Computational Exploration of Origami Tessellation Design:

Harnessing Shape Grammar for Flexible Folding Structures

by

Lingyi Qiu

Submitted to the Department of Architecture
in partial fulfillment of the requirements of the Degree of

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Abstract

Origami tessellation, with its intricate folding patterns, presents a unique blend of artistic expression and engineering application. However, the design process often proves daunting due to its complexity, limiting accessibility to enthusiasts and impeding its potential impact in engineering and architecture fields. This thesis aims to lower the barrier of origami tessellation pattern design by leveraging shape grammar principles. Shape grammar provides a systematic framework for generating and analyzing folding patterns, offering a more intuitive and structured approach to design. Through computational exploration and experimentation, this research demonstrates the efficacy of shape grammar in creating diverse and innovative origami tessellation patterns. By streamlining the design process, this approach not only enhances the experience for origami enthusiasts but also opens up new avenues for engineering and architecture applications, including deployable structures, flexible materials, and adaptive systems. The integration of shape grammar into origami tessellation design has the potential to catalyze advancements in both artistic expression and practical utility, fostering creativity and innovation in diverse fields.

Key words: computational origami design, origami tessellation, shape grammar

Thesis advisor: Terry Knight
Title: Professor of Design and Computation

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I would like to express my sincere gratitude to my advisor, Terry Knight, for her invaluable guidance, unwavering support, and insightful feedback throughout the course of this thesis. Her expertise, encouragement, and dedication have been instrumental in shaping the direction of my research and enhancing the quality of my work. I am truly grateful for the time spent providing constructive criticism, offering valuable suggestions, and inspiring me to strive for excellence.

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PART I: Introduction

Chapter 1:

Problem Statement

This thesis presents new computational strategies that encourage creativity in conceptual origami tessellation pattern design. The first chapter motivates this research with a discussion of current design approaches and available tools, critiquing existing methods and identifying the needs and opportunities that the research in this thesis addresses.

1.1 Overview of origami tessellation

Origami, the ancient art of paper folding, has captivated minds for centuries with its simplicity and elegance. Beyond its aesthetic appeal, origami has found applications in various fields, including mathematics, engineering, and design. Origami tessellation, in particular, has emerged as a fascinating subset of origami, characterized by the repetition of folded units to create intricate geometric patterns. Renowned tessellation artists such as Ilan Garibi (Figure 1), Erik Demaine, and Goran Konjevod have contributed significantly to the exploration and advancement of this art form.

***Figure 1**: Adulthood (Ilan, Garibi, 2015)*

1.2 Significance of tessellation pattern design

While origami tessellation offers immense creative potential and practical utility, the process of designing new tessellation patterns remains challenging. Traditional methods often rely on trial and error or manual exploration, which can be time-consuming and limiting in scope. As a result, the barrier to entry for origami enthusiasts and the broader engineering community has remained high, hindering innovation and exploration in this field. However, despite these challenges, there are compelling motivations driving both recreational enthusiasts and educators/researchers to engage with origami tessellation design.

1.2.1 Recreational use

Origami tessellation design offers a unique blend of artistic expression and intellectual challenge, making it a popular recreational activity for enthusiasts of all ages. For many hobbyists, the process of folding intricate tessellation patterns provides a sense of relaxation and mindfulness, allowing them to immerse themselves in a meditative and creative pursuit. The repetitive nature of tessellation folding can be therapeutic, offering a welcome escape from the stresses of everyday life and providing a sense of accomplishment as complex patterns emerge from a single sheet of paper. Additionally, the visual appeal of tessellations, with their mesmerizing symmetries and intricate geometries, captivates the imagination and inspires endless exploration. For recreational origami enthusiasts, the motivation to design tessellation patterns lies in the joy of discovery, the satisfaction of mastering new folding techniques, and the pleasure of sharing their creations with others.

1.2.2 Educational use

Origami tessellation design also holds significant appeal for researchers and educators seeking to explore the intersection of art, mathematics, and science. From a research perspective, tessellation patterns offer a rich terrain for investigating geometric principles, mathematical algorithms, and computational modeling techniques. By studying the underlying structure and properties of tessellations, researchers can gain insights into fundamental concepts such as symmetry, topology, and optimization, with applications ranging from material science to robotics. Moreover, the process of designing and analyzing tessellation patterns provides a valuable context for interdisciplinary collaboration, fostering connections between artists, mathematicians, engineers, and educators. From an educational standpoint, origami tessellation design offers a hands-on approach to learning mathematical concepts and principles. By engaging students in the process of folding and manipulating paper, educators can promote spatial reasoning, problem-solving skills, and geometric intuition, while fostering creativity and curiosity. Tessellation patterns serve as tangible representations of abstract mathematical ideas, making them accessible and engaging for learners of all ages and backgrounds. Through the exploration of tessellation design, students can develop a deeper appreciation for the beauty and elegance of mathematics, inspiring a lifelong passion for learning and discovery.

1.2.3 Practical

Origami tessellation design isn't just about recreation and education; it holds significant practical utility across various fields. Mechanical engineers and designers leverage tessellation patterns to create visually stunning and structurally sound metamaterials (Zhai et al., 2013). In addition, origami-inspired designs find applications in aerospace engineering, medical device design, and the development of innovative materials and manufacturing techniques. For instance, researchers have developed self-folding machines inspired by origami principles, enabling the creation of complex three-dimensional structures (Felton et al., 2014). Origami tessellation design has also been employed in the development of deployable membrane structures for space applications, showcasing its adaptability and versatility (Morgan et al., 2007). These applications highlight the versatility and adaptability of origami tessellation design, paving the way for innovative solutions to complex challenges in diverse industries.

1.3 Challenges of tessellation pattern design

Designing tessellation patterns presents various challenges that require careful consideration and problem-solving. One of the primary challenges is achieving balance between structural integrity and aesthetic appeal. Tessellation patterns often involve intricate folds and repeated motifs, which can introduce weak points or inconsistencies in the overall structure. Engineers and designers must find ways to reinforce these patterns without compromising their visual impact.

Another challenge is scalability and adaptability. While some tessellation patterns may work well on a small scale, scaling them up for larger structures can present difficulties. Factors such as material properties, folding techniques, and geometric proportions must be carefully adjusted to ensure the pattern retains its integrity and functionality at different sizes.

Additionally, tessellation pattern design often involves complex mathematical and computational modeling. Creating precise folding instructions and algorithms requires a deep understanding of geometry, topology, and spatial relationships. Moreover, translating these mathematical concepts into practical, real-world designs can be a daunting task that requires collaboration between mathematicians, engineers, and artists.

Furthermore, achieving symmetry and balance in tessellation patterns can be challenging, particularly when dealing with irregular shapes or non-linear folds. Ensuring that each element of the pattern fits seamlessly with its neighbors while maintaining overall coherence requires meticulous planning and attention to detail.

1.4 Introducing Shape Grammar

Shape grammar is a computational design methodology that formalizes the rules governing the generation and manipulation of shapes within a given design space. It provides a systematic framework for describing and generating complex geometric forms by specifying a set of rules or production sequences. These rules define the relationships between different elements of a shape, such as their size, position, orientation, and connectivity. By applying these rules iteratively, designers can generate a wide variety of shapes and patterns with desired properties and characteristics. Shape grammar has been widely used in various fields, including architecture, urban design, product design, and origami tessellation, to explore and create novel design solutions efficiently and systematically.

1.4.1 Concepts and principles of Shape Grammar

A shape grammar consists of *shape rules* and a *generation engine* that selects and processes rules. A shape rule defines how an existing (part of a) shape can be transformed. A shape rule consists of two parts separated by an arrow pointing from left to right. The part left of the arrow is termed the *Left-Hand Side* (LHS). It depicts a condition in terms of a shape and a marker. The part right of the arrow is termed the *Right-Hand Side* (RHS). It depicts how the LHS shape should be transformed and where the marker is positioned. The marker helps to locate and orient the new shape.

A shape grammar minimally consists of three shape rules: a *start rule*, at least one transformation rule, and a *termination rule*. The start rule is necessary to start the shape generation process. The termination rule is necessary to make the shape generation process stop. The simplest way to stop the process is by a shape rule that removes the marker.

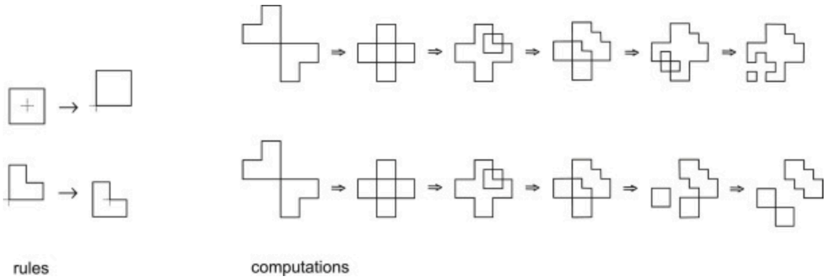


Figure 2a: *Rules and computations of 2D object (Terry Knight)*

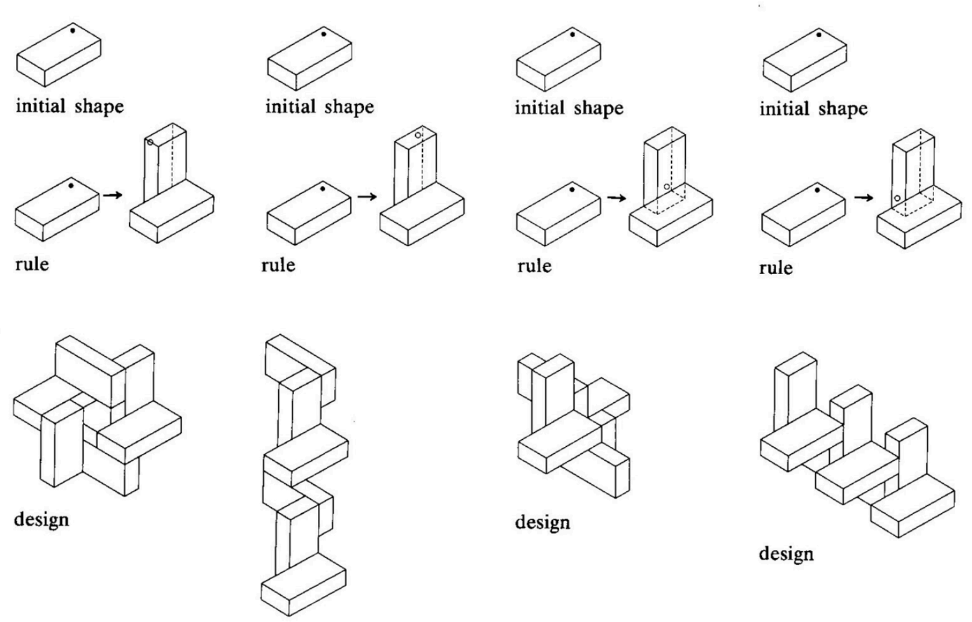


Figure 2b: *3D object computations under different Rule (Terry Knight, 1999)*

Shape grammar offers several advantages in the context of design. With its systematic approach, designers can follow clear rules and procedures for creating complex forms, ensuring consistency and coherence in their designs. Its generative power enables the exploration of a wide variety of shapes and patterns, fostering creativity and innovation. Additionally, shape grammar is flexible and adaptable, making it suitable for diverse design contexts and objectives. By automating certain aspects of the design process, such as pattern generation and exploration, shape grammar promotes efficiency and productivity, see diverse range of shapes and forms created using Shape Grammar in Figure 2a and 2b. Furthermore, its iterative nature facilitates design exploration and iteration, allowing designers to quickly generate and evaluate multiple design alternatives. The accompanying shape grammar computation images showcase the diverse range of shapes and forms that can be created using this approach, illustrating its potential to inspire and inform the design process.

