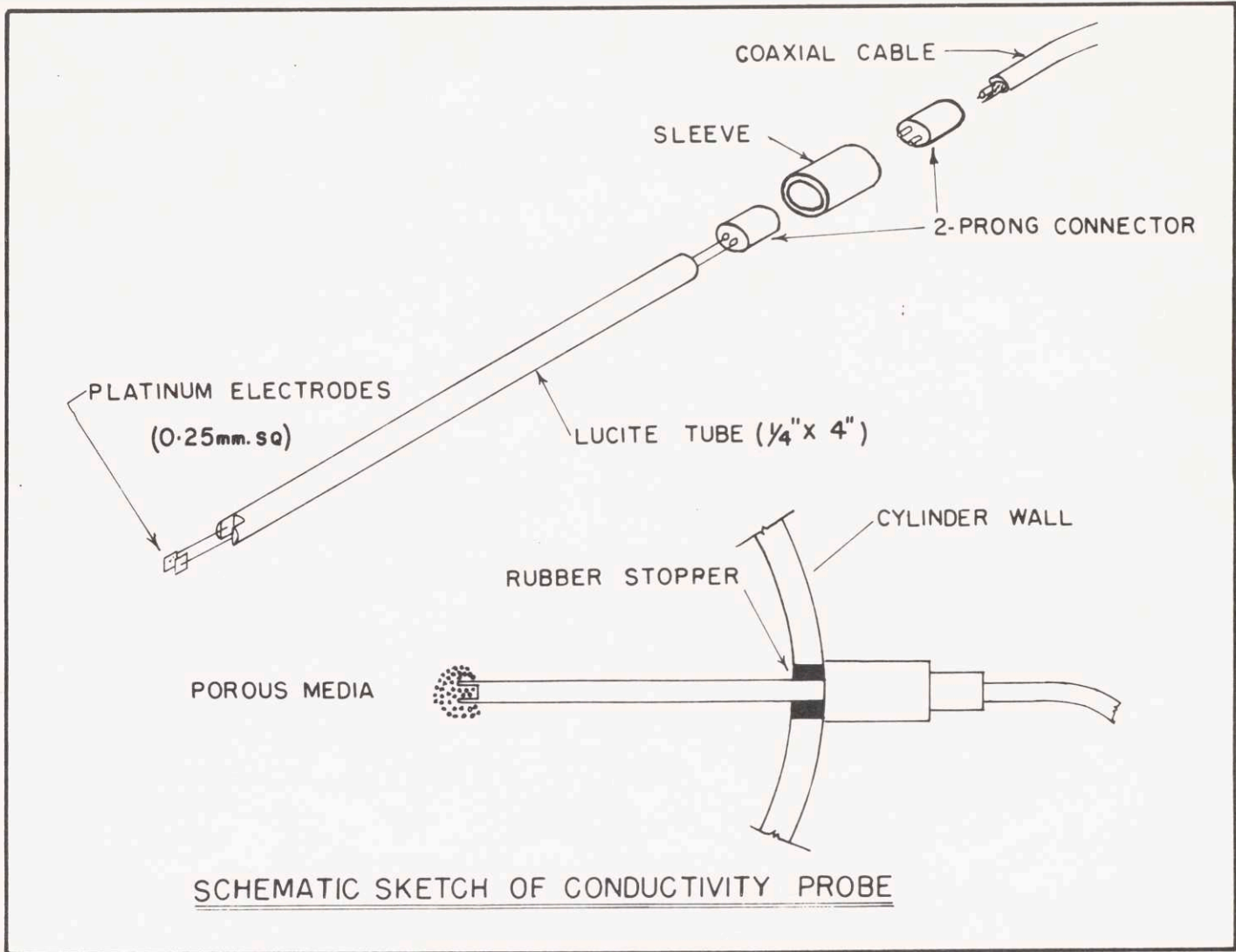
the best attributes of a tracer.⁽²⁴⁾

Conductivity Probe

The conductivity probe (Figure 10) used was similar to the one earlier used by Harleman, Rumer and Mehlhorn⁽³⁾. It essentially consists of 1/4" O.D. lucite tube through which two solid copper wires are led and connected to platinum electrodes. These electrodes (approx. 2.5 mm x 2.5 mm) were cemented to the inside faces of an open slotted lucite rod (Figure 10). The rod was cemented to the tube and a receiving socket was fixed to the other end with the help of a sleeve. This arrangement provided a compact waterproof conductivity probe. No moisture was ever noticed to enter the probe. The probe was inserted inside the column through a rubber stopper bushing at a distance of about 40 cms from the top. The probe was adjusted inside the column such that the electrodes were parallel to the walls and the space between them central inside the column. It may be noted that the probe was inserted before the column was packed.

Sanborn Recorder

Conductivity signals from the probe were fed into a four channel Sanborn recorder, model 154, equipped with an



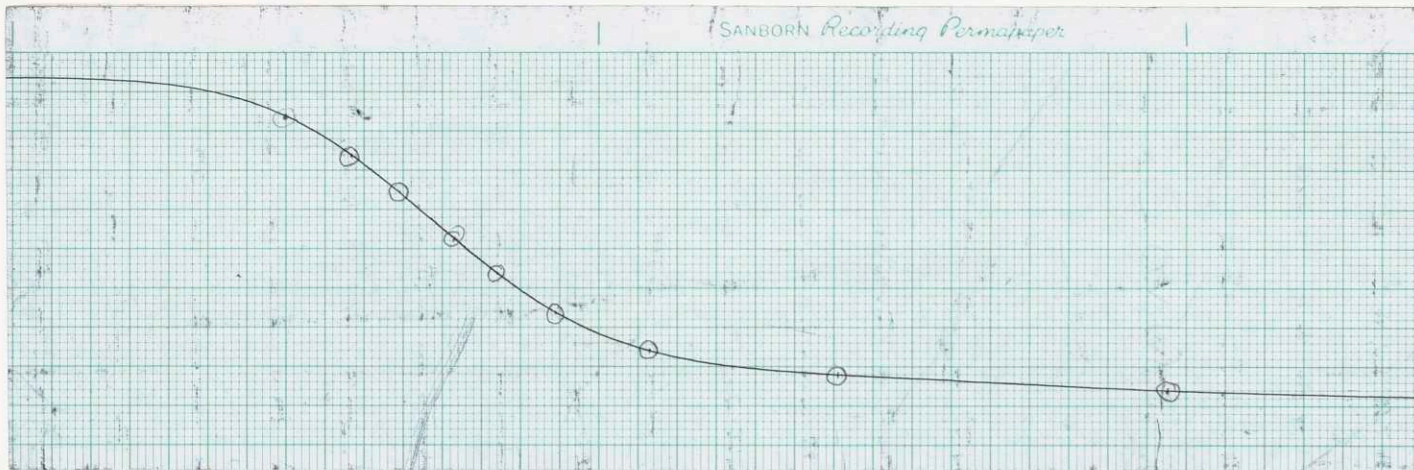
(FIG 10)

Schematic Sketch of Conductivity Probe

independent timer. The sensitivity of the amplifier is 8 microvolts per mm deflection of the recording styles, and a frequency response of 0-100 cycles with standard 2400 cps excitation. Paper speed of the recorder could be varied from 0.25 mm/sec to 25 mm/sec depending, of course, on the rate of flow in the column. Probe signals could also be attenuated over a wide range for suitable measurements.

Calibration of Probe

It has been shown^(1,2,3) that at low concentrations the plots of stylus deflection versus concentrations of NaCl are all linear. Few experiments with low solutions concentration of 0.05, 0.075, and 0.10 percent gave very nearly linear relations as stated above. Such a response facilitated calculations because the theoretical solution of the dispersion equation is given in terms of relative concentrations S/S_0 . Therefore, for any particular run, the relative concentration at any time could be directly read from the stylus recording by simply taking the ratio of the deflection at that instant to the deflection when the tracer front had passed the probe (Figure 11). There are two other factors that need to be considered. One regarding the initial concentration of NaCl in fresh water and the other of the solution concentration dependence upon temperature.



Sketch of the Sanborn Recording Paper
with a Typical Breakthrough Curve

(fig. 11)

The first of these was eliminated by balancing the Sanborn stylus for zero deflection in fresh water. The second factor did not present much of a problem since temperature for most part remained constant during a run. The validity of using S/S_0 versus time plots straight from the Sanborn recording has been well established^(1,2,3).

Experimental Runs

Dispersion runs

As discussed earlier, after complete saturation was obtained, constant head reservoir was drained. Also, the water level in the column was lowered to the upper confining screen. With all the rubber stoppers in the upper lid in place, tracer solution was gradually introduced into the constant head device. Simultaneously one of the stoppers was slowly lifted to enable air escape and let the tracer solution enter the column until all the air escaped and there was only tracer solution above the screen. Such a procedure fulfilled one of the initial conditions that at $t = 0$, $S/S_0 = 1$, for $x \leq 0$. The probe was connected to one of the channels of the Sanborn and balanced for zero deflection. Flow rate and Sanborn timer were started at the same time giving a time indication of $t = 0$ on the Sanborn paper. Flow rate was previously adjusted, therefore, at

