

Lattice Girders

Trudge

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1874

Graduated in 1875

Design for a Double Warren Iron Girder Bridge.

In a girder, the load is resisted by the counteractive power of the particles of the material, so, to have a structure of sufficient strength to resist the forces caused by the load, and to realize equilibrium, we must know what the forces are to be, because the proportions throughout, have to be determined with reference to the amount of force, either of extension or compression, which the material is capable of exerting in resisting the action of the load. It is necessary also in calculations fixing the dimensions of the pieces to distinguish between the action of the permanent load, and that of a rolling load, or that action

coming from a load which is applied and then removed, and which for the traffic of a railroad arises from passing trains. For a steady load, which for a bridge would principally be the weight of the bridge, produces from the effect of gravity, one unvarying action, while the result produced by a rolling load varies, causing a sudden stress in the parts of the structure.

Formerly the arched form of bridges was almost universally employed; then when iron began to be more employed in building, it was applied to bridges at first in the cast shape.

The limitation of span for which girder bridges were safely applicable had restricted their employment, about forty feet having been commonly

considered the maximum length to which single cast iron girders could safely be applied, liable to be loaded with railway trains or other heavy weights. The desire to retain this convenient form of structure, however, and to extend its use to longer spans, induced attempts to combine wrought iron with cast metal in such a manner as should impart to the compound structure the superior power to resist extension which wrought iron is well known to possess. But here must be noticed the defects of compound ^{iron} structures, as it points directly to the superiority of homogeneous fabrics, and this is the liability of strain being caused in different parts from the difference in the effect of the tempera-

ture on cast and wrought metal, and the difficulty of combination to take into account this difference. It is from this consideration, mainly, that in this design the cast iron form of bars is almost wholly rejected.

In the girder form of bridge the depth of the structure is reduced to that of the section of material due to the maximum load and hence the peculiar applicability of this form of railway bridge in which it is desirable to preserve a minimum distance from the under side or soffit of the girder to the level of the top above, and moreover depending for its strength upon the sectional area of the girder at that point in its length over which the weight or load acts, requires

abutments to resist vertical pressure only, while those of arched bridges have to resist the lateral thrust of the arch. In choosing this particular form of girder bridge, we are led by the consideration of extraordinary simplicity and economy which it has and the ease with which it is put together, each piece having dimensions different from the others, and therefore its own peculiar place, the holes for rivets and bolts always being punched before transportation which as the pieces are generally short is comparatively easy. As the design is particular and not general we shall employ general formulae as little as possible. From experiments on actual cases and examples given by

different authorities, it is seen that the passage of heavy weights produce no sensible deflection, so from this we will not consider camber, but make the estimates with the intention of having the bridge stiff enough to do without it. The principal dimensions are given us, namely span, 192 ft. divided into 16 panels, height 18 ft., 15 ft. clear between the two girders, and the bridge to be pivotted. As there are sixteen panels each panel is twelve feet long.

As the load is distributed symmetrically in regard to the middle point of the bridge it will be sufficient to consider the action of the forces on one half of the bridge only, the dimensions of every piece being such that there

shall be enough material to resist the greatest force liable to be brought to bear on that piece, and here as before stated we must distinguish between the action of the dead and that of the live load. Another requirement is that the bridge shall be designed to bear a dead load of eight hundred (800) lbs per ft. run, and a live load of twelve hundred (1200) lbs. per foot run.

Generally in designing a structure, it is customary to consider the breaking strength, and this is for a certain material a certain number of times larger than the working strength, and for wrought iron, for a dead load this number we will take as three and for a live load as six.

For certain reasons the calculations will be made both considering the working strength and the breaking strength using the numbers, or factors of safety. The weight extending over one panel is to be supposed as acting at one point, that at which the diagonals meet and the load is supported. The weight on half a panel at each end is not taken into consideration as it is supposed to go directly into the abutments.

For all our calculations respecting shear and the bending action at any plane of section vertically through the girder, we consider for simplicity all the forces acting at the left of the plane of section, because the resistance of particles at that section

must equal the forces applied there,
 for if they are not equal the bridge
 breaks. Regarding the action of
 the live load, suppose this load to
 come on to the bridge at the right
 till one point is loaded. Call the
 supporting force at the left P_1 , and it
 is evident that the force at that
 plane of section which acts at all is
 simply P_1 . Suppose two points are
 loaded, from the second point to the
 left hand point of support the only
 force to act is P_1 , remembering that
 now we are considering only the live
 load. We see that as the points go
 on being covered the force P_1 must
 grow larger. Referring to p. 10
 fig. 1 which is a skeleton outline
 of this form of lattice girder,

Fig 1.

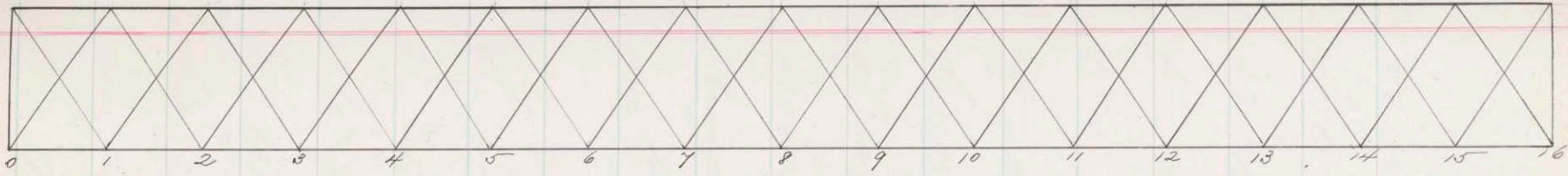


Fig 2.