1.4.2 Applications of Shape Grammar

Shape grammar finds applications in various fields. It's widely used in architectural design for generating and exploring design alternatives, creating complex building forms, and analyzing spatial configurations. An example is shown in Figure 3a where shape grammar is used to create a space enclosure.

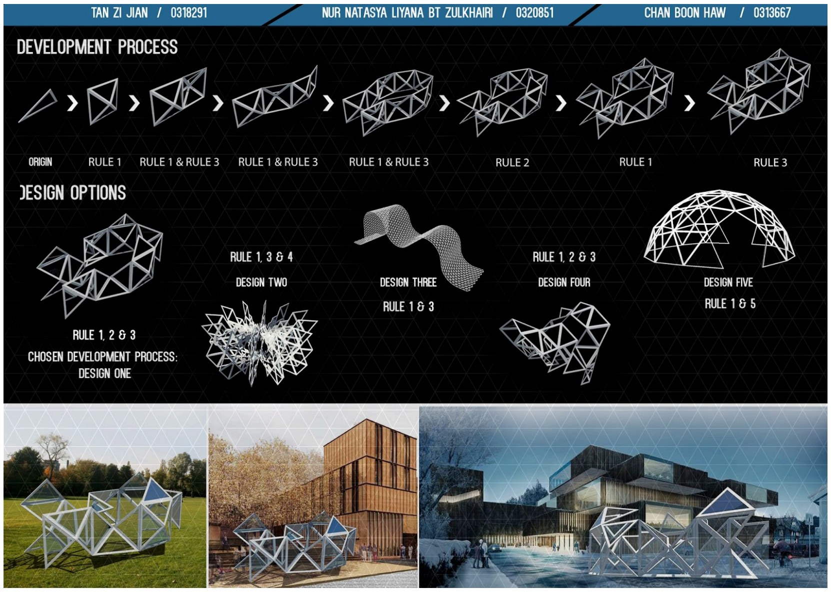


Figure 3a: *Shape grammar in architectural project design (Zijian Tan, 2016)*
(Full sized image in Appendix C)

Shape grammar serves as a valuable tool for teaching design principles and techniques in art and design education, helping students understand spatial relationships, form generation, and design logic. A book about shape grammar generated art form is pictured in Figure 3b.

Figure 3b: *Shape grammar art (Jannis Maroscheck)*

In addition, shape grammar has diverse applications across various domains. In urban design, it aids in generating street layouts, building facades, and urban landscapes, thereby facilitating the planning and development of cities and neighborhoods. In product design, it enables the creation of a wide range of product designs, spanning from furniture and consumer goods to industrial machinery and transportation vehicles. Moreover, in computational design, shape grammar is instrumental in developing algorithms and software tools for automating design processes, creating parametric models, and generating design variations.

Overall, shape grammar offers versatile applications, providing designers and researchers with a powerful tool for generating, exploring, and refining design solutions, thus presenting abundant opportunities for innovation and creativity.

1.5 Objectives of the Thesis

This thesis seeks to address the challenge of lowering origami tessellation pattern design barrier by proposing a novel approach to origami tessellation pattern design: the integration of shape grammar principles. Shape grammar provides a formal framework for describing and generating complex patterns by recursively applying shape transformation rules. By applying shape grammar concepts to origami tessellation, this thesis aims to streamline the design process, foster creativity, and lower the barrier of entry for enthusiasts and professionals alike.

1.6 Organization of the Thesis

This thesis comprises three main sections: *Introduction*, *Shape Grammar-Based Origami Tessellation Design Framework,* and *Conclusions*.

The *Introduction* section sets the stage by presenting the problem statement and conducting a critical literature review relevant to the research question addressed in this thesis.

The *Shape Grammar-Based Origami Tessellation Design Framework* section delves into the study of existing origami tessellation pattern designs, extracting underlying logic from them, and formulating a new shape grammar for origami tessellation pattern design.

Chapter 4 provides an introduction to Origami Simulator, an online tool for fold simulation, and demonstrates how it can be utilized to visualize and simulate the designed patterns.

Chapter 5 delves into the possibilities and challenges associated with integrating shape grammar into origami tessellation design, exploring both the potential benefits and limitations.

Chapter 6 outlines future directions for research in the field, identifying areas for further exploration and development.

Chapter 7 summarizes the specific intellectual contributions of the thesis and discusses their potential impact, envisioned applications, and important directions for future research.

Additionally, there are two appendices that document detailed results referred to in the Design Space Strategies chapters. A list of figures is sorted in Appendix D. References are also included in Appendix E.

Chapter 2:

Literature Review

2.1 Origami tessellation: history, development, and applications

Origami tessellation design represents a fascinating intersection of art, mathematics, and culture, with roots dating back centuries to traditional Japanese paper folding practices. However, it wasn't until the latter half of the 20th century that origami tessellations began to receive significant attention and exploration within the origami community. This section provides an in-depth review of the historical evolution of origami tessellation design, highlighting key milestones and influential figures in the field.

Robert J. Lang's groundbreaking work, "Origami Design Secrets: Mathematical Methods for an Ancient Art" (Lang, 2003), stands as a cornerstone in the development of modern origami tessellation techniques. In this comprehensive tome, Lang elucidates the mathematical principles underlying origami design, including the intricate tessellation patterns that have captivated artists and mathematicians alike. Through meticulous analysis and experimentation, Lang demonstrates how geometric concepts such as symmetry, proportion, and modulation can be leveraged to create stunning tessellation designs. Lang's pioneering insights laid the groundwork for subsequent advancements in the field, establishing him as a leading authority on origami tessellations.

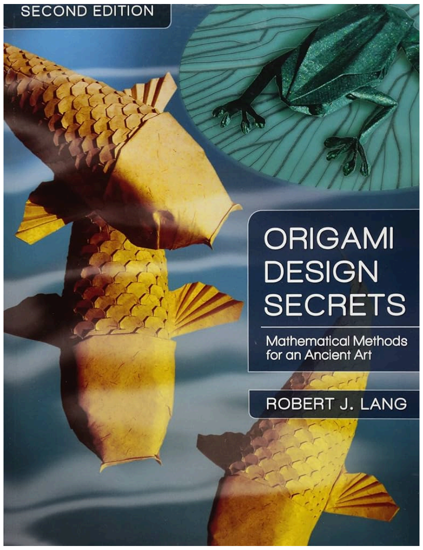


Figure 4: *Origami Design Secrets (Lang, 2003)*

A significant milestone in the computational aspect of origami tessellation design came with the publication of "From Pixels to Triangles: A Generalized Algorithm for Computing Crease Patterns" by Goran Konjevod and Robert J. Lang (Konjevod & Lang, 2004). This seminal paper introduced a revolutionary approach to origami design, leveraging computational algorithms to generate crease patterns for complex tessellations. By bridging the gap between mathematical theory and practical design, Konjevod and Lang's algorithmic framework enabled artists to algorithmically transform geometric shapes into origami folds, opening up new avenues for creativity and exploration in tessellation design.

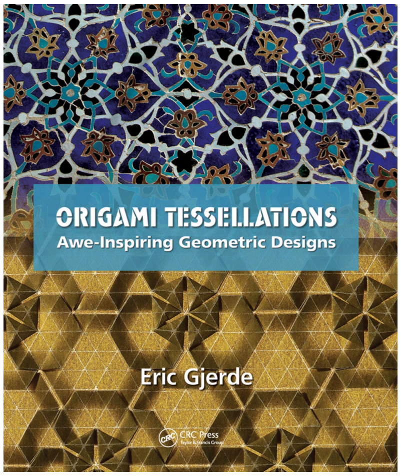


Figure 5: *Awe-Inspiring Geometric Designs (Gjerde, 2008)*

In "Origami Tessellations: Awe-Inspiring Geometric Designs" (Gjerde, 2008), Eric Gjerde provides a comprehensive survey of the artistic and creative potential of origami tessellations. Drawing upon centuries of origami tradition and contemporary innovations, Gjerde showcases a diverse array of geometric patterns and tessellation techniques employed by artists around the world. Through stunning visual examples and insightful commentary, Gjerde highlights the versatility and expressive power of origami tessellations as a medium for artistic expression and geometric exploration. His work serves as a testament to the enduring appeal and relevance of tessellation design in the modern origami landscape.

2.2 Previous research on shape grammar in origami

Shape grammar, a formal system for describing and generating geometric forms, has emerged as a powerful tool for analyzing and generating origami designs. This section provides an overview of previous research on shape grammar in origami, highlighting key studies and contributions that have shaped the field.

In "Computational Origami: From Flapping Birds to Space Telescopes" (Lang, 2001), Robert J. Lang introduces the concept of shape grammar as a fundamental framework for understanding and creating origami designs. Lang's seminal work explores the application of shape grammar principles to a wide range of origami models, from simple animals to complex tessellations. Through detailed analysis and experimentation, Lang demonstrates how shape grammar rules can be used to systematically generate and manipulate origami forms, offering valuable insights into the underlying structure and composition of origami designs.

Building upon Lang's foundational work, the paper "OPTIMAL DESIGN FOR DEPLOYABLE STRUCTURES USING ORIGAMI TESSELLATIONS" by Carolina Cardona, Andres Tovar, Sohel Anwar (Cardona et al., 2019) presents a computational approach to deployable structures based on origami tessellation principles. The paper presents innovative origami optimization methods aimed at designing unit cells for complex origami tessellations, with a specific focus on their potential application in the creation of deployable structures. The design approach leverages discrete topology optimization principles, typically used for ground structures, and applies them to origami crease patterns. The initial design space encompasses all potential creases, with input and output forces specified. By incorporating foldability constraints derived from Maekawa's and Kawasaki's theorems, the algorithm distinguishes between active and passive creases. Geometric constraints are defined based on the target 3D object. Through the periodic reproduction of the unit cell, tessellations are created and evaluated for structural integrity. The paper also discusses design requirements for robust tessellations and outlines future directions, including the exploration of origami-inspired mechanisms and self-deployable structures, with a particular emphasis on shelters for natural disasters.

In "Algorithmic design of origami mechanisms and tessellations (Walker & Stankovic, 2022), Walker and Stankovic investigate the integration of algorithmic and physical approaches to origami mechanisms and tessellation design. The paper introduces a novel approach to algorithmically design rigidly-foldable origami structures with a single kinematic degree of freedom, unlocking the full potential of origami for design practitioners and researchers. By leveraging generalized conditions for rigid foldability, the proposed method enables the creation of origami patterns of arbitrary size and complexity. Notably, the approach extends beyond regular origami patterns to encompass kirigami, generic three-dimensional panel-hinge assemblages, and their tessellations. This versatility offers an abundant source of foldable patterns with applications spanning from foldable structures to reconfigurable metamaterials, showcasing the transformative potential of origami-inspired design in various fields.

2.3 Review of existing computational tools for origami pattern design

The field of computational origami has seen significant advancements in recent years, with the development of various software tools aimed at facilitating origami pattern design and simulation. This section provides an overview of current computational tools for origami pattern design, highlighting their features, capabilities, and contributions to the field.

2.3.1 Tess

Tess stands out as a dedicated software tool specifically designed for creating origami tessellation patterns. Developed by origami artist and researcher Eric Gjerde, Tess offers a range of features for designing and manipulating tessellation patterns, including symmetry operations, grid generation, and pattern simulation. With its user-friendly interface and powerful tools, Tess has become a popular choice among origami enthusiasts and researchers alike for exploring the creative possibilities of tessellation design.

2.3.2 Origamizer

Origamizer represents another notable contribution to computational origami, offering an automated approach to generating origami crease patterns from 3D models. Developed by researchers at MIT, Origamizer leverages mathematical algorithms to algorithmically convert arbitrary shapes into origami folds, enabling designers to create complex origami structures with ease. By automating the process of pattern generation, Origamizer streamlines the design workflow and opens up new possibilities for exploring origami design space.