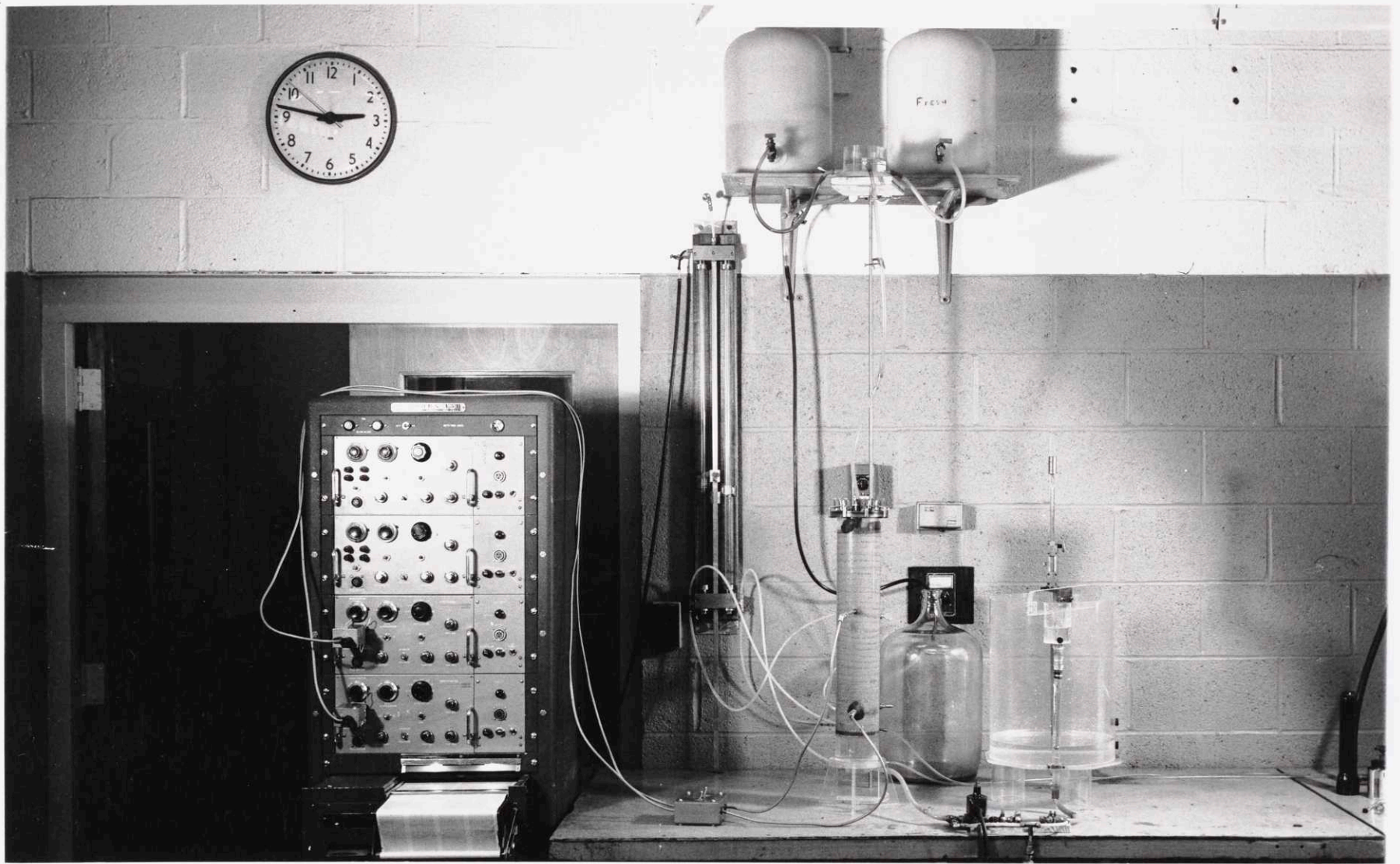
$t = 0$ flow commenced for a pre-set valve position. Measurements of temperature were taken frequently to see that no large variation occurred which might affect the conductivity dependent stylus recordings. Depending on the rate of flow each run took anywhere between five minutes and two and a half hours.

Permeability runs

Permeability measurements were taken independent of dispersion runs to avoid any possible flow distortions caused by manometric fluctuations. Recordings of both breakthrough and elution runs were made and analyzed.

The photograph on page 45 shows the general arrangement of the experimental equipment as set up in the Laboratory.

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Experimental Apparatus

PRESENTATION AND DISCUSSION

The correlations

$$1. \quad D = \alpha u_s^n \quad (27)$$

$$2. \quad D/\nu = \Psi R_{d50}^n \quad (20)$$

$$3. \quad D/\nu = \Phi R_k^n \quad (24)$$

for sands A, B, and C with uniformity coefficients of 1.35, 3.35, and 6.0 have been presented in Figures 12, 13, and 14, respectively.

All of the above parameters correlate well; however, the various coefficients assume different values for the three different sands. The value of the exponent "n" has remained constant for each sand in the three correlations, but progressively decreases toward unity with increasing non-uniformity.

Glycerol solutions of 30% and 60% concentration in water were used in the case of sand B to study the effect of viscosity on the various coefficients of equation (20) and equation (24), and results are presented in Figures 18 and 19.

The various media characteristics are given in Table I. While the values of seepage velocities and corresponding

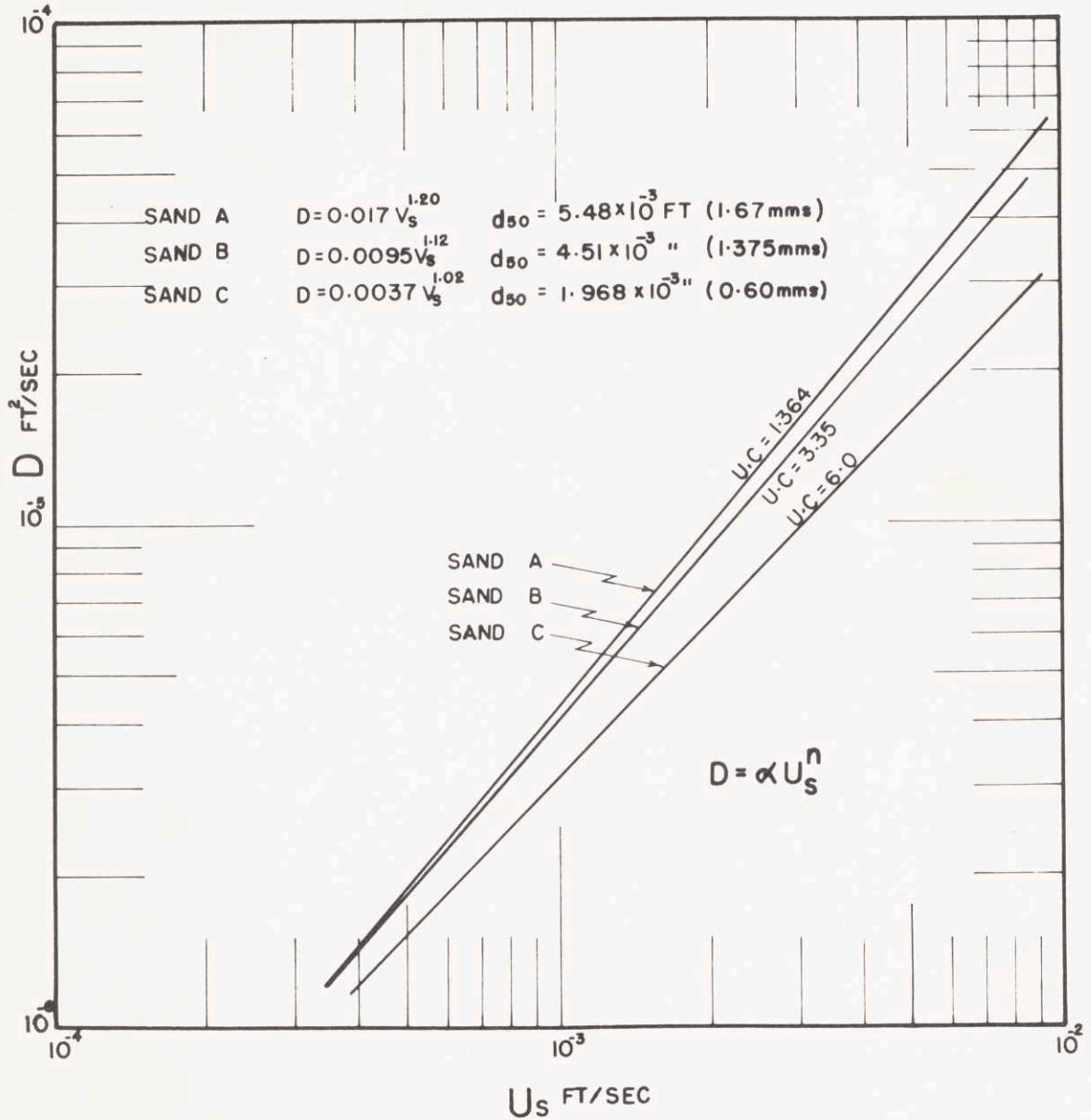
dispersion coefficient together with particle and permeability Reynolds numbers is given in Table II. The values which the coefficients in equations (20), (24), (27) assume for the different sands are given in Table III.

Since the exponent was found to decrease with increasing non-uniformity, an attempt to correlate n with increasing non-uniformity gave a constant trend as shown in Figure 15.

It was observed that variation of Ψ with U.C. could not be correlated in any meaningful manner.