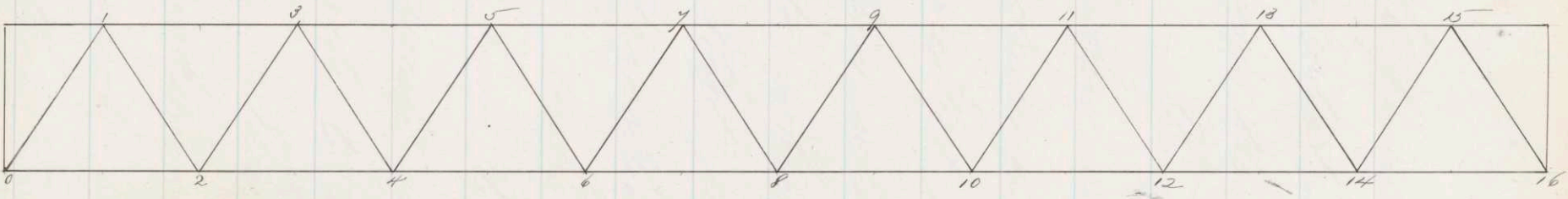
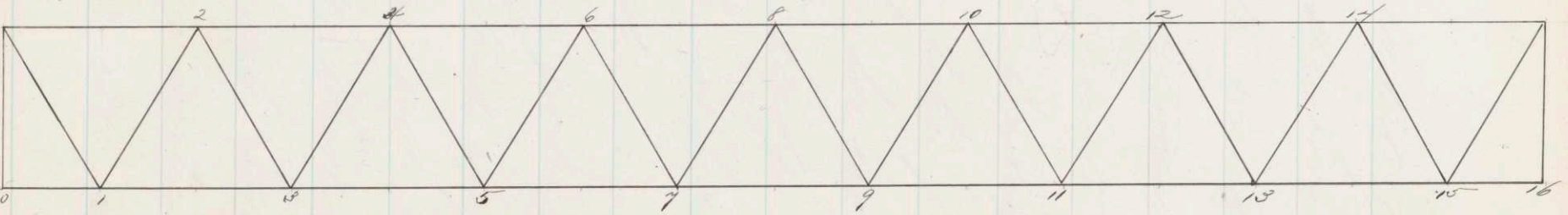


Fig 3.



Scale 20' to 1".

pro

when all the points are loaded the force P_1 is greatest, and therefore the shear between the points 0 and 1 is the greatest. The shear between 2 and 1 is greatest when all the points up to and including 2 are loaded, hence the greatest shear takes place at any section when the longer segment into which the bridge is divided by that section is loaded with the travelling load.

The double Warren girder consists of an upper boom to withstand compression, a lower to resist extensions, and two sets of diagonals to resist the shearing action of the load. These diagonals cross each other and make with the vertical, an angle which we will call θ , and is found thus.

$$\tan \alpha = \frac{12}{18} = \frac{2}{3}$$

$$\begin{aligned} \log 2 &= 0.301030 \\ \log 3 &= 0.477121 \\ \hline \log \tan \alpha &= 9.823909 \end{aligned}$$

$$\therefore \alpha = 33^\circ 41' 24'' 31$$

This subtracted from 90° gives 56° 18' 35'' 69 as the inclination of the diagonals, or the angle which they make with the horizon.

The girder is seen to be made up of two simple ones, so in computing the stresses, it is best to pursue the method of considering the two separate simple girders, and then combine the results. Hence taking the simple trusses we have the figures 2 and 3, p. 10.

First we will find the supporting force at each end, or the weight that the abutment at either end must bear.

Calling w the wt. per foot of the dead load and w_1 the wt. of live load per foot, and l the length we have $(w+w_1)\frac{l}{2} = (800+1200)\frac{22}{2} = 192000$

as the force resisted by either abutment.

To find the shear in the diagonals. First we will find the shear from the dead load, both with and without the factors of safety, considering the panels in the simple trusses (figs. 2 & 3), and a weight of $12 \times 800 = 9600$ lbs. at each point between the points of support.

The maximum action of the live load on each panel will alone be considered, the weight of the live load on a single point being $12 \times 1200 = 14400$ lbs. Pages 14 and 15 show the results.

At each plane of section, we have to consider only the forces at the left hand and these are the greatest shear, which we will call P_1 , at the ends and the load on each point of 9600 lbs up to that section. This is for a

Shearing force for fig. 2.

From Dead Load.

$$F = P_1 - \sum W$$

$$W = 12 \times 800 = 9600$$

Without factor of safety.

With factor $\frac{3}{2}$

From 0 to 2	$F_0 = P_1 = \frac{1}{2} \times 7 \times 9600 = 33,600$	100,800
" 2 to 4	$F_1 = 33600 - 9600 = 24,000$	72,000
" 4 to 6	$F_2 = 33600 - 19200 = 14,400$	43,200
" 6 to 8	$F_3 = 33600 - 28800 = 4,800$	14,400

Maximum Shear from Live Load.

$$F = P_1$$

$$W' = 12 \times 1200 = 14400$$

Without factor

With factor 6.