2.3.3 TreeMaker

TreeMaker, developed by origami artist and mathematician Robert J. Lang, is a software tool primarily used for designing origami crease patterns for tree-like structures. While TreeMaker is not specifically tailored for tessellation design, its powerful algorithms and intuitive interface make it a valuable tool for exploring geometric relationships and generating complex origami patterns. With its emphasis on mathematical precision and artistic expression, TreeMaker remains a popular choice among origami enthusiasts and researchers for creating intricate origami designs.

2.3.4 Rhino + Grasshopper

Rhino and *Grasshopper* represents a versatile platform for parametric design and computational modeling, offering a range of tools for generating and manipulating geometric forms. While not specifically designed for origami pattern design, Rhino and Grasshopper can be used in conjunction with custom scripts and plugins to create complex origami tessellations. By leveraging parametric design principles, designers can explore the creative possibilities of origami design space and generate intricate patterns with precision and efficiency.

2.3.5 Adobe Illustrator & FOLD

Adobe Illustrator and *FOLD* (Free Origami Layout Design) offer alternative approaches to origami pattern design, focusing on vector graphics and digital simulation, respectively. While Adobe Illustrator provides a powerful tool for creating and manipulating geometric shapes, FOLD offers a web-based platform for simulating origami folding processes and optimizing designs for manufacturability. Both tools offer unique advantages and capabilities for origami pattern design, catering to different workflows and design requirements.

2.4 Critical evaluation of challenges and opportunities in the field

The challenges of accessibility and interoperability in computational tools for origami pattern design underscore the importance of fostering collaboration and standardization within the field. By promoting open-source initiatives and developing standardized protocols for data exchange, researchers can enhance accessibility and facilitate seamless integration between different software platforms. Moreover, efforts to improve user interfaces and provide comprehensive documentation can reduce barriers to entry and empower a broader range of designers and researchers to leverage computational tools in their work.

Despite these challenges, the opportunities for innovation and advancement in computational origami are abundant. The growing interest in interdisciplinary research and the rapid development of computational algorithms offer exciting possibilities for automating design processes and pushing the boundaries of origami creativity. As researchers continue to explore new techniques and methodologies, the field of computational origami is poised to make significant strides in the coming years, driving innovation and inspiring new avenues of exploration in the art and science of origami pattern design.

PART II: Origami Tessellation Pattern Design Space Strategies

Chapter 3:

An Origami Tessellation Pattern Design Grammar

This chapter studies existing designs in the field of origami tessellation pattern, with a keen focus on uncovering the hidden principles that underpin existing designs. Through meticulous examination, these principles are distilled into a newly designed shape grammar - XXX, crafted to streamline and enrich the pattern design process. Subsequently, the grammar will be translated and implemented in Rhino/Grasshopper to automate the pattern generating process. Additionally, this chapter explores expanded variations of pattern design through case studies, shedding light on the significant differences that emerge from parameters within the grammar.

3.1 Designing origami tessellation patterns with Shape Grammar

Origami tessellation patterns can be based on various types of grids, including both rectangular grids ***(Figure 6a)*** and hexagonal/triangular grids ***(Figure 6b)***. The choice of grid type depends on the specific design and desired aesthetic of the tessellation pattern. Rectangle grids are commonly used and provide a more angular and geometric appearance to the pattern, while hexagon grids offer a more organic and flowing look. In the scope of this thesis, the focus will primarily be on tessellation design patterns based on rectangular grids.

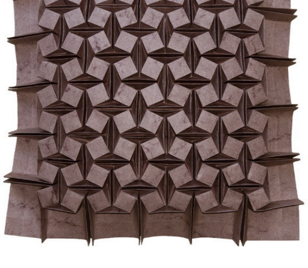


Figure 6a. *Rotated Cubes (Ilan Garibi, 2009)*

Figure 6b: *Bedrock (Creators.desk, 2022)*

3.1.1 Study of existing tessellation patterns

As stated earlier in this chapter, it is imperative to examine a selection of existing origami tessellation pattern designs. This section offers thorough analyses of these patterns.

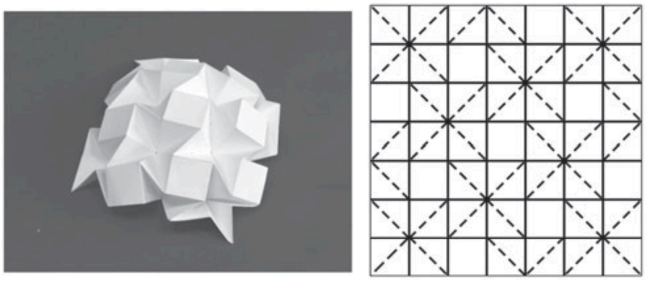


Figure 7a: *Waterbomb Tessellation and its grid pattern (Cheng et al, 2016)*

In Figure 7a, a folded tessellation form known as the waterbomb is presented alongside its corresponding grid pattern. Upon close examination of the pattern and its folded structure, a discernible repetition of squares becomes evident. To facilitate observation, a selection of four squares is chosen for further analysis in Figure 7b. Subsequently, lines are drawn from each vertex of the square, resulting in the formation of a "pinwheel" configuration (Figure 7c), delineated by blue lines. Following this, additional lines, depicted in red, extend from each vertex at a 45-degree angle, covering the dashed lines indicative of valleys. This process yields a unit of the pattern. Upon further scrutiny, it becomes apparent that the entire tessellation pattern can be constructed through the repetitive arrangement of this unit. Once the pattern is fully populated with these repetitions, the paper size can be delineated by cropping the pattern, as illustrated within the black box in Figure 7b.

Another observation emerges at the final stage of pattern formation: not all original gray lines from the initial pattern are obscured, indicating the need for further investigation. Upon closer examination (see Figure 7d), it becomes evident that the addition of cross lines of mountain folds is necessary to ensure the foldability of this pattern.

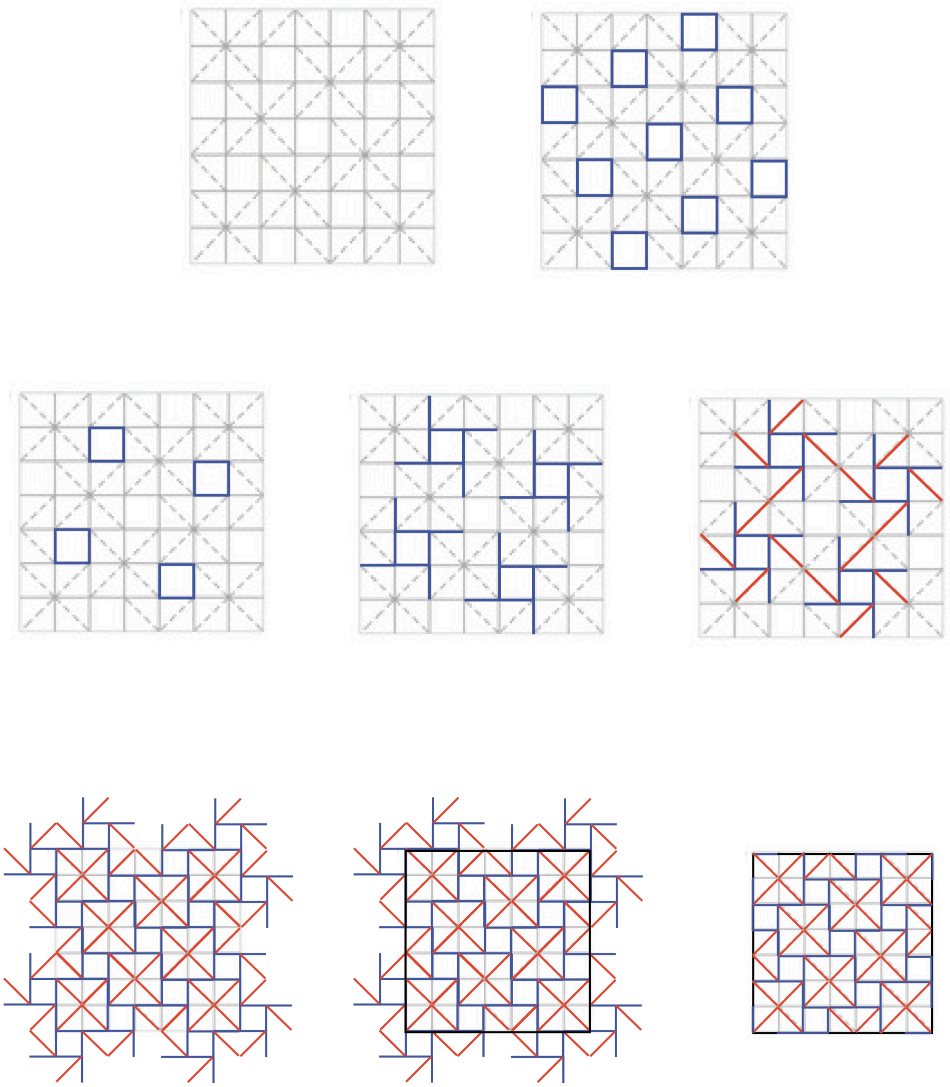


Figure 7b: *Sequences of pattern discovery for Waterbomb*

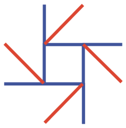


Figure 7c: *Pinwheel unit*

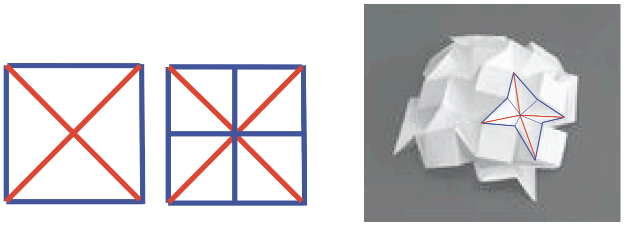


Figure 7d: *Additional mountain line refinement to ensure foldability*

Similarly, further examination of additional patterns (see Figure 8a and Figure 9a) reveals a common origin from a pinwheel unit, which then expands through repetition via either direct translation, as illustrated in Figure 8b or reflection, as depicted in Figure 9b.

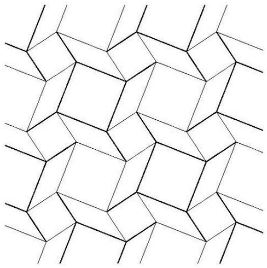


Figure 8a: Tessellation Pattern (Unknown)

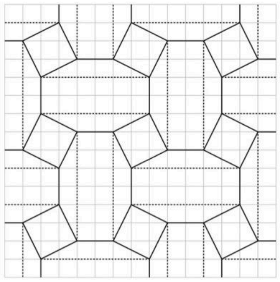


Figure 9a: Square Weave Tessellation (Eric Gjerde, 2006)