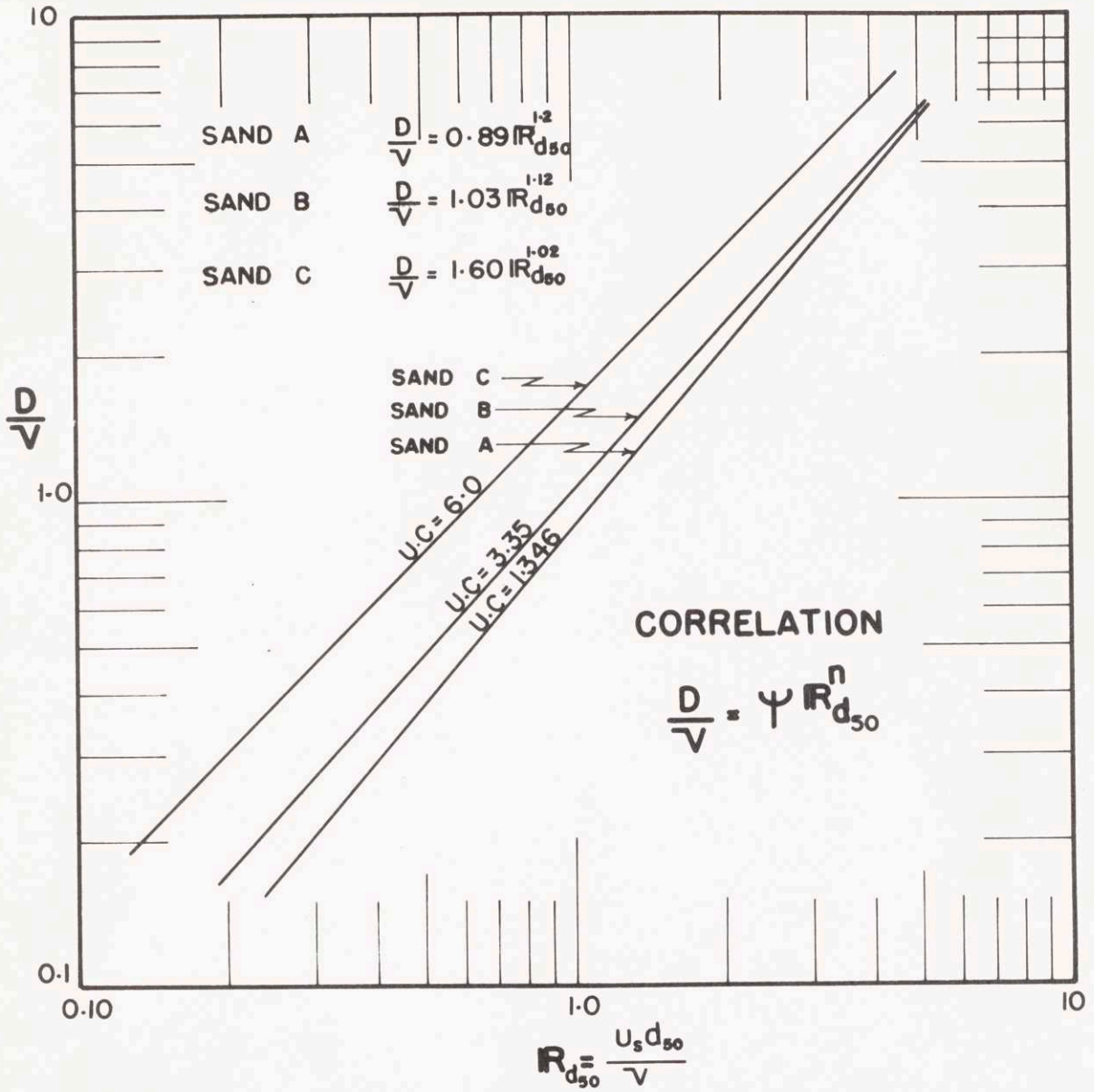
The constant Φ of equation (24), however, correlated well with uniformity coefficient (Figure 16) as would be expected, since it is believed that intrinsic permeability is a more representative length characteristic of the porous medium, especially when the medium is non-uniform.

A good correlation $P_k = 0.03(U.C.)^{-1.14}$ (Figure 17) between permeability Peclet number and U.C. seems to exist in a region where n is nearly unity. The correlation is substantiated by data from Raimondi's work⁽⁵⁾. It is seen that Rafai⁽²⁴⁾ and Harleman and Mehlhorn⁽²⁾ points do not fit well. This is presumably because of the non-normal and discreet distribution of grain sizes that they worked with. In the region where n is large, i.e. near U.C. of between one and two, the correlation breaks down and a scatter of values of



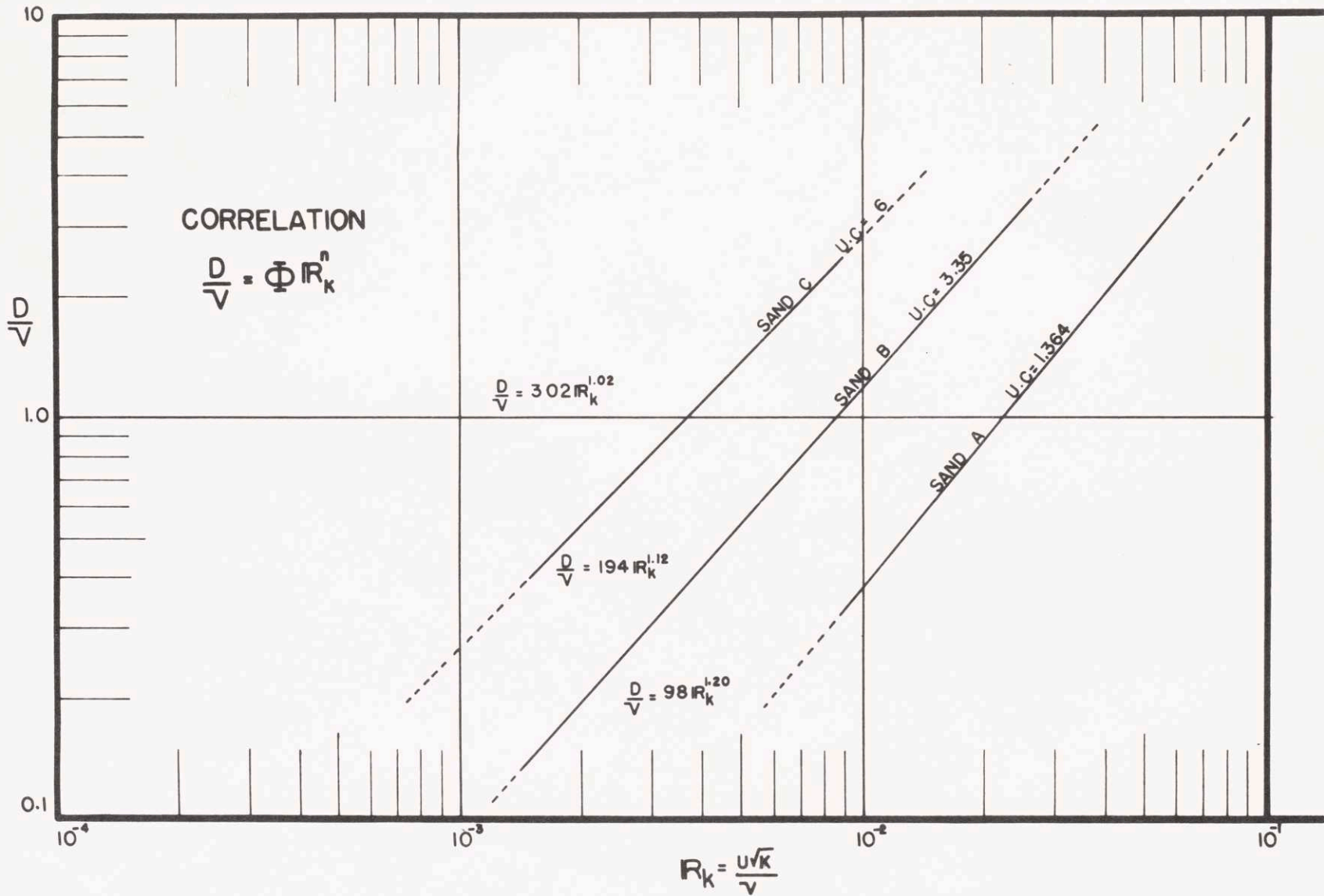
(FIG 12)

Variation of Dispersion Coefficient with Seepage Velocity



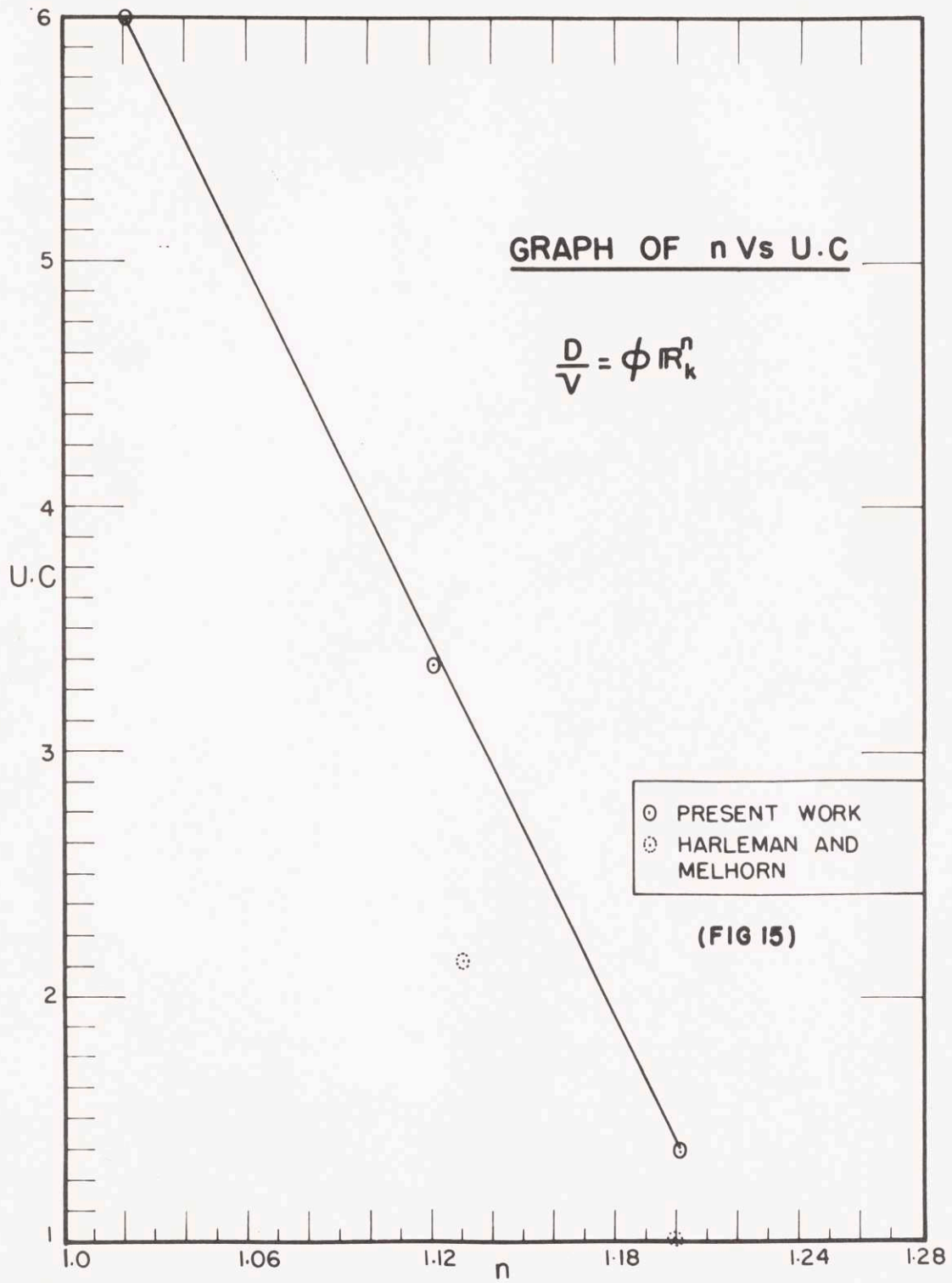
(FIG 13)