From 0 to 2	$F_0 = \frac{1}{2} \times 7 \times 14400 = 50400$	302,400
" 2 to 4	$F_1 = \frac{7}{16} \times 14400 \times 6 = 37,800$	226,800
" 4 to 6	$F_2 = \frac{6}{16} \times 14400 \times 5 = 27,000$	162,000
" 6 to 8	$F_3 = \frac{5}{16} \times 14400 \times 4 = 18,000$	108,000

Total Shear

Without factors

With factors

From 0 to 2	84,000	403,200
" 2 to 4	61,800	298,800
" 4 to 6	41,400	205,200
" 6 to 8	22,800	122,400

Shearing Force for Fig. 3. From Dead Load

$$F' = P_i - \sum W \quad \begin{matrix} \text{Without factor 3} & W = 9600 & \text{With factor 3} \end{matrix}$$

From 0 to 1	$F_0 = P_i = \frac{1}{2} \times 8 \times 9600 = 38,400$	115,200
" 1 to 3	$F_1 = 38400 - 9600 = 28,800$	86,400
" 3 to 5	$F_2 = 38400 - 19200 = 19,200$	57,600
" 5 to 7	$F_3 = 38400 - 28800 = 9,600$	28,800
" 7 to 9	$F_4 = 38400 - 38400 = 0$	0

Maximum Shear from Live Load.

$$F' = P_i \quad \begin{matrix} \text{Without factor 6} & W' = 14400 & \text{With factor 6} \end{matrix}$$

From 0 to 1	$F_0 = 8 \times 14400 \times \frac{1}{2} = 57600$	345,600
" 1 to 3	$F_1 = 7 \times 14400 \times \frac{7}{16} = 44100$	264,600
" 3 to 5	$F_2 = 6 \times 14400 \times \frac{6}{16} = 32400$	194,400
" 5 to 7	$F_3 = 5 \times 14400 \times \frac{5}{16} = 22500$	135,000
" 7 to 9	$F_4 = 4 \times 14400 \times \frac{4}{16} = 14400$	86,400

Total Shear

$$\begin{matrix} \text{Without factors 3 \& 6} & & \text{With factors} \end{matrix}$$

From 0 to 1	$38400 + 57600 = 96000$	460,800
" 1 to 3	$28800 + 44100 = 72900$	351,000
" 3 to 5	$19200 + 32400 = 51600$	252,000
" 5 to 7	$9600 + 22500 = 32100$	163,800
" 7 to 9	$14400 = 14,400$	86,400

Diagonals

Panel Fig. 1	Member.	Shear \times Sec. of i .		Stress.		Rivets
		Without Factors	With Factors	Without Factors	With Factors.	
1	Extension	96000 \times 1.202	460,800 \times 1.202	115,392 p.	553,882 p.	10
	Compression	84000 \times 1.202	403,200 \times "	100,968 t.	484,646 t.	8
2	Ex.	" \times "	" \times "	" p.	" p.	8
	Com.	72,900 \times "	351,000 \times "	87,626 t.	421,902 t.	8
3	E.	" \times "	" \times "	" p.	" p.	7
	C.	61,800 \times "	298,800 \times "	74,284 t.	359,158 t.	6
4	E.	" \times "	" \times "	" p.	" p.	6
	C.	57,600 \times "	252,000 \times "	62,023 t.	302,904 t.	5
5	E.	" \times "	" \times "	" p.	" p.	5
	C.	41,400 \times "	205,200 \times "	49,763 t.	246,650 t.	4
6	E.	" \times "	" \times "	" p.	" p.	4
	C.	32,100 \times "	163,800 \times "	38,584 t.	196,888 t.	3
7	E.	" \times "	" \times "	" p.	" p.	3
	C.	22,800 \times "	122,400 \times "	27,406 t.	147,125 t.	2
8	E.	" \times "	" \times "	" p.	" p.	2
	C.	14,400 \times "	86,400 \times "	17,309 t.	103,853 t.	2

dead load, and for a live load P ,
 is alone to be considered, P , being
 in this case the greatest shear at the
 end when the longer segment is loaded.
 P , is an upward force and the other are
 downward, hence we have for the
 shear at any section $F = P - \Sigma W$ for a
 dead load and $F = P$, for a live load.

$W = 12 \times 800 = 9600$ lbs. Pages 14 and
 15 give the shears. To find the
 amount of stress either of extension or
 compression in the diagonals, multiply
 the shear by the sec of i as shown
 in the table p. 16. Looking at
 a single panel and considering that
 alone, and the action of the load on it,
 it is evident that in the diagonal
 sloping up towards the centre there is
 compression and in the diagonal

sloping up, from the centre there is compression. Or this might be seen by taking the shearing force in any panel and as it is upwards, F , being always the greater, resolved into components gives the same as before.

When the longer segment is loaded with the travelling load the diagonals act alternately as struts and ties.

When the shorter segment is loaded the shearing force acts downward, opposite to that from the dead load, and in order to have any diagonal act as strut and tie both, the shear from the travelling load on the shorter segment must be greater than the shear from the dead load.

In fig. 3 on 7-8 & 8-9 the only load acting is the live load, as the shear from the dead is nothing.