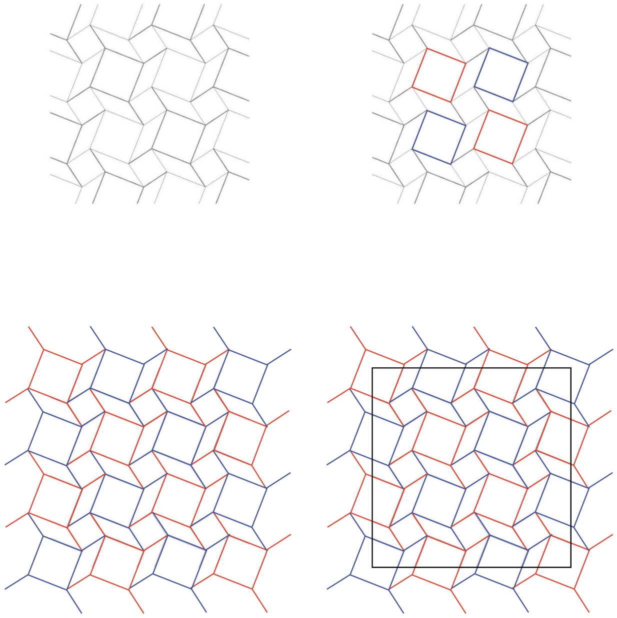


Figure 8b: *Sequences of pattern discovery for waterbomb*

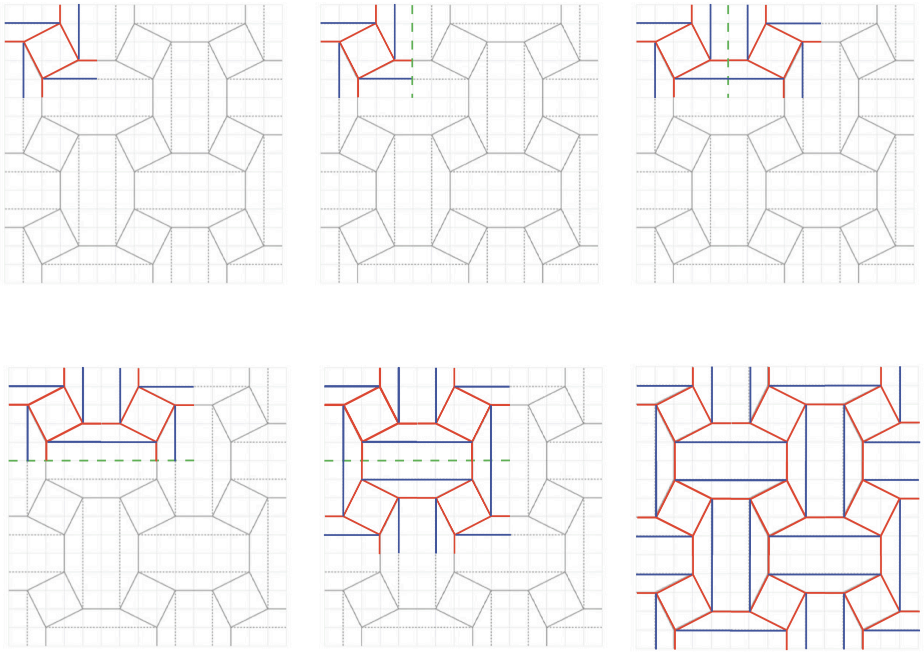


Figure 9b: *Sequences of pattern discovery for Square Weave Tessellation*

In the concluding example, the thesis examines "*Adulthood*" (see Figure 10a), a tessellation design created by the esteemed origami expert Ilan Garibi. The folding pattern for this design (see Figure 10b) is sourced from Happyfolding, a YouTube channel dedicated to origami tutorials.

(a) Adulthood (Ilan, Garibi, 2015)

(b) Folding tutorial (Happyfolding, 2019)

Figure 10: *Tessellation pattern "Adulthood" in folded and unfolded form*

Through observation, it becomes apparent that a pinwheel remains at the center, albeit with a subtle alteration: a zigzag pattern emerges along each side of the square. This arrangement, when folded, imparts the illusion of curvature. These findings are meticulously documented and illustrated in Figure 10c.

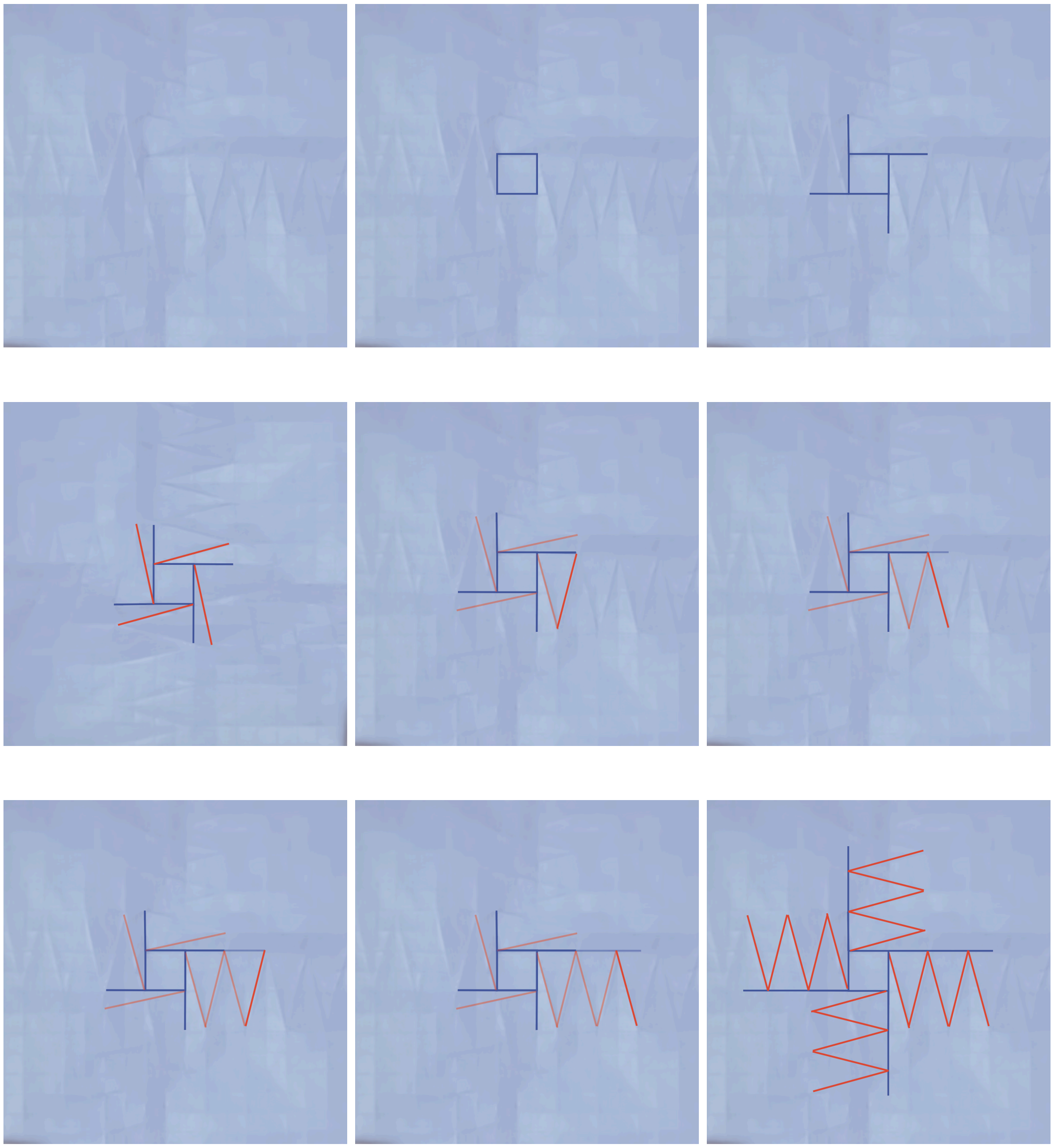


Figure 10c: *Sequences of pattern discovery for Adulthood*

Upon comprehensive examination of various examples, it becomes evident that the tessellation pattern design process can be modeled as that begins with a square base, which then transforms into a pinwheel configuration. This foundational structure is subsequently enriched with creative variations, such as 45-degree angle rotations or elongated zigzag triangular shapes, resulting in the formation of intricate tessellation patterns. Notably, the repetition of these fundamental units, often accompanied by mirror reflections or direct translations, ultimately yields a cohesive and holistic tessellation pattern.

Section 3.1.2 will discuss the translation of the above findings into integration within a shape grammar for designing tessellation patterns.

3.1.2 Tessellation pattern grammar

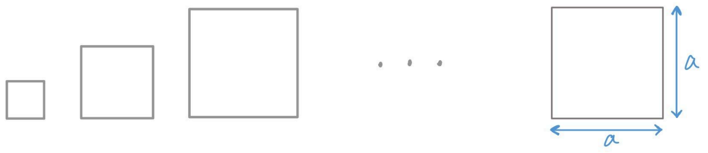
As revealed in Section 3.1.1, the entirety of the pattern design process can be conceptualized through four distinct stages:

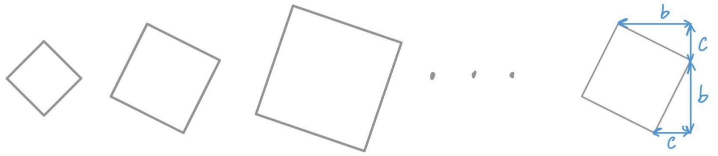
Step 1. **Foundational Square Selection**:
Establishing the foundation with square dimensions and rotation parameters
Step 2. **Pinwheel Expansion**:
Extending the square to form the initial pinwheel structure
Step 3. **Creative Embellishment**:
Unleashing creativity to expand the pattern from the pinwheel, enriching tessellation variations
Step 4. **Iterative Reproduction**:
Repeating translation or reflection of the unit pattern until achieving desired completion and ready for folding

The following content delves into how shape grammar can be employed in each of the four steps.

Step 1: Foundational Square Selection
In this step, square dimensions and rotational parameters are established. Size and rotational angles are determined by parameters, as illustrated below.

Initial square determination (**Rule1):**





Select an initial square from above two styles:

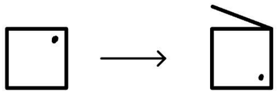
Style 1: no rotation (top), Style 2: with rotation (bottom)

Note 1: a, b, c are integers, which indicates the number of grid(s) it occupies.

Note 2: b, c can be negative, indicating reverse rotating direction.

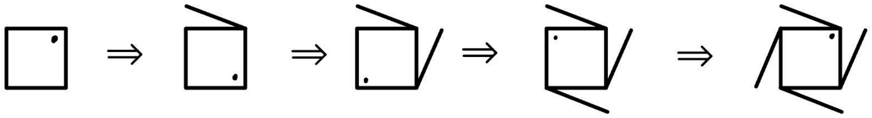
Step 2. **Pinwheel Expansion**

Pinwheel Creation Rule (**Rule 2**):



The initial shape is a square, with the starting label positioned at the top-right corner. The rule dictates that a diagonal line is drawn from the vertex of the square adjacent to the label, followed by clockwise shifting of the label to the next vertex.

Pinwheel computation:



This completes a full cycle of computation based on this rule. In a finished computation, the end label is erased, although it is retained here for future step usage.

The square identified in step 1 is consequently extended to shape the initial pinwheel structure. Refer to Figure 11 for visual representations of pinwheel styles and additional details.

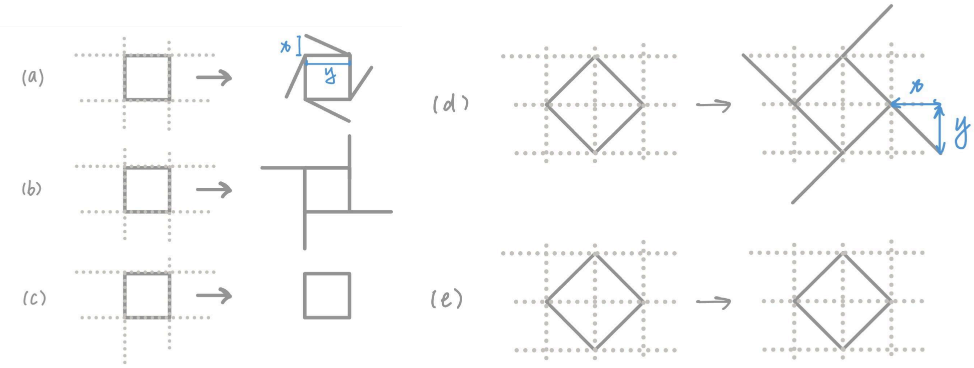
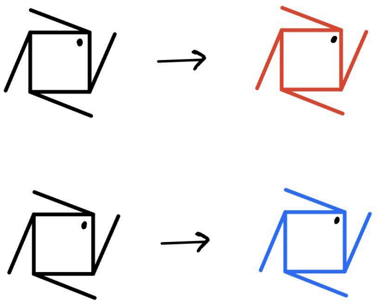


Figure 11: *Pinwheel styles generated from 2 initial square styles.*
No rotation from style 1 (on the left), with rotation from style 2 (on the right)

In pinwheel (a) and (d), $x,\ y\ >\ 0$;
In pinwheel (b), $x\ =\ 0,\ y\ >\ 0$;
In pinwheel (c) and (e) , $x\ =\ 0,\ y\ =\ 0$;

To ensure compatibility with origami foldability, the determination of mountain and valley folds is essential from the outset. This requirement is addressed by rule 2*.