Effect of Uniformity Coefficient on the Correlation $\frac{D}{v} = \Psi(u_s d_{50})/v$

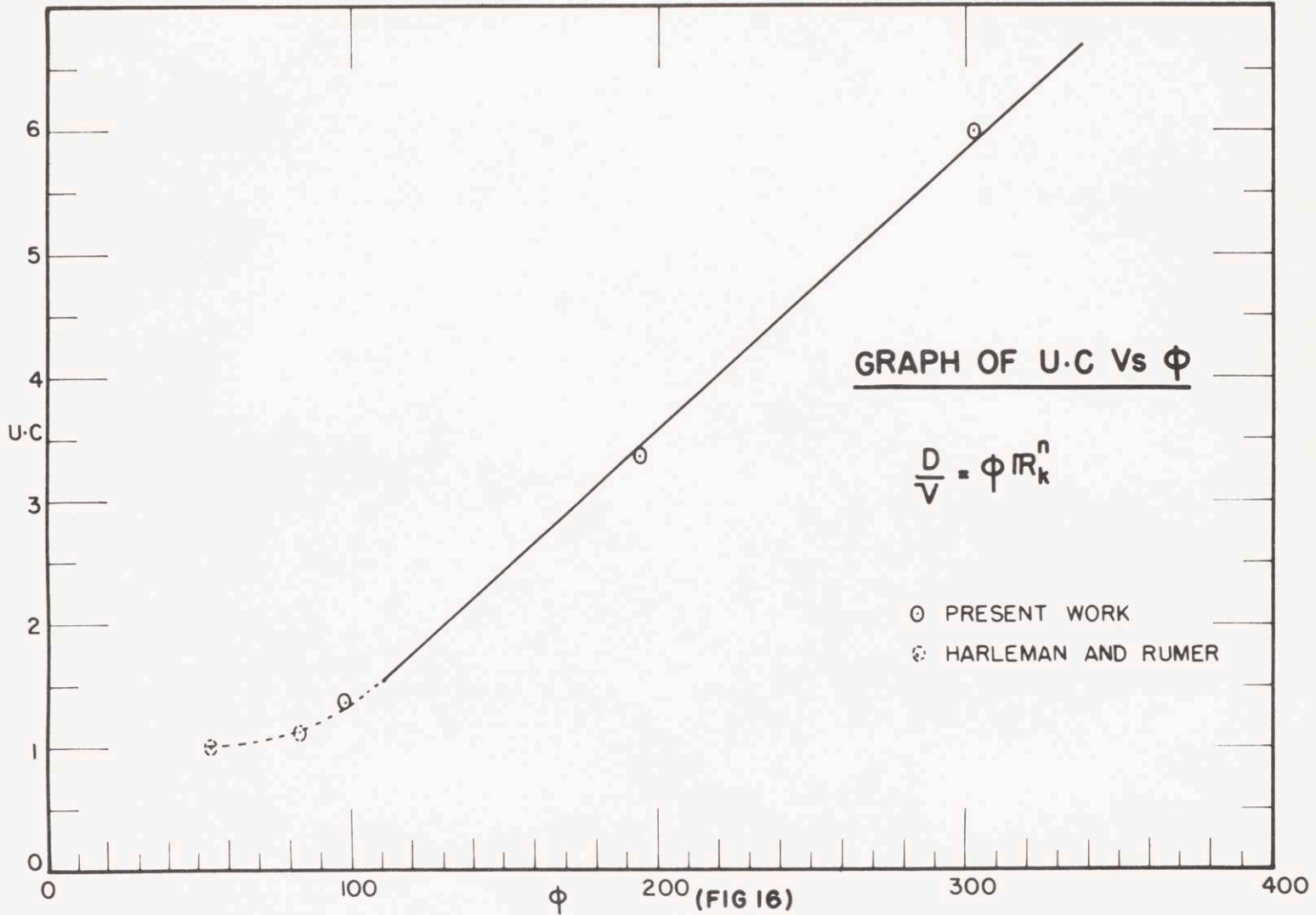


(FIG 14)

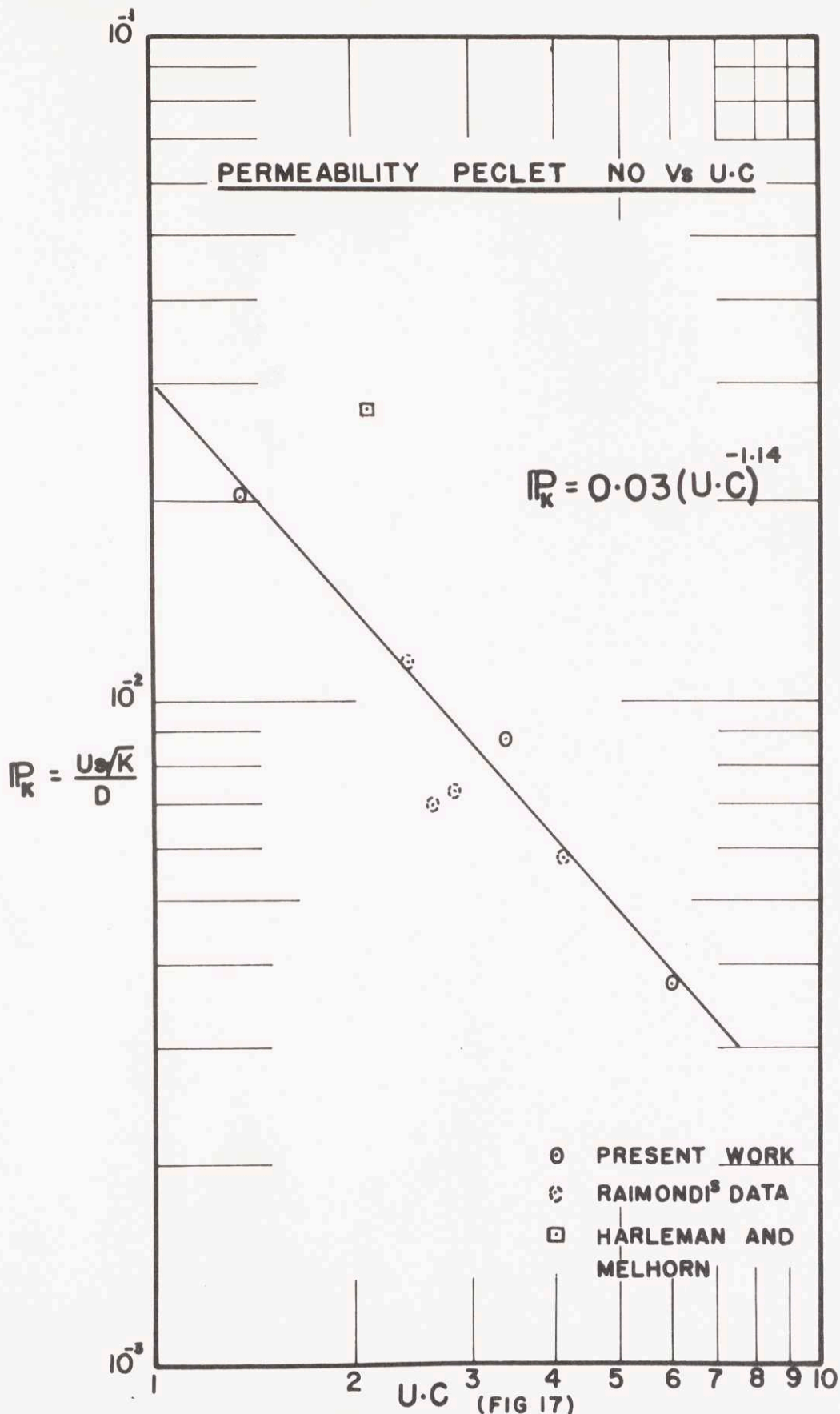
Effect of Uniformity Coefficient on the Correlation $D/v = \Phi (u_s \sqrt{k})/v$