Hence 7,8 acts only as a tie.

Calculating $P_{1,2}$ ^{fig. 2} for the several cases when 1, 2, & 3 points are covered we find that there is for 3 points, on 67-78 a downward shear of $10800 - 4800 = 6000$ without factors and with $64800 - 14400 = 50400$ and these diagonals are the only ones which act alternately as struts and ties.

For the bending moments we have when the load covers the whole bridge the maximum effect.

Pages 20 and 21 give the results, and the method of finding the moments.

Dividing each moment by 18 the height of the girder, we have the horizontal stresses as given in page 22; - and p. 23 gives the total horizontal stress for any panel.

Pending Amounts.

Fig. 2.

$$M = P_1 \times X - \sum w \Delta X = P_1 \times X - w \sum \Delta X$$

- At 1 lower boom $84000 \times 12 = 1,008,000$
- " 2 upper " $84000 \times 24 = 2,016,000$
- " 3 lower " $84000 \times 36 - 24000 \times 12 = 2,736,000$
- " 4 upper " $84000 \times 48 - 24000 \times 24 = 3,456,000$
- " 5 lower " $84000 \times 60 - 24000 \times 48 = 3,888,000$
- " 6 upper " $84000 \times 72 - 24000 \times 72 = 4,320,000$
- " 7 lower " $84000 \times 84 - 24000 \times 108 = 4,464,000$
- " 8 upper " $84000 \times 96 - 24000 \times 144 = 4,608,000$

Without factors

-
- 1 lower " $403,200 \times 12 = 4,838,400$
 - " 2 upper " $403,200 \times 24 = 9,676,800$
 - " 3 lower " $403,200 \times 36 - 115,200 \times 12 = 13,132,800$
 - " 4 upper " " $\times 48 - 115,200 \times 24 = 16,578,800$
 - " 5 lower " " $\times 60 - " 48 = 18,662,400$
 - " 6 upper " " $\times 72 - " 72 = 20,736,000$
 - " 7 lower " " $\times 84 - " 108 = 21,424,200$
 - " 8 upper " " $\times 96 - " 144 = 22,118,400$

With factors

Bending Moments.

Fig. 13.

$M = P \cdot x - w \cdot x^2$ Shear at end = $P = 96000$
 without factor

At upper beam	96000×12	= 1,152,000
" 2 lower "	$96000 \times 24 - 12 \times 24000$	= 2,016,000
" 3 upper "	" $\times 36 - 24 \times$ "	= 2,880,000
" 4 lower "	" $\times 48 - 48 \times$ "	= 3,456,000
" 5 upper "	" $\times 60 - 72 \times$ "	= 4,032,000
" 6 lower "	" $\times 72 - 108 \times$ "	= 4,620,000
" 7 upper "	" $\times 84 - 144 \times$ "	= 4,608,000
" 8 lower "	" $\times 96 - 192 \times$ "	= 4,608,000

Without Factor

At 1 upper	46800×12	= 5,529,600
" 2 lower "	" $\times 24 - 12 \times 115200$	= 9,676,800
" 3 upper "	" $\times 36 - 24 \times$ "	= 13,924,000
" 4 lower "	" $\times 48 - 48 \times$ "	= 16,558,000
" 5 upper "	" $\times 60 - 72 \times$ "	= 19,053,600
" 6 lower "	" $\times 72 - 108 \times$ "	= 20,736,000
" 7 upper "	" $\times 84 - 144 \times$ "	= 22,118,400
" 8 lower "	" $\times 96 - 192 \times$ "	= 22,118,400

With Factor

Upper Room

At Points	Pending Moment		$\frac{NL}{12}$	Horizontal Stress		Between Points
	Without Factors	With Factors		Without Factors	With Factors	
1	$96000 \times 12 = 1,152,000$	$460800 \times 12 = 5,529,600$	64,000	307,200		20 2
2	$84000 \times 24 = 2,016,000$	$403,200 \times 24 = 9,676,800$	112,000	537,600		17 4
3	$96000 \times 36 - 24 \times 24000 = 2,880,000$	$440800 \times 36 - 24 \times 115200 = 13,824,000$	160,000	768,000		12 4 2
4	$96000 \times 48 - 24 \times 24000 = 3,456,000$	$403200 \times 48 - 24 \times 115200 = 16,588,800$	192,000	921,600		5 5
5	$96000 \times 60 - 72 \times 24000 = 4,032,000$	$460800 \times 60 - 72 \times 115200 = 19,353,600$	224,000	1,075,200		4 10
6	$84000 \times 72 - 72 \times 24000 = 4,320,000$	$403200 \times 72 - 72 \times 115200 = 20,736,000$	240,000	1,152,000		10 14
7	$96000 \times 84 - 144 \times 24000 = 4,608,000$	$460800 \times 84 - 144 \times 115200 = 22,118,400$	256,000	1,228,800		10 14
8	$84000 \times 96 - 144 \times 24000 = 4,608,000$	$403200 \times 96 - 144 \times 115200 = 22,118,400$	256,000	1,228,800		10 14