<u>**Rule 2***: Mountain/Valley Determination:</u>

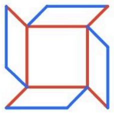


With the pinwheel created, set its mountain/valley property. Red color indicates maintain fold, blue color indicates valley fold.

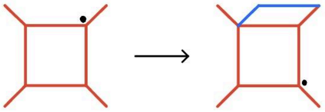
Step 3. **Creative Embellishment**

With the pinwheel formation established, the stage is set for a creative exploration. This step focuses on unleashing creativity to expand the pattern from the pinwheel into a unit ready for repetitive reproduction in step 4, thereby expanding tessellation variations. Drawing insights from past findings, the incorporation of techniques such as forming parallelograms, zigzag shapes, or other innovative approaches is encouraged. Examples of rules and computations are provided below.

Example 1: "Adding a parallelogram"



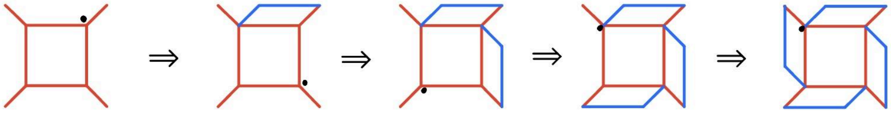
Example **Rule 3**:



In this example, the initial square is selected without rotation, featuring a pinwheel-style extension of 45-degree lines. The starting label is positioned at the open corner of the top-right vertex. As the blue line is drawn to create the parallelogram, the starting label undergoes a 90-degree clockwise rotation to the right side of the bottom-right vertex.

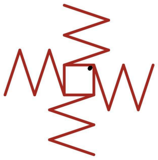
Note: the added parallelogram is drawn in blue, which indicates valley fold contrasts to the base pinwheel red color - mountain fold, to encompass foldability in origami tessellation. For more information on mountain/valley assignment in origami tessellation pattern design, please see paper "Mountain-Valley Crease Assignment and Reconfigurability of 4-Crease Origami Vertices and Their Tessellations" (W. Liu, S. Cao, Y. Chen, 2024) https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4710435

Computation:

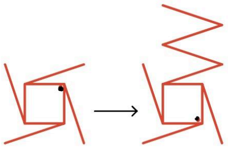


This completes a full cycle of computation based on this rule. Ready for repetitive reproduction for step 4.

Example 2: “Zigzag”

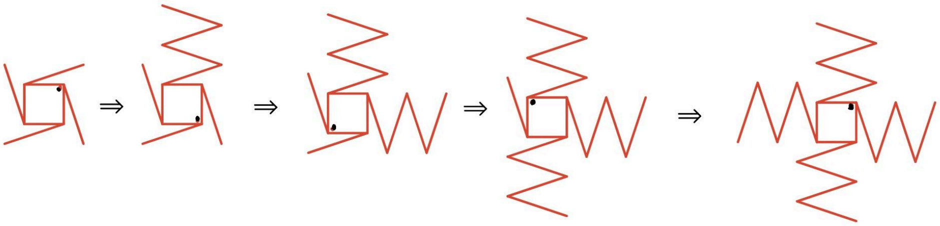


<u>Example **Rule 3**:</u>



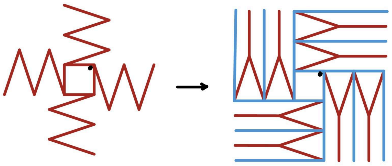
In this example, the initial square is selected without rotation, featuring a pinwheel-style extension of 30-degree lines. The starting label is positioned at the open corner of the top-right vertex. As the zigzag line is drawn, the starting label undergoes a 90-degree clockwise rotation to the right side of the bottom-right vertex.

<u>Computation:</u>



This completes a full cycle of computation based on this rule. Ready for repetitive reproduction for step 4.

Rule 3*: <u>Mountain/Valley Addition</u>

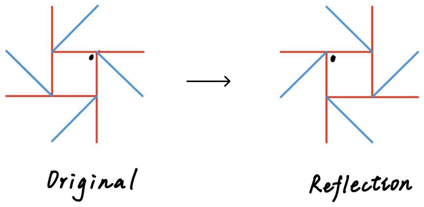


Note: After computation, additional valley lines need to be incorporated to ensure foldability prior to repetitive reproduction.

Step 4. **Iterative Reproduction**

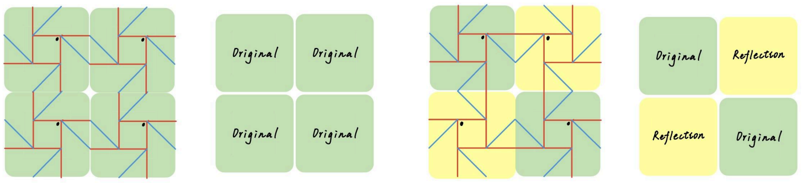
This step focuses on iteratively reproducing the previous unit through translation or reflection of the unit pattern until achieving the desired completion and readiness for folding.

Rule 4A: Reflection Rule:



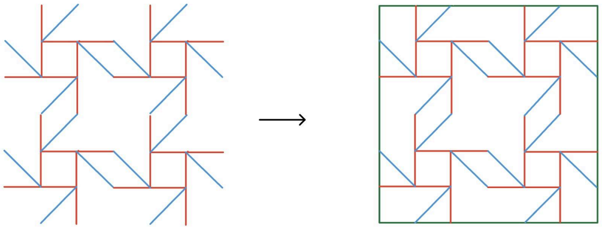
This rule describes the reflection of the unit pattern under symmetrical reflection.

Rule 4B: Reproduction Rule:



Rule 4B describes 2 ways to iteratively reproduce the unit pattern (2 spatial relations) - either directly translate the original unit pattern (shown on the left), or mismatch the original and the reflection of the unit pattern (shown on the right) to form a simple four-unit pattern.

Rule 4C: Boundary Capturing Rule



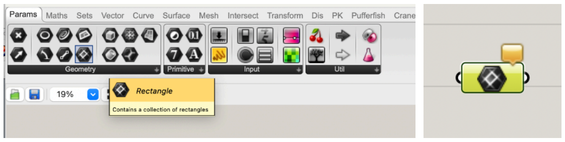
Once satisfied with the pattern design, Rule 4C necessitates capping the final design with a box, denoted in green, to signify the paper shape for folding.

The next section aims to implement the shape grammar rules outlined in Section 3.1.2 using Rhino/Grasshopper for parametric optimization purposes.

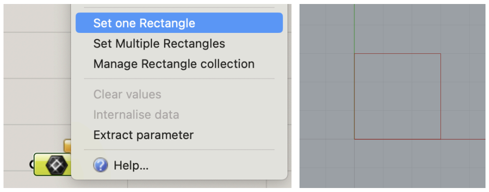
3.2 Implementation of Origami Tessellation Grammar rules in Rhino/Grasshopper

With the above rules defined by shape grammar, this section will introduce the implementation of the rules in Rhino/Grasshopper.

Step 0: Set Initial Square

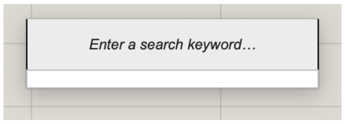


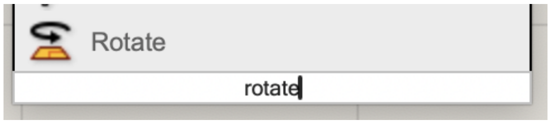
In grasshopper interface, under “Params”, select component “Rectangle”

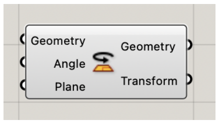


Right click to select “Set one Rectangle”. Then in the Rhino interface, draw a square from the origin. Initial square is thus created.

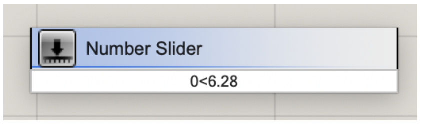
Step 1. **Foundational Square Selection**



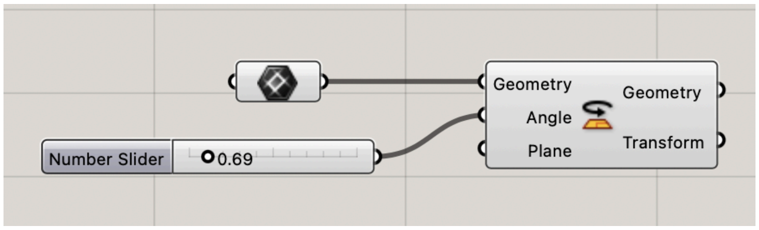




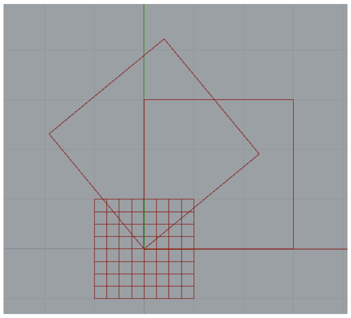
Press space bar, a prompt will show up. Type “Rotate”, and select this icon Rotate.



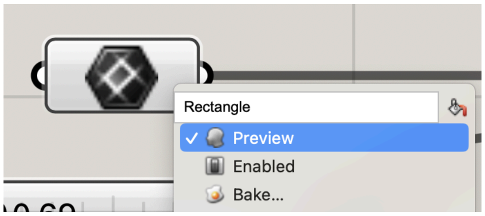
Then press space bar again, find “Number Slider”, type “0< 6.28”, this creates a number slider ranges from 0.00 to 6.28 (equals to 2π).



Connects current components as shown.

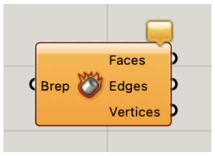


Now drag the slider, the rotational angle of the square is dictated by the value of the number slider. The size of the square can be reset by repeating “Set one Rectangle” by right clicking the rectangle component. This completes step 1.

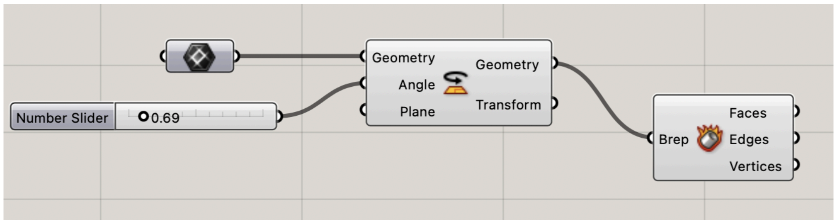


For visual purpose, the original square can be hidden by right clicking the component and deselect "Preview"

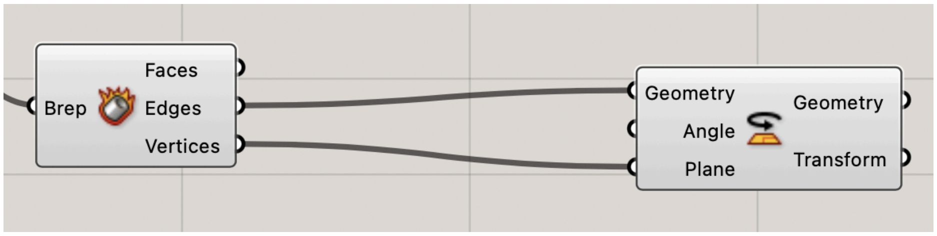
Step 2. Pinwheel Expansion



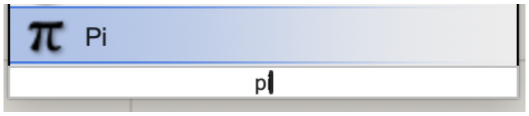
Input a new component called "Deconstruct Brep".