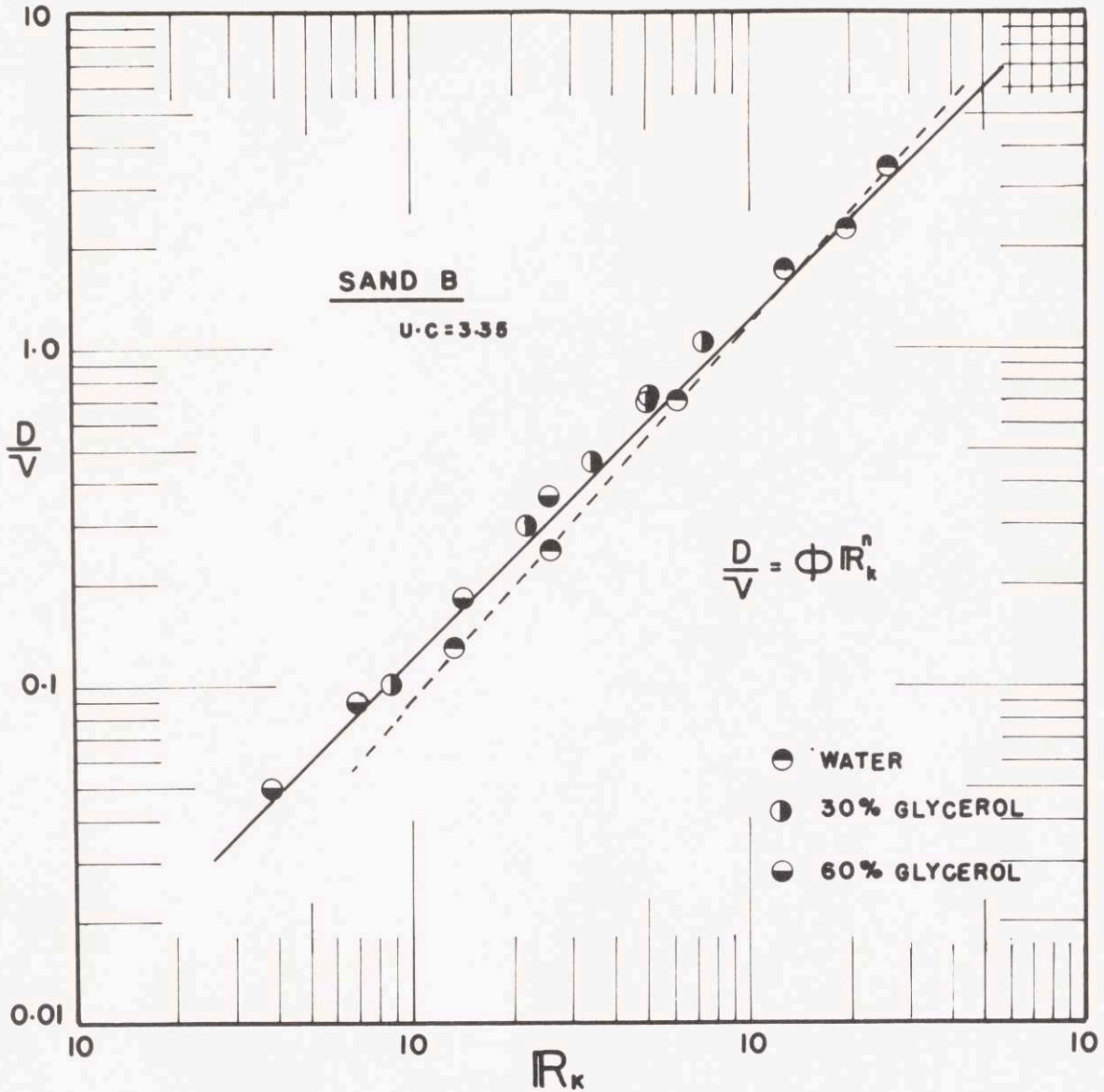
Effect of Uniformity Coefficient on the Exponent n



Effect of Uniformity Coefficient on the Constant ϕ



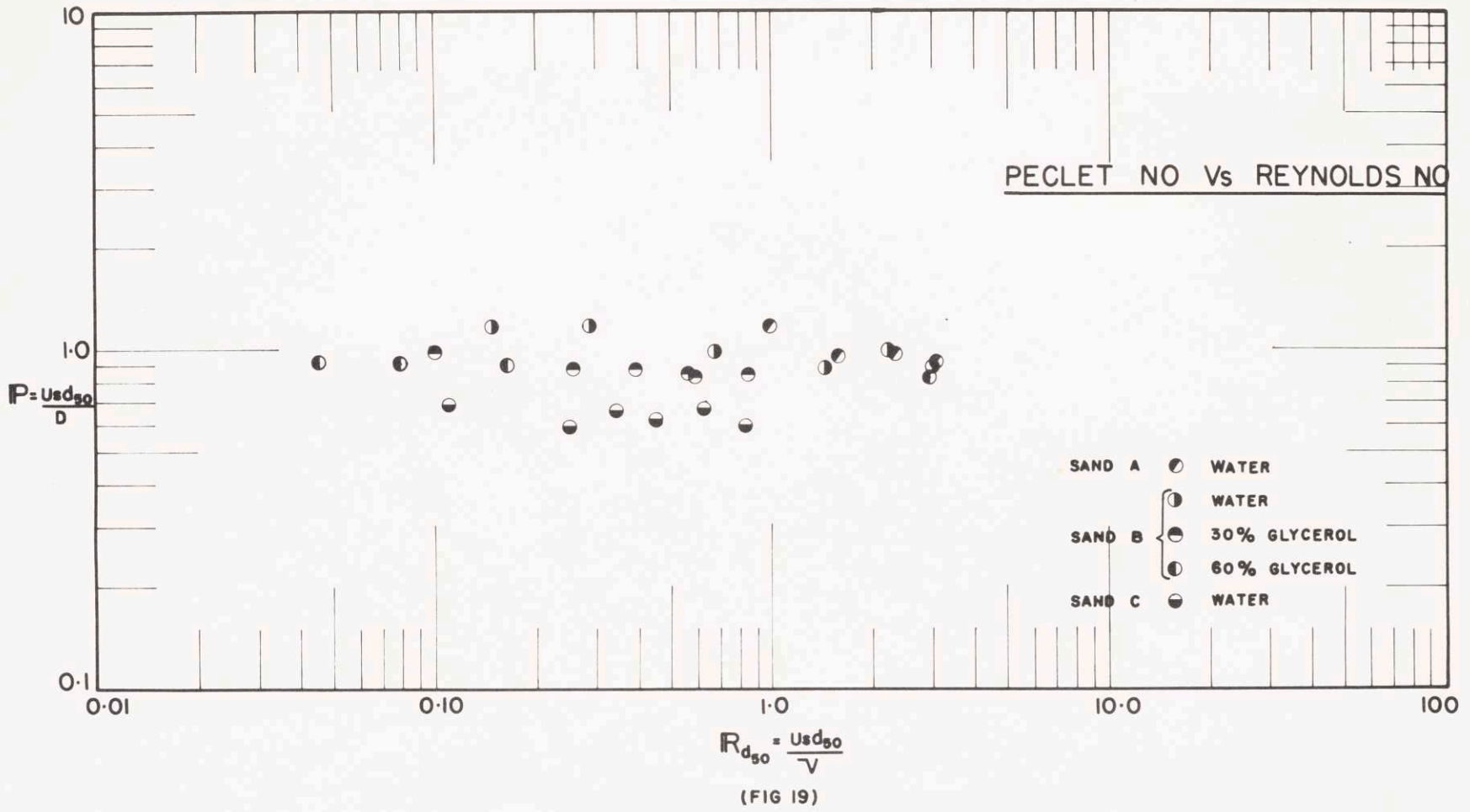
Effect of Uniformity Coefficient on the Permeability Peclet Number (P_k)



(FIG 18)

Effect of Viscosity on the Correlation $\frac{D}{v} = \phi R_k^n$

-55-



Variation of Particle Peclet Number with Particle Reynolds Number

TABLE I

<u>Sand</u>	d_{50} ft	U.C.	θ	k ft ²
A	5.41×10^{-3} (1.67 mm)	1.35	0.354	1.167×10^{-8}
B	4.51×10^{-3} (1.375mm)	3.35	0.29	1.509×10^{-9}
C	1.968×10^{-3} (0.6 mm)	6.0	0.287	1.4×10^{-10}

TABLE II

Sand	U.C.	Fluid	U_s ft/sec	D ft ² /sec	d_{50}	k
A	1.35	Water	3.035×10^{-3}	1.74×10^{-5}	1.57	3.095×10^{-2}
			4.50×10^{-3}	2.57×10^{-5}	2.33	4.593×10^{-2}
			6.14×10^{-3}	3.70×10^{-5}	3.18	6.26×10^{-2}
			8.58×10^{-4}	3.58×10^{-5}	0.44	8.75×10^{-2}
			5.98×10^{-3}	3.62×10^{-5}	3.01	6.098×10^{-2}
			1.93×10^{-3}	0.91×10^{-5}	0.97	1.968×10^{-2}
B	3.35	Water	1.61×10^{-3}	0.75×10^{-5}	0.68	5.88×10^{-3}
			6.51×10^{-4}	0.25×10^{-5}	0.29	2.55×10^{-3}
			3.51×10^{-3}	1.83×10^{-5}	1.46	1.30×10^{-2}
			5.31×10^{-3}	2.43×10^{-5}	2.22	1.95×10^{-2}
			7.15×10^{-3}	3.69×10^{-5}	2.99	2.60×10^{-2}
			3.71×10^{-4}	0.139×10^{-5}	0.15	1.33×10^{-3}
		30% glycerol	1.44×10^{-3}	0.75×10^{-5}	0.26	2.21×10^{-3}
			3.27×10^{-3}	1.79×10^{-5}	0.59	5.04×10^{-3}
			2.21×10^{-3}	1.16×10^{-5}	0.40	3.41×10^{-3}
			5.65×10^{-4}	2.60×10^{-5}	0.10	8.71×10^{-4}
			4.78×10^{-3}	2.61×10^{-5}	0.85	7.37×10^{-3}
			3.22×10^{-3}	1.74×10^{-5}	0.57	4.96×10^{-3}
		60% glycerol	6.83×10^{-3}	3.76×10^{-5}	0.30	2.57×10^{-3}
			3.77×10^{-3}	1.9×10^{-5}	0.16	1.42×10^{-3}
			1.05×10^{-3}	0.52×10^{-5}	0.046	3.95×10^{-4}
			1.86×10^{-3}	0.93×10^{-5}	0.082	7.02×10^{-4}
C	6.0	Water	4.46×10^{-3}	1.49×10^{-5}		4.98×10^{-3}
			3.34×10^{-3}	0.99×10^{-5}		3.73×10^{-3}
			2.42×10^{-3}	0.78×10^{-5}		2.70×10^{-3}
			1.86×10^{-3}	0.56×10^{-5}		2.07×10^{-3}
			1.43×10^{-3}	0.45×10^{-5}		1.50×10^{-3}
			5.89×10^{-4}	0.17×10^{-5}		6.58×10^{-3}

TABLE III

<u>Sand</u>	<u>U.C.</u>	<u>α</u>	<u>Ψ</u>	<u>Φ</u>	<u>n</u>
A	1.35	0.017	0.89	98	1.20
B	3.35	0.0095	1.03	194	1.12
C	6.0	0.00368	1.60	320	1.02

permeability Peclet numbers for the same U.C. may be obtained. Such a trend is essentially manifested because Peclet number is not independent of the Reynolds number when the exponent n is not unity. In other words, the porous medium is no longer a completely random ensemble.