Lower Room

1	$84000 \times 12 = 1,008,000$	$403200 \times 12 = 4,838,400$	56,000	268,800		20 2
2	$96000 \times 24 - 12 \times 24000 = 2,016,000$	$460800 \times 24 - 12 \times 115200 = 9,676,800$	112,000	537,600		17 4
3	$84000 \times 36 - 12 \times 24000 = 2,736,000$	$403200 \times 36 - 12 \times 115200 = 13,182,800$	152,000	729,600		12 4 2
4	$96000 \times 48 - 48 \times 24000 = 3,456,000$	$460800 \times 48 - 48 \times 115200 = 16,588,800$	192,000	921,600		5 5
5	$84000 \times 60 - 48 \times 24000 = 3,888,000$	$403200 \times 60 - 48 \times 115200 = 18,662,400$	216,000	1,036,800		4 10
6	$96000 \times 72 - 108 \times 24000 = 4,320,000$	$460800 \times 72 - 108 \times 115200 = 20,736,000$	240,000	1,152,000		10 14
7	$84000 \times 84 - 108 \times 24000 = 4,464,000$	$403200 \times 84 - 108 \times 115200 = 21,427,200$	248,000	1,190,400		10 14
8	$84000 \times 96 - 192 \times 24000 = 4,608,000$	$460800 \times 96 - 192 \times 115200 = 22,118,400$	256,000	1,228,800		10 14

Total Horizontal Stresses in
Upper + Lower Booms
Combining Horizontal Stresses

Plane	Thrust in Upper Booms	
	Without factors	With Factors
1 to 1	64000	307200
1 to 2	64000 + 112000	307200 + 537600
2 to 3	112000 + 160000	537600 + 768000
3 to 4	160000 + 192000	768000 + 921600
4 to 5	192000 + 224000	921600 + 1,075,200
5 to 6	224000 + 240000	1,152,000 + 1,075,200
6 to 7	240000 + 256000	1,152,000 + 1,228,800
7 to 8	256000 + 256000	1,228,800 + 1,228,800
Pull in Lower Booms		
1 to 1	56000	268800
1 to 2	56000 + 112000	268800 + 537600
2 to 3	112000 + 152000	537600 + 729,600
3 to 4	152000 + 192000	729,600 + 921,600
4 to 5	192000 + 216000	921,600 + 1,036,800
5 to 6	216000 + 240000	1,036,800 + 1,152,000
6 to 7	240000 + 248000	1,152,000 + 1,190,400
7 to 8	248000 + 256000	1,190,400 + 1,228,800

Total.

Page 16 the stress in the diagonals is given, "p" representing pull, and "t" thrust. To find the sectional area to stand the stresses, we divide the amount of pull (this being for pull only) by the intensity of the stress, or the amount which one square inch of the metal will withstand. Page 25 gives the calculated ^{sectional} area of the ties and the area given, using 10000 lbs. as the working tensile strength of ^{wrought} iron. Calling the bars 1 inch thick the widths are given in the column marked "area given". For the struts we use Mr. Gordon's formula for the ultimate strength of wrought iron, deduced from Mr. Hodgkinson's experiments, which is $\frac{P}{S} = 36,000 \div 1 + \frac{al^2}{h^2}$. Making

Mistake. The areas of upper chord should be reversed, the 1st coming last.

$$\text{Sec. Area} = S = \frac{P}{10000}$$

Panel	$S = \frac{P}{F}$ Stress	Calculated Area of 'T'ies	Sectional Area Given	Horizontal Stress in Lower Boom without factor	Cal. Area	Sectional Area Given
1	115,392	11.5	11	56,000	5.6	4 7.5 12.5 24
2	100,968	10.09	11	168,000	16.8	11.5 25 36.5
3	84,626	8.46	8.5	264,000	26.4	11.5 25 36.5
4	74,284	7.42	8.5	344,000	34.4	11.5 25 36.5
5	62,023	6.20	8.5	408,000	40.8	36.5 15 51.5
6	49,763	4.97	6	456,000	45.6	51.5
7	38,584	3.85	6	488,000	48.8	51.5
8	27,406	2.74	3	504,000	50.4	51.5
		Cal. Area of Struts	Sectional Area Given	$S = \frac{P}{36000}$ Upper Boom with factor		
1	484,646	13.8	14	307,200	8.5	$4 \times 18 \times \frac{3}{4} = 54$ $8 \times 3 \times \frac{3}{4} = \frac{18}{\frac{1}{2}}$
2	421,902	12.1	12	844,800	23.5	$4 \times 16 \times \frac{3}{4} = 48$ 66 15
3	359,158	10.3	12	1,305,600	36.3	$15 \times 3 = 45$ 63
4	302,904	8.7	12	1,689,600	46.9	$13 \times 3 = 39$ 15 57
5	246,650	7	8	1,996,800	55.2	$10 \times 3 = 30$ 15 48
6	196,888	5.7	6	2,227,200	61.8	$8 \times 3 = 24$ 15 42
7	147,125	4.2	6	2,380,800	66.1	42
8	103,853	3	3	3,457,600	68.2	42

$a = \frac{1}{3000}$ and using the radius of gyration, we have $\frac{P}{S} = 36000 \div 1 + \frac{l^2}{36000 r^2}$

As the cross section is rectangular

$r^2 = \frac{h^2}{12}$ Call $h = 2$ then $r^2 = \frac{1}{3}$

$l = 12^2 + 18^2 = 468 \quad \therefore \frac{P}{S} = \frac{36000}{1 + \frac{468}{12000}} =$

$\frac{36000}{1.039} = 34648.7$

$\therefore S = \frac{P}{34648.7}$ p. 25 giving sectional areas.