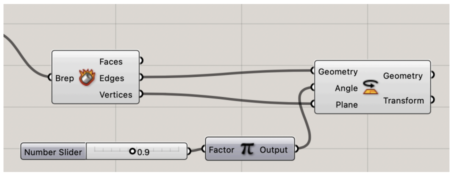
And connect to previous components like so. The logic here is to deconstruct the square created in step 1, and rotate their edges with centers being corresponding vertices.



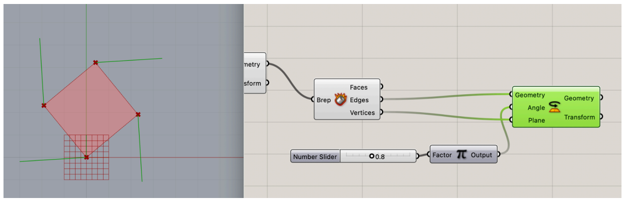
Create a new Rotate component, connect "Edges" to "Geometry", and "Vertices" to "Plane".



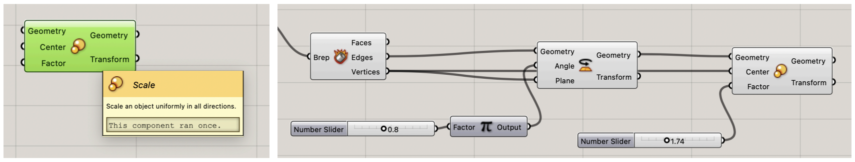
Press spacebar, enter "Pi" to get a π component, and create a number slider from 0.0 to 2.0 (rad). This allows the edges to be rotated freely from 0 to 360 degrees.



Connect all components like such.



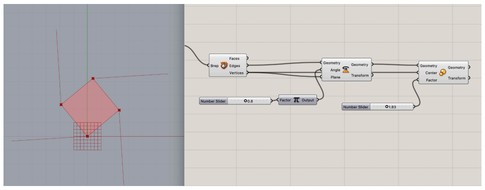
Dragging along the number slider results in rotation of the pinwheel arms.



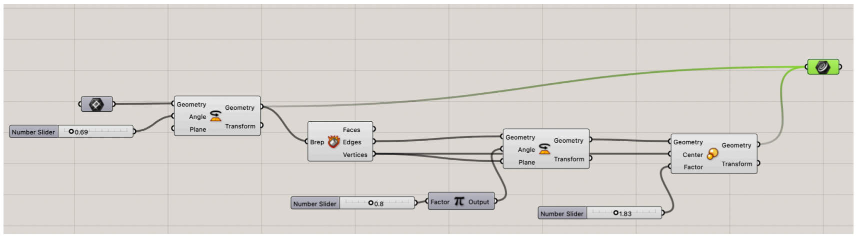
Scale an object uniformly in all directions.

This component ran once.

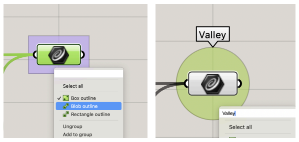
To change the length of the pinwheel arms, input a "Scale" component, connect geometry from "Rotate", and connect "vertices" from "Deconstruct Brep" to "Center". Create a number slider from a reasonable range, connect to "Factor"



Extended pinwheel arm can be seen from the Rhino interface.



For step 2*, determining mountain/valley, use a curve component to include outlines as shown above,

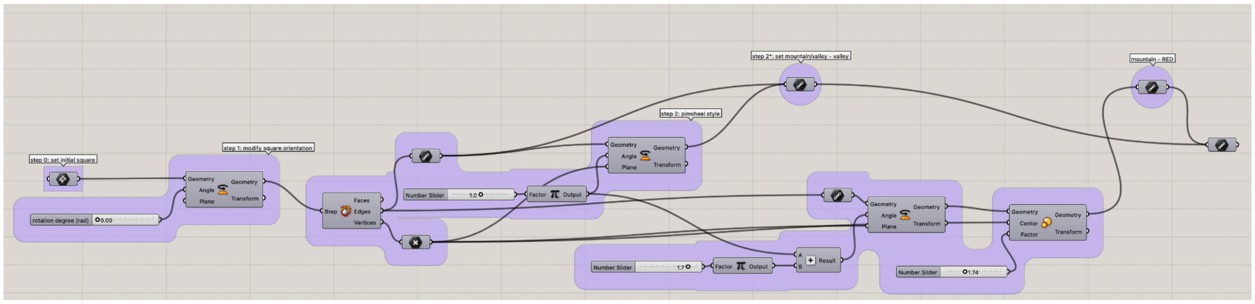


For good practice, right click the component to "Group", and right click again to change to "Blob outline", and label as "Valley" or "Mountain".

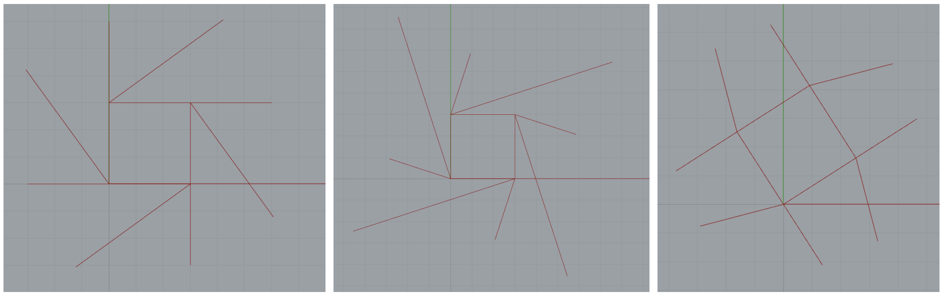
Step 2 Pinwheel expansion is completed.

Step 3. **Creative Embellishment**

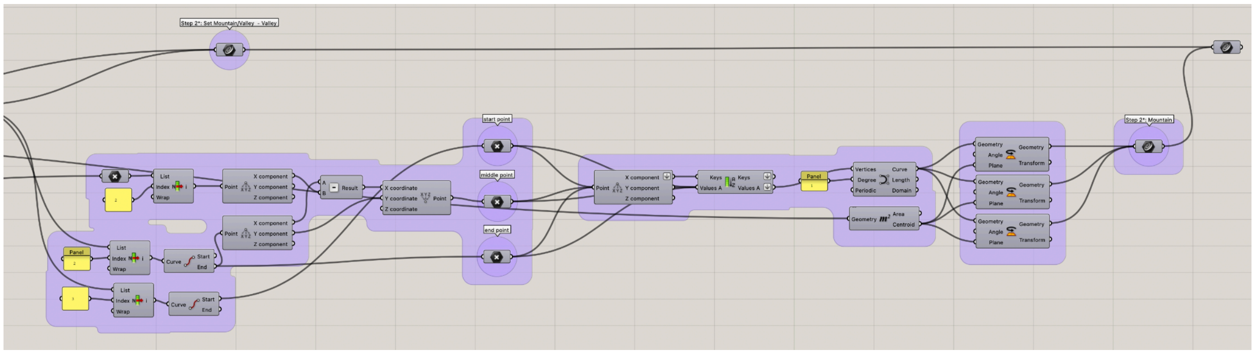
This section covers 3 implementations of creative formations. Normal pinwheel with line segments pivoting from square vertices, parallelogram formation, and the zigzag design mentioned in 3.1.1 and 3.1.2.



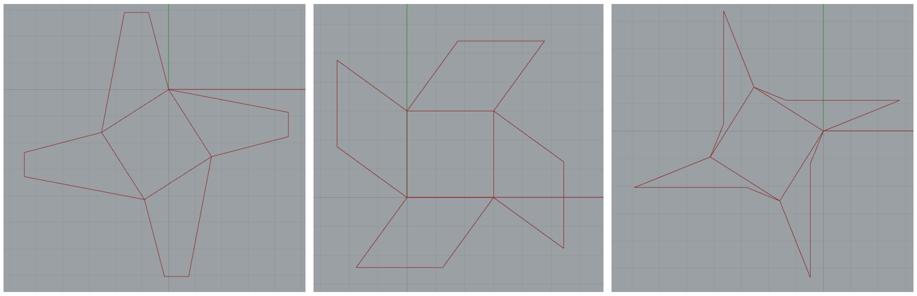
For the style of "Normal pinwheel with line segments pivoting from square vertices", a new section of components is introduced, utilizing line segments originating from square vertices. These components facilitate the extraction of line segments from the unit square, utilizing the square vertices as rotating centers. Users have the flexibility to adjust the rotation angle to their preference and scale the segments as needed. The newly generated lines are appropriately labeled as "Mountains," as shown.



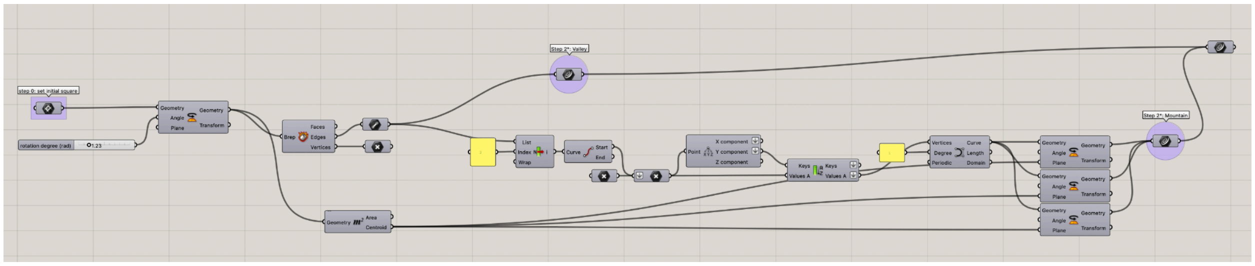
Variations of the unit pattern in this style is achieved by changing the initial square rotation angle, pinwheel arm rotation angle, pivoting line rotation angle, and length.



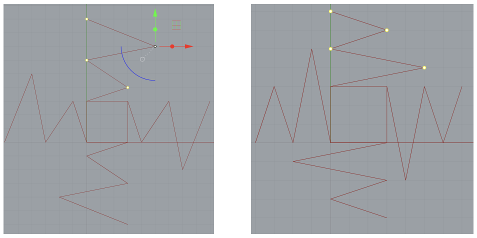
In the "Parallelogram Formation" style, as discussed in Section 3.1.2, Step 3, Example 1, parallelograms are generated by examining one of the four sides of a square. The endpoint of the pinwheel arm serves as the reference point to determine the fourth point of the parallelogram. Subsequently, a "Nurbs Curve" component is utilized to construct a 1-degree polyline connecting the dots, followed by four rotations around the centroid of the square.



Variations of the unit pattern in this style is achieved by changing the initial square rotation angle, size, and the program generated parallelogram midpoint location.



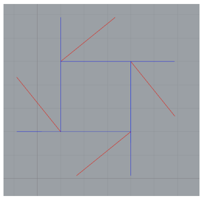
The "zigzag" style, outlined in Section 3.1.2, Step 3, Example 2, involves importing a user-selected set of points from Rhino located on one of the square's four sides. These points, along with the starting vertex, are sorted and utilized to create a 1-degree polyline using the "Nurbs Curve" command, with evaluation based on their y-axis positions. The resulting polyline is then rotated four times to ensure that each side of the square is filled with this pattern.