It has been pointed out earlier that for any particular sand, the exponent n remains constant in all the equations (20, 24, and 27). Such a tendency is the outcome of the fundamental nature of equation (27). In the derivation of equation (20) and equation (24), the development is started by assuming equation (16) from Scheidegger's⁽³¹⁾ postulation. In effect equations (27) and (16) are identical when $n = 1$. The present investigation, however, confirms that the value of the exponent in equation (20) and equation (24) will always be the same as in equation (27).

It will be shown later that n actually measures the randomness of the medium and is reflected in the power of the seepage velocity. Since all three equations involve directly the functional dependence of the dispersion coefficient on seepage velocity, n would, therefore, remain the same and is a constant of the medium.

Since equation (27) is so fundamental, it might be useful to investigate what values of n and α have been

reported in literature for different media. Scheidegger⁽²¹⁾ has pointed out that there are theoretically two possibilities to correlate dispersion with velocity:

$$D = \alpha u_s$$

$$D = \alpha u_s^2$$

However, he agrees that experiments seem to indicate that the first of these forms corresponds more to physical reality. This is borne out by the work of Ebach and White⁽⁹⁾, who have reported $n = 1.06$ (uniform material): Rafai, Kaufman, and Todd⁽²⁴⁾ reported $n = 1.02$ for uniform Ottawa sand; Raimondi, Petrick, and Gardner⁽⁵⁾ reported $n = 1.0$ (uniform and non-uniform materials). Ebach⁽¹⁹⁾, in his Ph.D. thesis, reports $n = 1.068$ (uniform materials); Harleman and Rumer⁽¹⁾ report $n = 1.10$ (uniform material); Harleman, Mehlhorn, and Rumer^(2,3) report $n = 1.20$ (uniform material). It has also been established⁽³⁾ that n is unaffected by particle size, packing, shape, etc., and therefore the above values of n can be generally compared without defining the size of the materials.

Though theory predicts a value of $n = 1$ or 2 , the truth really lies somewhere in between. Different numbers have been reported but little effort has been made in the

past to investigate what essentially causes changes in n . The present investigation indicates that n is a strong function of the uniformity coefficient.

Saafman⁽²⁰⁾ and Dejong⁽³¹⁾, like Scheidegger, have also considered the random nature of the porous medium in one form or another and have theoretically predicted a dispersion coefficient which is linearly related to the velocity.

It is difficult to imagine a porous medium composed of uniform spheres, raschig rings, berl saddles, and intalox saddles, etc., without distribution in sizes, to be a truly random system. Just the uniformity of size is enough to break the element of complete disorder and as such the randomness that is almost invariably assumed in theoretical models. Extending the argument, therefore, it may be said that higher non-uniformity is an approaching condition to complete randomness. It would, therefore, be predicted that the higher the non-uniformity the more nearly the medium approaches ideal randomness and n would tend to assume values close to unity. Such a prediction is borne out from the findings of the present investigation.

The above approach enables one to conclude that the difference between n and unity is in effect a measure of the randomness of the porous medium; that higher uniformity

coefficients indicate a closer approximation to a completely random system.

Decreasing value of n with increasing non-uniformity may also be explained from an analogy between laminar flow in porous media and completely turbulent flow in bulk fluids. The dispersion of individual particles in a porous media was first interpreted in such a manner by Yuhara⁽²⁵⁾.

The fluctuation of velocities of the particles can be claimed as being analogous to the fluctuations of velocities during eddy motion in completely turbulent flow. As such, the flow path of particles in flow through a porous medium can be regarded as analogous to the trajectory of a particle in turbulent hydraulic motion. It might, therefore, be expected that results which hold good in the case of turbulent flow through pipes, etc., will also yield reasonable results if applied to laminar flow in porous media.

Since the flow path of a fluid particle in a macroscopically identical porous media is a probability phenomenon (over the ensemble), it can be thought of as corresponding to the random flow path of a fluid particle during turbulent motion in a bulk mass of fluid. The appearance of a "dispersivity" in hydrodynamics in porous media may be thought of as analogous to the diffusivity of eddies, etc.,

in turbulent motion. In fact it is by this analogy with turbulence that diffusivity equations have been suggested for laminar flow through porous media without recourse to a proper statistical treatment.

It is known that eddy diffusivity is linearly related to shear velocity. Therefore, in the completely turbulent range when the friction factor f becomes independent of the Reynolds number, eddy diffusivity is a linear function of the average velocity. Consequently, it would be expected that for a completely random porous media, the dispersivity is also linearly related to the average velocity. Since the only difference in the two cases is that in one there is a random variation in velocity at a point with respect to time, and in the other there is a random variation of velocity in space with respect to time and these two situations are quite analogous. Based on our basic premise that complete randomness in porous media cannot be achieved unless there is enough pore size distribution, it is concluded that the higher the non-uniformity, the closer will be the value of n to unity.

In Figure 12, it is seen that α decreases as the non-uniformity increases. This is contrary to what would be expected. It is not immediately obvious to draw conclusions

from this before examining individually the factors that affect α . Particle shape, size, permeability, and grain size distribution all affect α ^(2,5,6), and in the present investigation all the factors except shape have changed simultaneously. Therefore, it is difficult to interpret the variation of α as a function of uniformity coefficient alone. However, it has been established that for constant grain size distribution, α almost increases linearly with the square root of the intrinsic permeability⁽⁵⁾. Also keeping permeability constant, α increases with increasing uniformity coefficient⁽⁵⁾, though not as steeply. Harleman and Mehlhorn⁽²⁾ have established the effect of grain size on α . It has been shown that α again increases linearly with mean particle diameter. In the present investigation, the mean particle size and permeability have been continually decreasing with increasing uniformity coefficient. It appears, therefore, that the effect of non-uniformity on α has been outdone by the dominant effects of particle size and permeability, and hence accounts for progressively decreasing values of α with correspondingly increasing non-uniformities.

It is interesting to see that when the effects of particle shape and permeability are taken into account together with the functional relation of dispersion and

velocity, as in equations (20) and (24), the results obtained are as would be expected. Dispersion increases with increasing non-uniformity (Figures 13 and 14). In other words, within the range studied, dispersion is more for non-uniform media for constant values of particle and permeability Reynolds numbers.

It might then be concluded that dispersion behavior is more appropriately described by the form of equations (20) and (24).

Intrinsic Permeability as Characteristic Length Parameter for Description of Porous Media

Several investigators⁽²⁶⁾ have analytically and empirically related permeability to the various media properties. Not all are general and few give exact results because of the difficulty of including all of the possible variations of porous media. For an assemblage of uniform spheres, permeability can be accurately determined⁽²⁶⁾ from known porosity and packing conditions. The results even when applied to selected sands need much modification.

A formula developed from dimensional considerations by Fair and Hatch⁽²⁷⁾ can be considered typical of many of the contributions. It has been verified experimentally and gives good indication of the factors which determine permeability.

The intrinsic permeability can be expressed as

$$k = \frac{1}{m \frac{(1-\theta)^2}{\theta^3} \left(\frac{F}{100} \sum \frac{P}{d_m} \right)^2} \quad (28)$$

where θ is the porosity. The factor m is a packing factor, found experimentally to be about 5; F is the sand shape factor, varying from 6 for spherical grains to 7.7 for angular grains; P is the percentage of sand held between adjacent sieves; and $d_m = \sqrt{d_1 d_2}$ is the geometric mean of rated sizes, d_1 and d_2 of adjacent sieves. The equation is dimensionally correct so that any system of units may be used.

It is evident, therefore, from the above that k (intrinsic permeability) is a function of the particle size, shape, packing, and the uniformity coefficient, or the geometric standard deviation of the grain size distribution.

Therefore

$$k = f(\theta, m, F, d_{50}, U.C.) \quad (29)$$

Also, it has been established that dispersion is a function of the physical characteristics of the porous medium and of the average seepage velocity^(1,2,3,5) and, therefore, the dispersion coefficient can be expressed as

$$D = f(\theta, m, F, d_{50}, U.C. \text{ and } u_s) \quad (30)$$

From equation (29) it is observed that equation (30) can be

expressed as

$$D = f(k, u_s) \quad (31)$$

Present experimental investigation, however, indicates that D is more appropriately represented by a function of the form

$$D = f(k, u_s, U.C.) \quad (32)$$

It might not be obvious why $U.C.$ should not be reflected in k , and, therefore, the functional dependence of equation (31) be adequate to describe dispersion. There are many reasons and probably the most important is the fact that \bar{k} represents only an average characteristic pore dimension and does not give enough idea of the particle size distribution that reflects pore sizes distribution also. Since dispersion is essentially dependent upon the randomly varying velocity distribution within the pores, intrinsic permeability alone is insufficient to describe the dispersion process. However, together with some measure of the distribution of sizes, it suitably describes this mixing process. It might then be argued, though in a lesser measure, why k describes other media properties adequately for purposes of dispersion. This may, however, be explained from the experimental findings of others. It has been established⁽²⁾ that equations (20) and (26) are unaffected by changes in

particle size, porosity, and packing. Porosity and packing are in any case strong functions of permeability, and may not be taken into account when a permeability is considered. Change in shape, however, changes the coefficients Ψ and Φ . By considering sand of the same shape, such effect is eliminated in the present investigations (Figures 4,5, and 6), but it is difficult to discount altogether the variations caused by shape change. Particle shape definitely affects the friction behavior of the porous media. The extent to which such an effect is reflected in the permeability is also difficult to evaluate but it is evident that intrinsic permeability still remains a better parameter than any other characteristic of the porous media to take account of shape effects.

It may be said, therefore, that the functional dependence of dispersion coefficient is suitably described by equation (32).

Having established the validity of equation (32), the value of σ/k as a non-dimensionalizing characteristic length parameter is evident, and, therefore, it might be concluded that the square root of intrinsic permeability is a more suitable and representative length characteristic of the porous media than the mean particle size, especially when

the media are non-uniform. That the mean particle size alone is not sufficient to represent the characteristic length of the porous media is also apparent from the fact that Ψ and U.C. could not be correlated in any meaningful way. On the other hand, Φ and U.C. gave excellent correlation (Figure 16).