For convenience we make two struts, 7 by 1 for the first, two 6 by 1 for the second, third, & fourth, till the last which is made single 3 by 1 and acts only as a tie. For lower boom the

sectional area is found as for diagonal ties.

For the upper boom Mr. Gordon's formula is used, the section of the boom in finding the radius square of gyration, being considered a thin square cell. This form of section is chosen because it is the stiffest.

The formula becomes calling $l = 12$ &

$$r^2 = \frac{h^2}{6} \quad \frac{P}{S} = \frac{36000}{1 + \frac{144}{36000 \times \frac{h^2}{6}}} \quad \text{calling } h$$

$$= 8 \quad \text{we have} \quad \frac{P}{S} = \frac{36000}{1 + \frac{144}{6000 \times 64}} = \frac{36000}{\frac{8503}{8000}} = 36000 \text{ nearly}$$

$$\therefore S = \frac{P}{36000}$$

p. 25 gives the results for the different panels. Making the middle chord piece at the centre

18" on a side, we have the minimum area, as $4 \times 18 \times \frac{3}{4} = 54 + 8 \times 3 \times 3 = 18 + 54 = 72$

where 68 is required. Call the next from the centre 16" on a side, the next 15" the next 13" then 10" then 8" to the end. The mid chord piece is 18 ft. long.

The tower chord is made up as in the figure, on the plate of details, the thickness of iron in flange at the centre = 2", flange 20" wide.

Fig. 2 shows the method of obtaining the required sectional area at any point, in the distribution of the plates making the flange.

In the upper chord the size, or the width of a side diminishes from the centre of one panel to the centre of the next. The iron is $\frac{3}{4}$ " thick, and the lower, ^{part} at the points where the diagonals are fixed has a depth of 10".

For the end post we take a square cell cut at the top to fit the upper boom, and fastened to the lower boom by angle irons. The compressive force at the end is the sum of the forces at the end of each simple truss legs 273 and is $403200 + 460800 = 864000$.

$$\frac{P}{S} = \frac{36000}{1 + \frac{l^2}{36000r^2}} \quad r^2 = \frac{h^2}{6} \quad h = 11 \quad h^2 = 121$$

$$\therefore r^2 = \frac{121}{6} \quad b = 18 \quad b^2 = 324$$

$$\therefore \frac{P}{S} = \frac{36000}{1 + \frac{324}{36000 \times \frac{121}{6}}} = \frac{36000 \times 121}{121.054} = 36000 \text{ nearly}$$

$$\therefore S = \frac{P}{36000} = \frac{864000}{36000} = 24$$

The inside dimensions are 11", and thickness of iron is $\frac{3}{4}$ ", making total area of metal given as 51 sq. in.

The I beams are to be suspended from the bottom flange, and rest on iron castings, as shown in the details.

To calculate the size, we assume a beam of 15" ht. 15' length weighing 200 lbs. to the yd. Now we consider the weight of stringers ties and rails with the load from a heavy locomotive to act through two points, each five feet from ~~either~~ ^{the} ends. Two stringers are really to act 6 inches from the end but it will make no appreciable difference to take them acting as above.

From many examples we see that a locomotive has ~~its~~ the greater part

of its weight resting on the driving wheels, and twenty tons over twelve feet is taken for our live load in calculations for the beam. The action of the live load is intermediate, between that of an absolutely sudden load and a perfectly gradual load, but in practice the additional strain from the swift motion of the train is provided for in the factor of safety, six.

	No.	Dimensions	Cu. Ft.
Stringers	4	12' x 8" x 14"	37
Ties	9	13' x 8" x 8	52
Rails	2	8 yds.	52
		65 lbs. per yd.	520
		One I beam weighs 1000 lbs.	520
			5415 dead wt.
			Call 5400

$$M = \frac{1}{8} Wl = \frac{3000}{8} \times 12 \times 15 = 67500$$

$$3843000$$

$$15 \overline{) 3910500}$$

$$4000 \overline{) 260700}$$

Live load 2700 through pt. 5 ft. ^{from end.}

$$\frac{2700}{3} \text{ factor} = 900$$

$$120000$$

$$\frac{120000}{8100} = 128100 \text{ wt. on 1 ft. length of arm.}$$

$$3843000 \text{ as there are two I beams.}$$

6.5 area of 1 flange. $2 \overline{) 7686000}$
 6.5
 7.2
 20.2 area of I beam.

Because there are two equal couples at the ends of the beam, making the bending moment the same between the points referred to, we add this moment to the greatest moment from the weight alone and divide by 15 the height to obtain the horizontal stress. To find the sectional area of bolts to sustain I beams. Wt. at each end is 23700

2000	I beams...	
40000	Locomotive	
5400	Two etc.	
2)44400		10000)23700
23700		2.37

Take 2 bolts $1\frac{1}{2}$ diam. or $\frac{3}{4}$ " radius.
Area of cross section is therefore 1.77 sq. in.

$$2 \times 1.77 = 3.54 \text{ where } 2.37 \text{ is required.}$$

The bolts are strong enough to bear the greatest weight that will ever come on them

The bolts to hold the I beams are held by the flange of the lower boom, and the mode of arranging the bolts is shown on the plate of details, fig. 4.