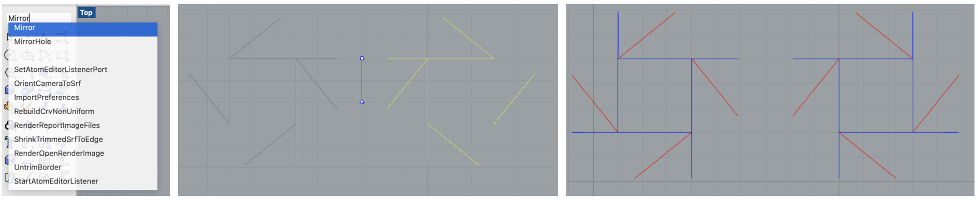
Variations of the unit pattern in this style is achieved by changing the initial square rotation angle, size, and user input on the selected points.

When all is ready, bake the mountain and valley lines as organized into Rhino, set mountain lines in one layer, with RGB color (255,0,0); and valley lines in another layer, with RGB color (0,0,255).

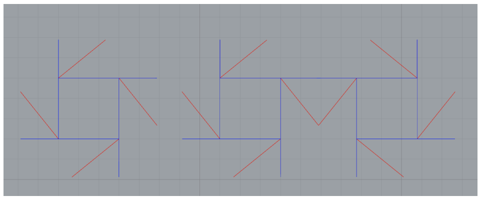
Step 4. **Iterative Reproduction**



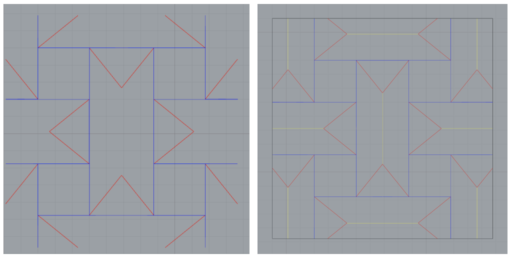
This section will utilize the unit pattern as a demonstration. To streamline the process, select all lines and group the pattern using the "Group" command in the Rhino interface for enhanced operational efficiency.



To apply the reflection rule (Rule 4A), initiate the "Mirror" command in the Rhino command bar. Follow the prompted instructions to define the reflection axis, resulting in the display of the reflected version.



For reproduction, the example here will follow the reflective reproduction method, shown in the spatial relation above.



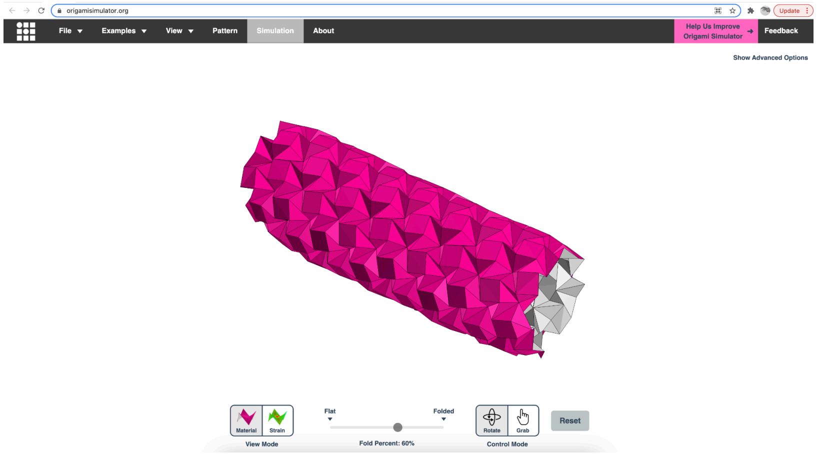
Direct computational output derived from the grammar is depicted on the left. A black boundary box serves to indicate paper size limitations, while yellow lines aid in facilitating a smoother folding process. This concludes chapter 3.

Chapter 4:

Fold Simulation

This chapter concentrates on visualizing the folding of the tessellation pattern designed in Chapter 3, utilizing a folding simulator known as the Origami Simulator.

4.1 Overview of Origami Simulator and its features



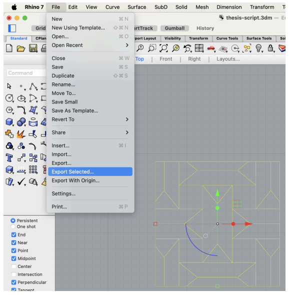
Fold Percent: 60%

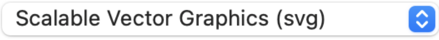
Origami Simulator, available at origamisimulator.org, is a web-based application that allows users to simulate the folding process of origami models digitally. It provides a virtual environment where users can interactively fold virtual paper and explore various origami techniques and designs. The simulator offers a range of features, including the ability to manipulate paper size, fold angles, and crease patterns, providing users with a realistic and intuitive experience of origami folding.

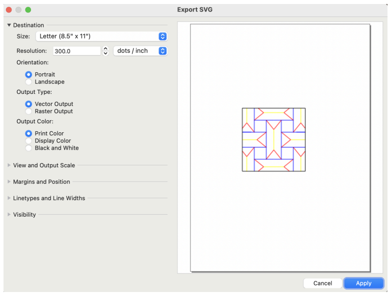
Additionally, Origami Simulator supports the import and export of origami diagrams and models in SVG and FOLD format, enabling users to share their designs with others and collaborate on projects.

The following sections explore ways to export SVG files from Rhino/Grasshopper and Adobe Illustrator, and the complete file importing process to OrigamiSimulator.

4.2 Exporting Rhino/Grasshopper designs to SVG format






Export SVG

Destination

Size: Letter (8.5" x 11")

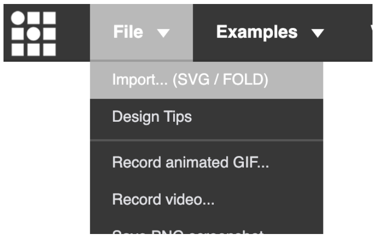
Resolution: 300.0 dots / inch

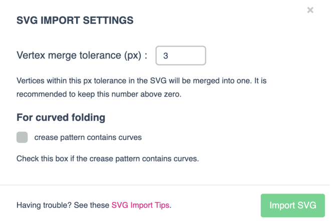
Orientation:
● Portrait
○ Landscape

Output Type:
● Vector Output
○ Raster Output

Output Color:
● Print Color
○ Display Color
○ Black and White

Continuing from the finalization of the pattern in Step 4 of Section 3.2, in Rhino, select the final line drawing you wish to be folded, navigate to "File" → "Export Selected," opt for SVG format, and verify that the file exclusively contains the single fold pattern. Upon confirming, proceed to click "Apply."




SVG IMPORT SETTINGS

Vertex merge tolerance (px) : 3

Vertices within this px tolerance in the SVG will be merged into one. It is recommended to keep this number above zero.

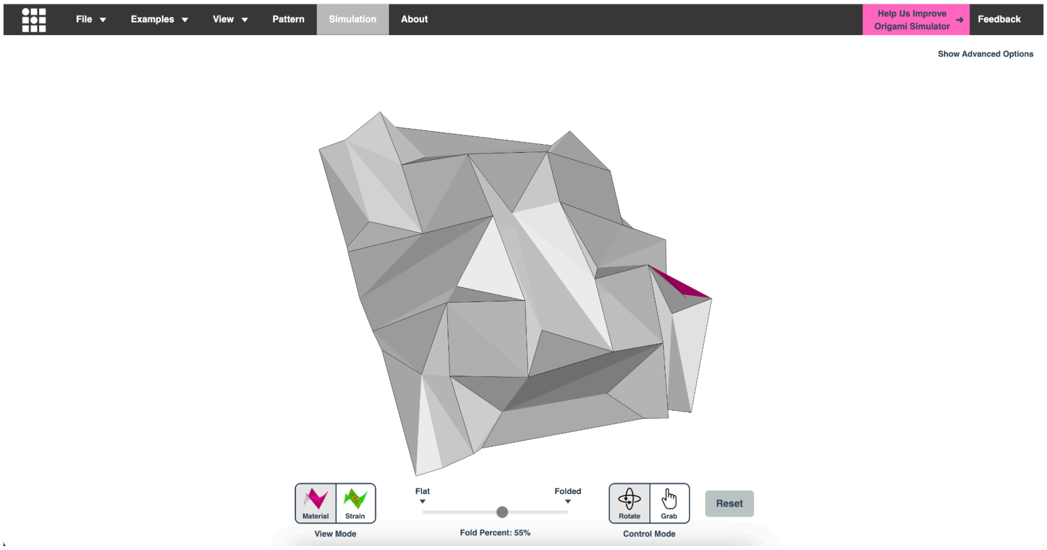
For curved folding

☐ crease pattern contains curves

Check this box if the crease pattern contains curves.

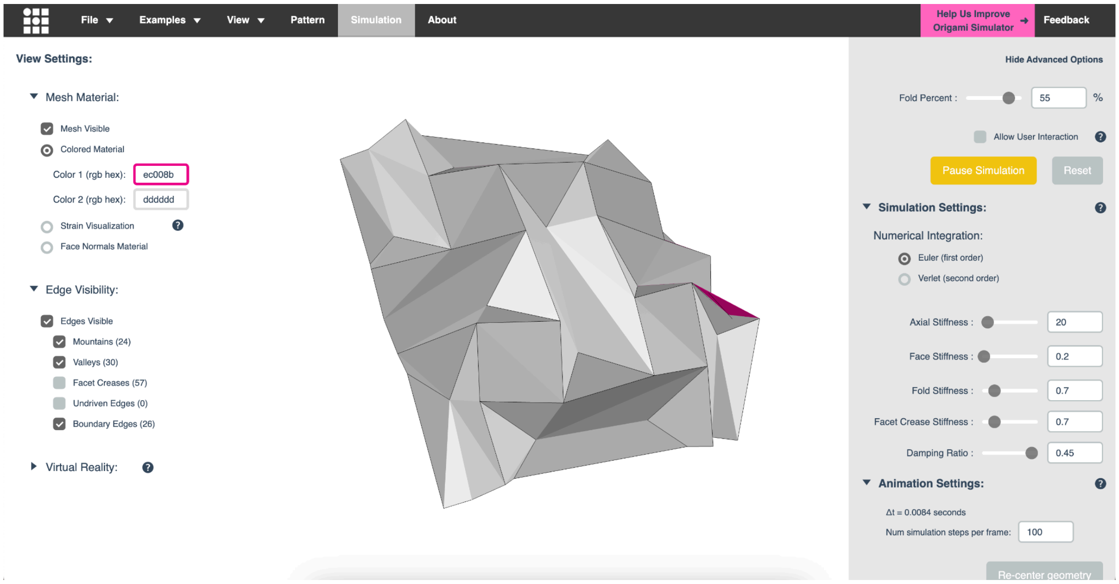
Having trouble? See these SVG Import Tips.

Go to Origami Simulator, on top left, click “File” → “Import SVG” → “Accept”



Fold Percent: 55%

The program automatically simulates the folding process. To observe this process, users can toggle along the slider between the Flat and Folded stages.



View Settings:

Mesh Material:

- ☑ Mesh Visible
- ● Colored Material
  - Color 1 (rgb hex): ec008b
  - Color 2 (rgb hex): dddddd
- ○ Strain Visualization
- ○ Face Normals Material

Edge Visibility:

- ☑ Edges Visible
  - ☑ Mountains (24)
  - ☑ Valleys (30)
  - ☐ Facet Creases (57)
  - ☐ Undriven Edges (0)
  - ☑ Boundary Edges (26)

Virtual Reality:

Fold Percent : 55 %

☐ Allow User Interaction

Simulation Settings:

Numerical Integration:

- ● Euler (first order)
- ○ Verlet (second order)

Axial Stiffness : 20

Face Stiffness : 0.2

Fold Stiffness : 0.7

Facet Crease Stiffness : 0.7

Damping Ratio : 0.45

Animation Settings:

Δt = 0.0084 seconds

Num simulation steps per frame: 100

For more detailed examination, navigate to “Show advanced options” on the top right

4.2.1 Handling compatibility issues and re-exporting SVGs from Adobe Illustrator

Sometimes the SVGs exported from Rhino/Grasshopper might not fold at the first time in OrigamiSimulator. The best solution is to reopen and save the SVG in Adobe Illustrator, and re-import into OrigamiSimulator.