Effect of Viscosity

Since it has been shown that the square root of intrinsic permeability is a more appropriate length characteristic and that equation (24) is a very suitable form of representing the dispersion process in a porous media, all the data for different fluids used in the case of sand B have been plotted in the form of the ratio of dispersion coefficient to kinematic viscosity versus permeability Reynolds number (Figure 18). It is seen that even a tenfold change in the viscosity does not produce a significant change in the form of equation (24). The dotted line in Figure 18 indicates the results for water as the fluid and the firm line is the best for all the data (water, 30% glycerol, and 60% glycerol). The difference in the two lines is exceedingly small considering a tenfold change of viscosity. It seems therefore, that equation (24) is a good means of representing dispersion when viscosities are changing. In any case, Ebach and White⁽⁹⁾ have shown that dispersion is not significantly

affected by viscosity changes. In their experimental findings, they observed that dispersion coefficient changed negligibly by changing viscosity from 0.95 to 27 cp (centipoises), keeping velocity constant, in a bed of 1 mm spheres. (Reynolds range $0.6 > d_{50} > 0.02$). Chemical Engineers⁽¹⁰⁾, on the other hand, have shown that though viscosity may have small effects on axial dispersion in the laminar flow, these effects become very significant at high Reynolds numbers. It has also been shown⁽¹⁰⁾ that particle Peclet number, $(ud_{50})/D$, remains constant with respect to particle Reynolds numbers for all viscosities. This is in fair agreement with the results of the present investigation (Figure 19).

The slight upward displacement of data for glycerol in Figure 18 can be explained by an analogy between laminar flow in Capillary tubes and laminar flow in porous media. Taylor⁽²⁸⁾ has analytically shown that effective dispersion coefficient in laminar flow in Capillary tubes can be expressed as $D = (a^2 u_0^2) / 192 D_m$ where u_0 is the maximum velocity at the axis; a is the radius of the tube; D is the effective dispersion coefficient; and D_m is the molecular diffusion coefficient. Thus for constant velocity, dispersion is inversely proportional to molecular diffusion coefficient. With increasing viscosity D_m decreases and therefore D increases.

As such, qualitatively, slight upward shift of data for higher viscosity fluids may be explained. It might be concluded, therefore, that correlation expressed by equation (24) is valid for a large range of viscosity levels in the laminar flow region.

Effects of Molecular Diffusion and Range of Validity of Results

Dispersion has been generally described by the equation of the form

$$D = D_m + \alpha u_s$$

to be valid for the entire viscous flow region⁽⁵⁾. The term D_m (molecular diffusion coefficient) is often neglected for higher velocities. Limits in terms of Reynolds numbers and velocity rates have been suggested by various investigators to define the point at which molecular diffusion becomes as important as dispersion and can no longer be neglected. It is of interest to see the lower limit of the validity of the equation (24). Harleman and Rumer⁽²⁾ in comparing their longitudinal and lateral dispersion data with Raimondi et al.'s work showed that molecular diffusion becomes important at a particle size Reynolds number of about 10^{-3} (for glass spheres $d_{50} = .0115$ cm). But the effect of molecular diffusion on longitudinal dispersion depends not

only on the magnitude of the coefficient of diffusion but also on the surface area of contact between fresh and salt water. It is, however, not easy to separate these two effects until surface area of all sand grains could be measured. Such an attempt in a non-uniform, non-spherical sand particle is a tremendous task. Therefore the validity of this lower limit of 10^{-3} cannot be generalized.

In the derivation of the coefficient of molecular diffusion Beran⁽²⁹⁾ distinguished between three different cases: (a) negligible molecular diffusion effects; (b) dominant molecular diffusion effects; and (c) when the random arrangement of the sand grain and movement due to molecular diffusion are of approximately equal importance in determining dispersion. These three cases are distinguished by different values of $L = (U d_{50})/D_m$, U being average velocity of flow; d , a characteristic sand grain diameter; and D_m , the molecular diffusion coefficient. When $L > 1$, it is case (a); when $L < 1$, it is case (b); and (c) is for intermediate values of L . The values of L corresponding to the minimum values of velocity for the three sands investigated are 12.9, 3.75, and 2.58. Thus by the above criterion the molecular effects are negligible. In the absence of a better criterion it may be concluded that molecular diffusion effects

have been negligible in the present work and that the dispersion process is adequately described by equation (27), which in turn leads to a more appropriate representation in the form of equation (24).

As a final observation with respect to molecular diffusion, it may be interesting to note the general convergence of the lines in Figure 12. Garrel, et al.⁽³⁰⁾ found experimentally that dispersion in porous media due to molecular diffusion only was identical with dispersion of a tracer in a stationary liquid, and is independent of pore size except for pores approaching the diameters of ions. Such may be the case in Figure 12 also, as the velocity decreases the effect of molecular diffusion becomes more and more felt and it seems that the point at which these lines meet may indicate incipience at which molecular diffusion is as important or more than dispersion. It is not clear what shape and form these lines would assume at exceedingly low velocities but the convergence certainly suggests that media properties begin to become less and less important. This is in agreement with Garrel's work.

Some of the Practical Uses of the Present Findings

As final closing remarks it might be of interest to indicate some of the uses of the present findings.

As has been observed earlier, different values of exponent n in the fundamental equation (27) have been reported and there has been nothing in literature from which to determine quantitative or even qualitative trends the exponent n assumes under a given set of conditions. It has now been established that n essentially is a function of the uniformity and may be regarded as a measure of the randomness of a medium. Figure 16 defines n for a given medium from grain size distribution.

Also it has been established that the square root of the intrinsic permeability is a more appropriate representation of the characteristic length of the porous media and as such equation (27) is a very suitable form for representing the dispersion process between two miscible fluids in such a medium. With the help of equation (16) and knowing the permeability in the field, dispersion coefficient is totally defined and hence we obtain a complete solution of equation (15).

Having established the fact that \sqrt{k} is a better correlative parameter, hydraulic studies in fibrous and consolidated materials, where the usual media characteristics are impossible to obtain, might find a useful tool to operate with in the absence of other media characteristics such as

mean grain size.

The unique qualities of \sqrt{k} suggested to correlate dispersion parameters in the form of equation (24) for different viscosities. It has been found that this is a suitable way of presenting data and that for a tenfold change in viscosity no significant change from that of water has been observed. As such, Chemical and Petroleum Engineers might find such correlation very useful in reactor designs and oil extraction by miscible displacements in porous media, respectively.

CONCLUSIONS

1. The correlations

$$D = \alpha u_s^n \quad (\text{Fig. 12})$$

$$D/\nu = \Psi R_{d50}^n \quad (\text{Fig. 13})$$

$$D/\nu = \Phi R_k^n \quad (\text{Fig. 14})$$

hold for non-uniform media. There are, however, changes in the various coefficients.

2. For any particular sand, n remains constant in all the above correlations. It decreases with increasing uniformity, approaching unity for highly non-uniform media (Figure 15).

3. α is found to decrease with increasing non-uniformity (Figure 12). The influence of non-uniformity on α cannot be singled out in the present investigation since mean size, intrinsic permeability, and grain size distribution, all of which are known to affect α , have been changed simultaneously. In any event, α is not a very meaningful parameter unless $n = 1$, otherwise it has very peculiar dimensions.

4. It has been established that \sqrt{k} is a more representative length characteristic of the porous media

than the geometric mean size -- especially when the medium is non-uniform.

5. The value of Ψ in equation (20) increases with increasing non-uniformity (Figure 13). No meaningful correlation between Ψ and n or U.C. could be established. As such, equation (20) is not a very suitable way of correlating dispersion parameters for non-uniform media.

6. The value of Φ in equation (24) increases with increasing non-uniformity (Figure 14). A good correlation between Φ and U.C. has been obtained (Figure 16), which enables prediction of dispersion coefficient from a knowledge of permeability and grain size distribution, which are measurable quantities in the field.

7. It has been found that a tenfold change in viscosity did not significantly alter equation (24).

8. For non-uniform porous media (when $n = 1$), good correlation between the permeability Peclet number and U.C. has been obtained (Figure 17). It is substantiated by data from Raimondi's⁽⁵⁾ work. It leads to an expression of the form $P_L = 0.03(U.C.)^{-1.14}$.