There is no danger of the bolt head being pulled through the flange.

For, the diameter of head of bolt is 3 in. \therefore the outer circumference of bolt head = $2\pi r = 2 \times 3.14159 \times \frac{3}{2} = 9.42477$

Multiplying this by $\frac{1}{2}$ " = the distance from the bolt to outside of head, we have an area of 4.7124 sq in. giving ample resistance to any force that may be applied to the bolt. The flange of the lower boom, holding the I beams by means of the bolts, and thus supporting the load, is fastened to the web by "angle irons" $4" \times 4" \times \frac{1}{2}"$, as shown in fig. 3^{fig. 1}, plate of details.

Now, to see if these are rivets enough, if placed 3" from centre to centre, and the rivets being $1\frac{1}{8}$ " in diameter, to hold the flange with its dependent load. It either gives way by tearing off the rivets in lower part of the angle iron or by shearing off those in upper part. Weight at one end of 1 beam is about 25,000 lbs. A rivet of $1\frac{1}{8}$ " diam. has an area of about 1 sq. in. Now 1 sq. in. will bear 10,000 lbs., so it would take less than three. Shear of one rivet 1 sq. in. in cross section is 8000 lbs. therefore it would take only about 4 to hold, reckoning shear in upper part of angle iron. It may likewise give way by the rivets in upper part of angle iron, bearing through the $\frac{1}{2}$ " in.

now composing the angle iron.

Bearing area = $1\frac{1}{8} \times \frac{1}{2} = \frac{9}{16}$ 8000 lbs for bearing of 1 sq. in. gives about 4500 for bearing of one pivot. We thus see that it will take at least six, to hold.

Having pivots at every three inches gives more than enough, for the weight at one point is distributed by the angle irons through ^{the length of} one panel or twelve feet.

To support the I beams, the bolts are placed between the two at each point, and support the castings shown in fig. 4, which gives the method of hanging the I beams.

Rivetting.

One sq. in. bears or shears 8000 lbs.

First calculate the number of rivets for the diagonals.

Diameter of rivets = $1\frac{1}{2}$.

A rivet of this diameter bears 12,000 if the thickness of the diagonals is 1 in.

$$\text{Shear} = \pi r^2 \times 8000 = 3.14159 \times \frac{9}{16} \times 8000$$

$$= 14000. \quad \text{Take bearing.}$$

In this calculation we use the stress in diagonals without using the factors of safety, as it gives a larger number of rivets than stress with factors.

Page 16 gives the number of rivets in each diagonal, the disposition of the rivets being shown in the elevation of the half bridge on the sheet of details.

Fig. 1 shows the arrangement, in the first strut, a small piece of iron being inserted between the two members

of the strut, in order to accommodate the required number of rivets.

For the upper boom, we use a less number of rivets than the calculation would give, because the parts of the boom abut one against the other and it is only necessary to keep them in place, which is done by the arrangement shown in fig. 7, using angle irons of $\frac{1}{2}$ in thickness and rivets of $1\frac{1}{2}$ diam. For the lower boom we use covering plates of $\frac{5}{8}$ " iron.

Bearing of $1\frac{1}{2}$ " rivet = $\frac{3}{2} \times \frac{5}{8} \times 8000 = 7500$.

Dividing the pulls in each panel as shown on p. 23 we have as the required number of rivets respectively 7.4, 22, 35, 46, 53, 60, 65, 68. Now as we do not have to depend entirely on the covering plates, because there are angle irons extending the length of the girder, of

different lengths from the plates as cut, we are safe in calling the number of rivets at each point, 12, 18, 30, 42, 48, 54, 60, & 66 respectively.

The plates composing the flange of the lower boom are cut in the following manner, from the end to the centre.

The first is composed of iron $\frac{5}{8}$ " thickness, lengths 18'7", 24'; 24'; 24'; 12' which brings it to 6' beyond the centre point.

The second plate is $\frac{5}{8}$ " iron, and commencing at a point, 2' distant from the first bolt, is cut in lengths of 20'; 24'; 24'; 12'5 and 11' carrying it 5'5 beyond the centre. The third and lower plate is $\frac{3}{4}$ " in. which makes the total thickness of flange 2". The plate is cut in lengths of 20'; 24' and 12' commencing 2' nearer the supports, than the 4th

point of suspension of T beams.

The end posts are rivetted to the bottom flange as shown in fig. 1. and is cut at the top so as to admit of the upper boom fitting into it, the boom having as the side of the square a measurement of 8 in. The end of the boom projects three inches from the post and is fastened to the end post by angle irons similar to those holding the joints of the upper boom and shown in fig. 7. A square plate of 5/8" iron, 11" on a side, is capped over the end of the boom and held in place by the angle irons.

The diagonals are rivetted where they cross each other which makes the bridge stiffer.

For the upper and lower horizontal bracing we proceed as follows.

When a train of cars is on the bridge, the surface acted on by the wind, would be nearly of the area, $192' \times 18'$, or considering one panel only the area acted on is $12' \times 18' = 216 \text{ sq. ft.}$ or in round number 200 sq. ft.

A wind causing a pressure of more than 25 lbs to the sq. ft. would be a hurricane, and its effect would be to sweep away the structure from its points of support, or would be likely to overturn it. A train would never be run, when the wind causes such a pressure, so, for calculations regarding horizontal bracing we use a pressure of 25 lbs per sq. ft. For the lower horizontals, we consider them all ties because of the I beams, and the wind blowing from one side or the other, and

for this latter reason we see that the upper horizontals act alternately as struts or ties. We use for the lower the tensile strength of wrought iron, 10,000 lbs per sq. in. and for the upper we consider them as struts, using Gordon's formula for the ultimate strength of wrought iron. We calculate for struts because wrought iron gives way easier by crushing. On each point we have acting $12 \times 18 \times 25 = 5400$ lbs.

Sec. $i = \frac{19.2}{15} = 1.28$ Pull or thrust = shear \times sec. i .

Increment	Panel	Shear	Pull	Sec. Area	Diam. given	Thrust	Req. Rivets	Given	Req. Area	Given
5400	1	40500	51,820	5	2"	155,520	4.5	4	4.8	4
	2	35100	44,928	5	2"	134,784	3.7	4	4.2	4
	3	29700	38,016	4	1 3/4"	114,048	3.1	4	3.5	4
	4	24300	31,104	3	1 3/4"	93,312	2.5	3	2.9	3
	5	18900	24,192	2	1 1/2"	72,576	2.	3	2.2	3
	6	13500	17,280	2	1 1/2"	51,840	1.4	3	1.6	3
	7	8100	10,368	1	1 1/8"	31,104	.8	2	.9	3
	8	2700	3,456	1	1 1/8"	10,368	.03	2	.3	3

Ties
Lower Bracing

Struts
Upper Bracing

Gordon's formula becomes, calling $h=1$
and consequently $r^2 = \frac{h^2}{12} = \frac{1}{12}$ also $l^2 = 12^2 + 15^2$

$$= 369 \quad \frac{P}{S} = \frac{36000}{1 + \frac{369}{\frac{36000}{12}}} = \frac{36000}{1 + \frac{369}{3000}} = \frac{36000}{\frac{3369}{3000}}$$

$$= \frac{108}{.003369} = \frac{36}{.001123} \quad \therefore S = \frac{P \times .001123}{36}$$

$$\frac{155,520 \times .001123}{36} = 4.85$$

$$\frac{134,784 \times .001123}{36} = 4.20 \quad .65$$

$$\frac{114,048 \times .001123}{36} = 3.55 \quad .65$$

$$\frac{93,312 \times .001123}{36} = 2.91 \quad .64$$

$$\frac{72,576 \times .001123}{36} = 2.26 \quad .65$$

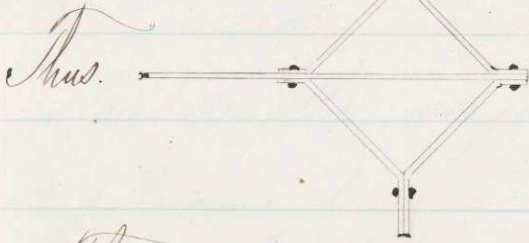
$$\frac{51,840 \times .001123}{36} = 1.61 \quad .65$$

$$\frac{31,104 \times .001123}{36} = .96 \quad .65$$

$$\frac{10,368 \times .001123}{36} = .31 \quad .65$$

These upper horizontals pass through the upper and lower divisions of the cell that is used for the upper boom and are rivetted by the same rivets that keep

the parts of the boom together.



The diagonals 4" wide are widened to 6" where they meet & pass through the parts of the cell, in order to make surface enough for rivets. Rivets throughout are placed 3" from centre to centre. ^{d 1 1/2 diam.}

The horizontal tie rods are 21.5 ft. in length, fastened as shown in plate of details.

A horizontal, perpendicular to the booms of 4" x 1" cross section, and length 14' 8", is put from end post to end post.

The angle irons for the lower booms are cut in the following manner.

First 14 1/2' from end, next 18' and all the rest except the remaining end, which is 14.5'

Approximate Weight

Consider the middle panel, and calculate the weight nearly. 1 cu. in. of iron weighs $\frac{5}{18}$ lbs.

2 I Beams. $l=17$	=	2267
Stringers ties etc	=	5415
2 Separators	=	150
2 Supporters	=	500
Wire rods	$2 \times 2 \frac{7}{8} \times 1 \times 12 \times \frac{5}{18}$	144
4 diag.	$4 \times 20 \times 1 \times 3 \times 12 \times \frac{5}{18}$	800
flange lower boom	$2 \times 14 \times 12 \times 12 \times \frac{5}{18}$	1120
Web	$2 \times 2 \times 20 \times 12 \times 12 \times \frac{5}{18}$	3200
Upper boom	$(72 \times 12 \times 12 + 7 \times 72 \times 12) \frac{10}{18}$	
	$(576 + 56) 10 = 6320$	6320
Ang. iron	$(4 \times 4 \times 12 \times 12 \times \frac{1}{2} + 3 \times 12 \times 12 \times \frac{1}{2} \times 4) \frac{5}{18}$	
	$7 \times 288 \times \frac{5}{18} = 560$	560
		20476 lbs.

The total dead load used for calculating the size of the pieces is 19,200 per panel. The 20,476 is actually larger than real weight because the dimensions are not taken exactly. Hence the bridge is safe.

H. K. Purvison

C. C.

May 10 1874