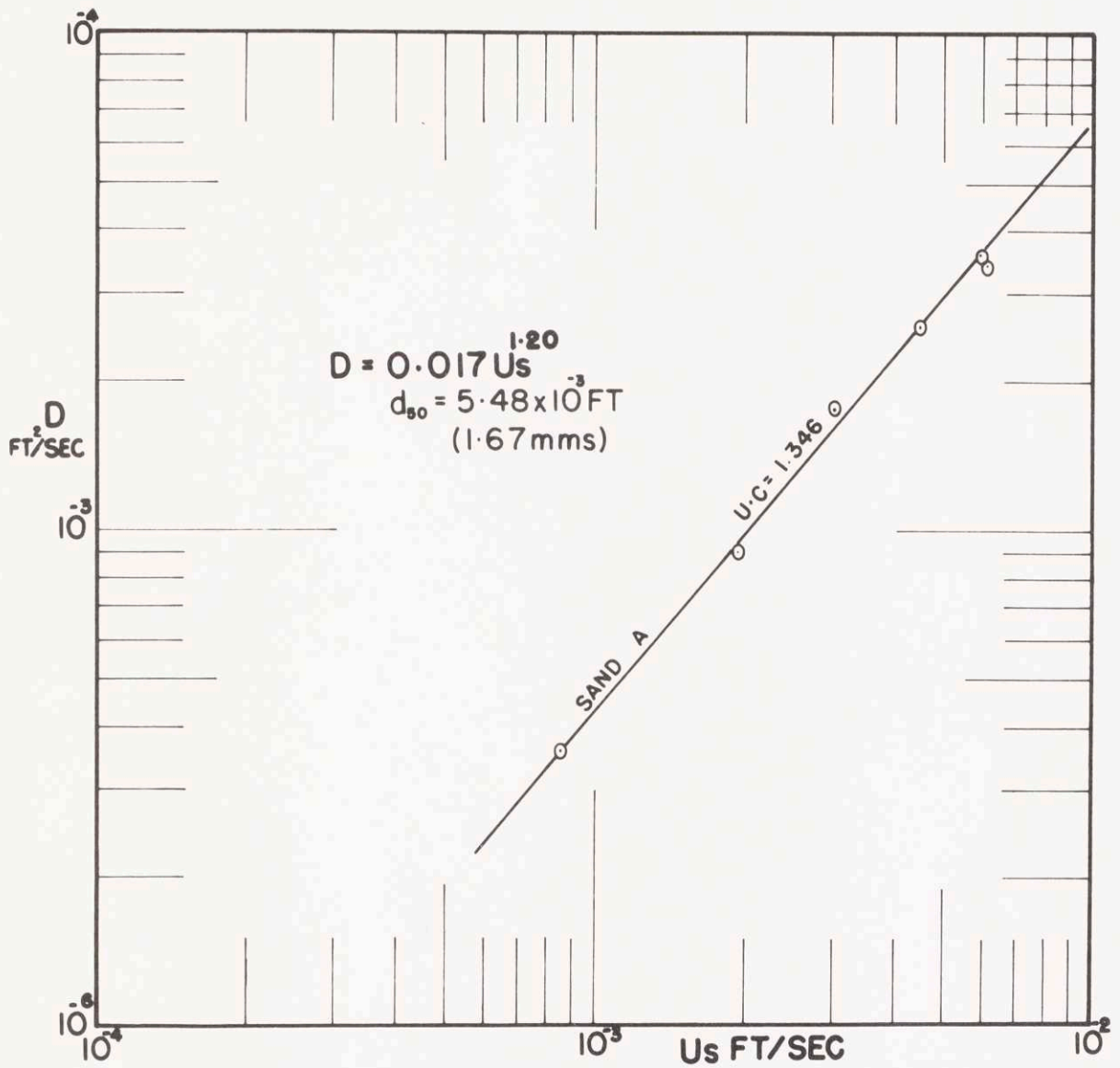
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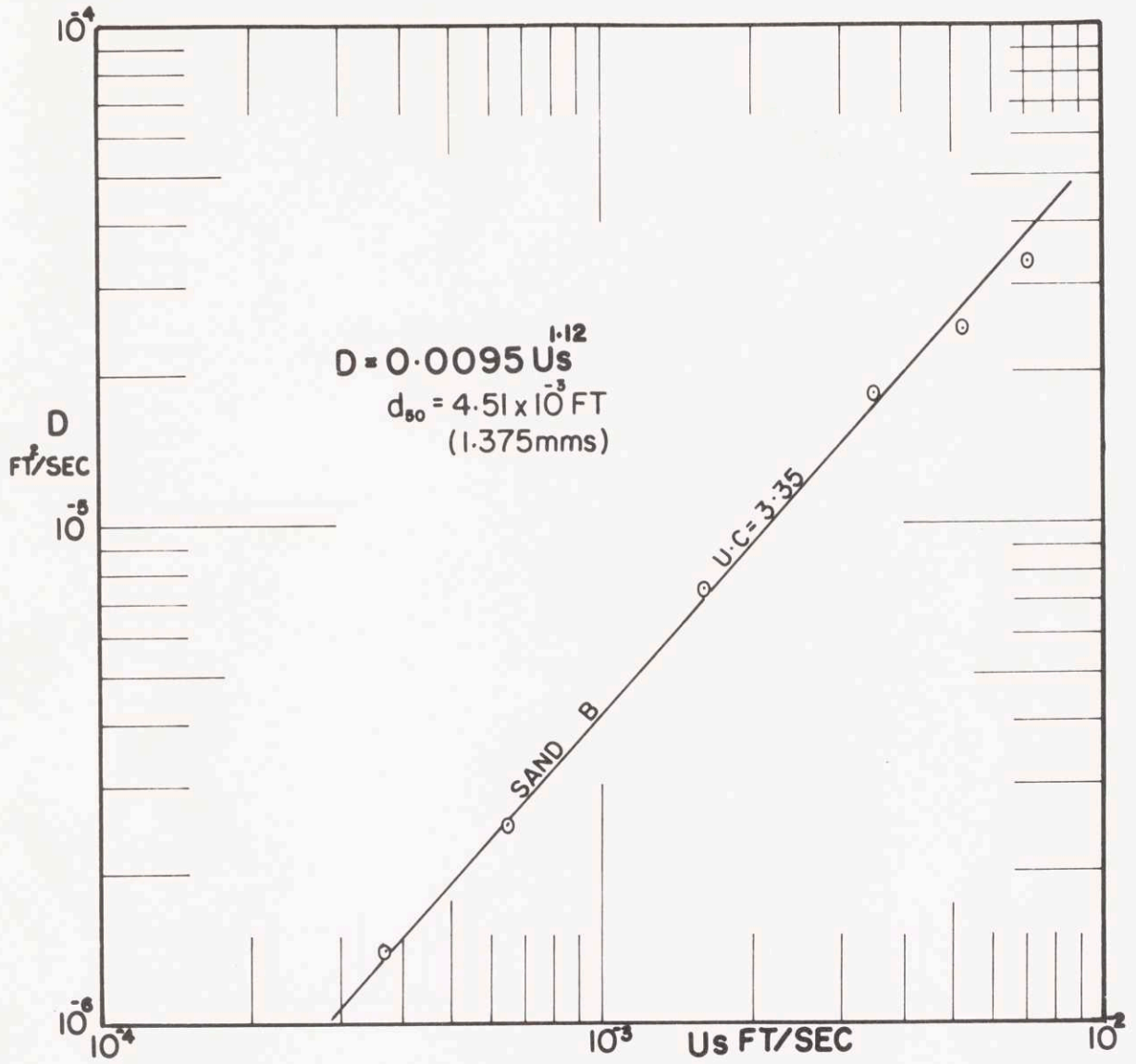
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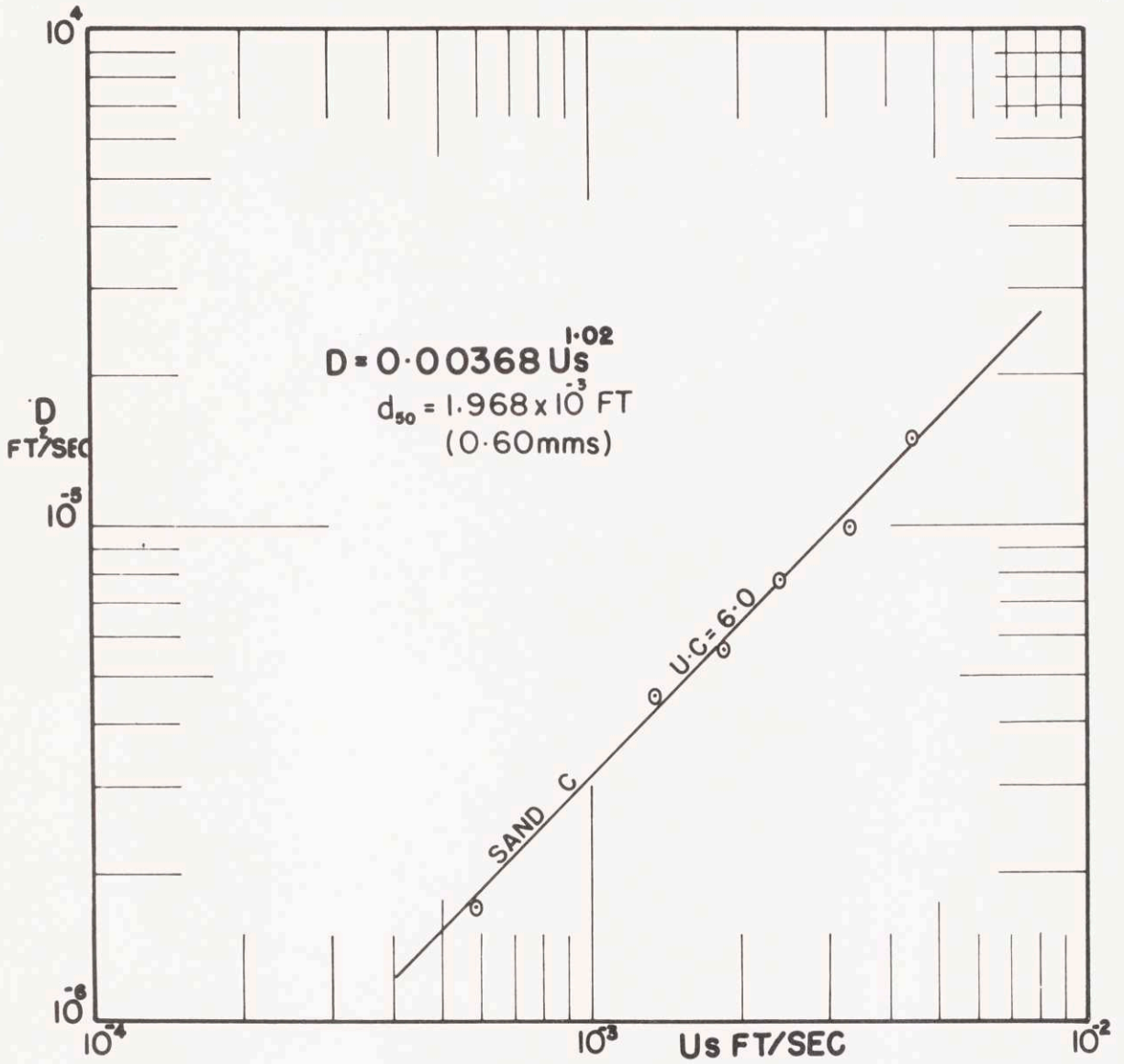
(FIG 12a)

Variation of Dispersion Coefficient with Velocity



(FIG 12b)

Variation of Dispersion Coefficient with Velocity



(FIG 12c)

Variation of Dispersion Coefficient with Velocity