PART III: Conclusions

Chapter 5:

Evaluation and Validation

5.1 Usability testing of the design workflow

Presently, the design framework and workflow function adequately, acknowledging the inherent challenge of encompassing all possible spatial relations within a single rule or script. While the existing model inevitably harbors some limitations, it remains effective in fulfilling its intended purpose.

5.2 Evaluation of pattern accuracy and simulation results

Upon comparison with hand-folded samples, the simulated result closely mirrors the manual folding process (compared to that of Figure 9a), with only minor discrepancies observed (see Figure 12), primarily in certain corner intersections.

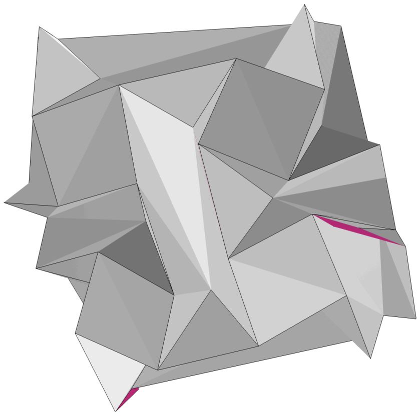


Figure 12: *Simulated fold of Square Weave Tessellation in Origami Simulator (*(Lingyi Qiu, 2024)

5.3 Comparison with manual design approaches

This method provides a direct and efficient approach to origami tessellation design. Unlike traditional methods that involve drawing out patterns on paper and visualizing them mentally, this approach offers immediate visualization of the pattern as soon as a change is made. The time saved can be redirected towards refining the design, allowing for more thoughtful exploration of tessellation variations.

Moreover, in terms of folding efficiency, manually folding a 10x10 origami tessellation design can typically take anywhere from 1 to 3 hours for someone with moderate experience in origami. However, with this approach, the time required for folding is drastically reduced to mere seconds.

Chapter 6:

Practical applications and future directions

6.1 Potential applications of the design workflow

This section delves into the diverse range of potential applications enabled by the developed design workflow, which harnesses the power of shape grammar principles and computational techniques to revolutionize the origami tessellation design process. By leveraging these innovative approaches, designers can streamline and accelerate the creation of intricate tessellation patterns, bypassing the laborious manual folding process by simply adjusting parameters within Rhino/Grasshopper. This not only enhances efficiency but also opens up new avenues for creative exploration and experimentation.

Beyond the realm of origami tessellation, the development of this shape grammar holds implications for a wide array of artistic and visual disciplines. By providing a structured framework for generating complex geometric patterns, the shape grammar methodology offers opportunities for creativity and expression across various art forms and design fields. From architectural façades to textile design and digital art, the principles underlying shape grammar can inspire novel approaches to visual expression and spatial design, enriching the creative landscape and fostering innovation in diverse creative domains.

6.2 Challenges and limitations of the current approach

While significant progress has been made in origami tessellation design and simulation, several challenges and limitations persist:

Limited grid variability: Presently, the approach primarily focuses on square-based grids, limiting the exploration of other grid types such as triangular or hexagonal. Future research could delve into these alternative grid structures to broaden the scope of tessellation design possibilities and enhance creativity.

Interface complexity and accessibility: While the current process utilizes Rhino/Grasshopper for parametric modeling, there is room for improvement in terms of user accessibility and ease of use. Further development could involve integrating the entire workflow into a simplified web interface, allowing users to achieve the same design outcomes with minimal technical expertise.

Expansion of Shape Grammar Rules: Although the current shape grammar provides a foundational framework for tessellation design, there is potential for further exploration and refinement of rules, particularly in step 3 of the grammar. By identifying or creating additional rules, designers can unlock new possibilities and avenues for innovation in tessellation design.

Transition to 3D Printing and Fabrication: While the current approach focuses on single-layer tessellations, there is growing interest in fabricating these patterns using 3D printing or other fabrication techniques. To facilitate this transition, new tools and strategies need to be developed to adapt the design process for multi-layer or three-dimensional tessellations, enabling crafters to explore new dimensions of creativity and expression.

Resource and Time Constraints: The tight timeline of the project may pose challenges in fully exploring the potential of the developed shape grammar and integrating it into practical applications. Further skill development and resource allocation may be necessary to optimize the design workflow and realize its full potential in real-world scenarios

6.3 Future enhancements and research directions

As discussed in section 7.2, exploring origami tessellation patterns within hexagonal and triangular grid structures presents a promising avenue for future research. Additionally, the development of user-friendly interfaces and web-based tools would significantly streamline the origami tessellation design and folding process, enhancing accessibility and efficiency.

Chapter 7:

Conclusion

7.1 Summary of key findings and contributions

In conclusion, this thesis has demonstrated the potential of integrating shape grammar principles and computational techniques in origami tessellation design. Through a thorough examination of basic origami tessellation steps and a comprehensive review of relevant literature, a set of shape grammar rules was formulated to encapsulate the underlying principles and techniques. Implementing these rules within a computational framework facilitated the generation of a diverse range of origami tessellation patterns. Integration with online origami simulators provided visual validation of the designs, while the development of a Rhino/Grasshopper script model streamlined the pattern generation process. The results highlight the efficacy of these computational methodologies in democratizing origami tessellation design, fostering accessibility and innovation in engineering and design fields. This research not only contributes to advancing origami tessellation but also sets the stage for future interdisciplinary exploration and collaboration.

7.2 Implications of origami tessellation design and computational geometry

The implications of this work extend beyond the realm of origami tessellation design. By harnessing the power of computational geometry, this research has demonstrated the broader applicability of shape grammar principles in various engineering and design disciplines. The integration of shape grammar into computational workflows not only streamlines the design process but also unlocks new avenues for creativity and innovation. This convergence of traditional origami techniques with modern computational tools holds immense potential for revolutionizing design practices across industries.

7.3 Closing remarks and suggestions for further study

This thesis underscores the transformative impact of computational methodologies on origami tessellation design. By democratizing access to advanced design tools and techniques, this research paves the way for greater collaboration and interdisciplinary exploration. Moving forward, continued research and development in origami tessellation design and computational geometry hold possibility to unlock new frontiers of creativity and innovation.

PART IV: Appendices

Appendix A:

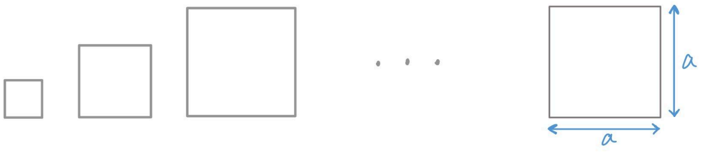
Tessellation pattern design grammar details

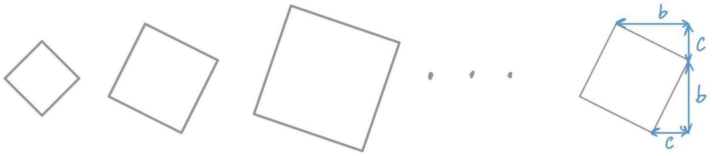
This appendix includes the rules for the tessellation pattern design grammar introduced and summarized in Chapter 3.

Origami Grammar Rules:

Step 1. Foundational Square Selection

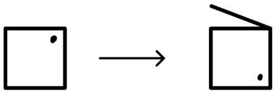
Initial square determination (**Rule1):**



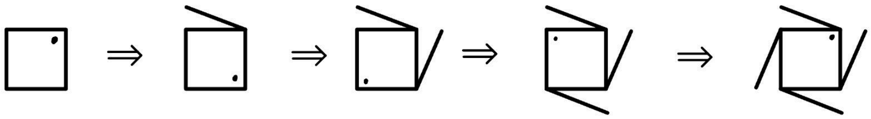


Step 2. Pinwheel Expansion

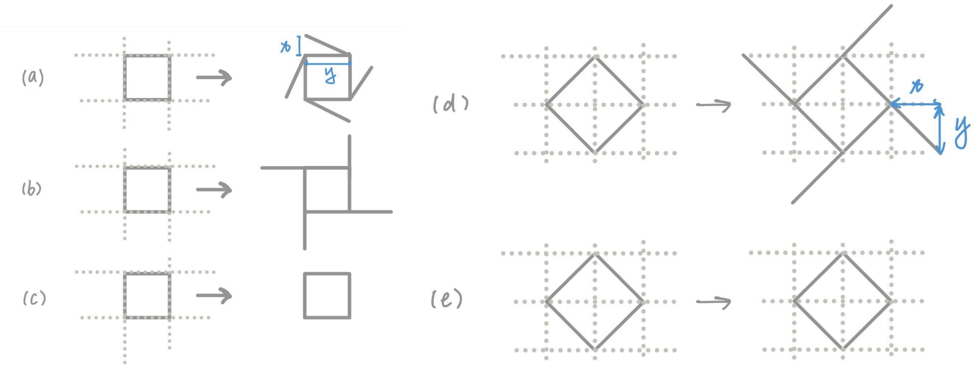
Pinwheel Creation Rule (**Rule 2**):



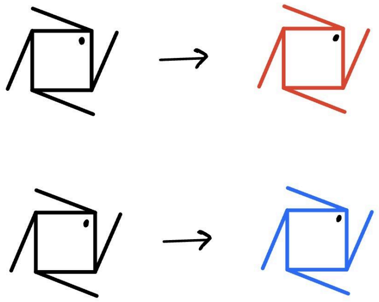
Pinwheel computation:



Pinwheel variation:

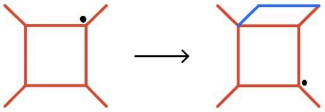


Rule 2*: Mountain/Valley Determination:

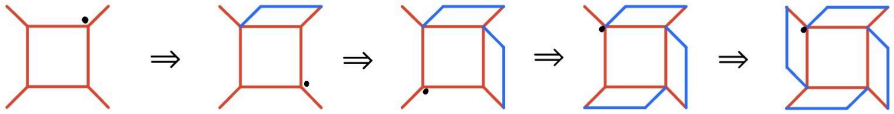


Step 3. **Creative Embellishment**:

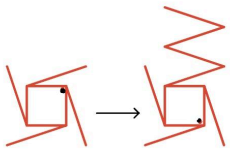
Example **Rule 3**:



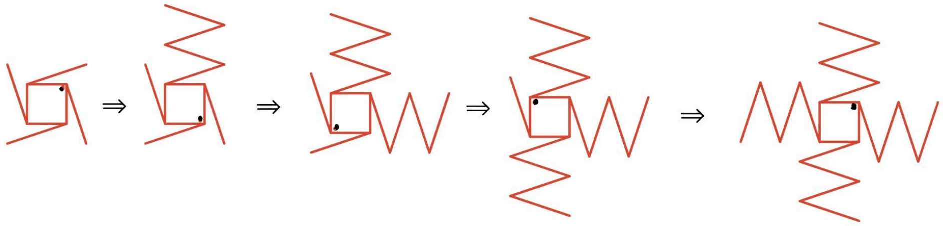
Computation:



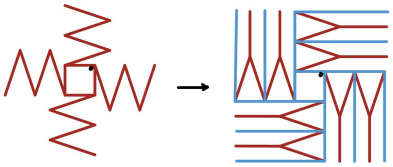
Example **Rule 3**:



Computation:

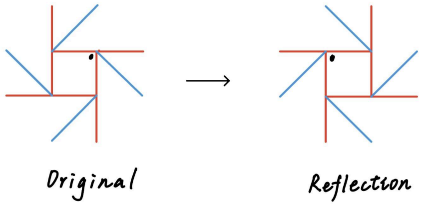


Rule 3*: Mountain/Valley Addition

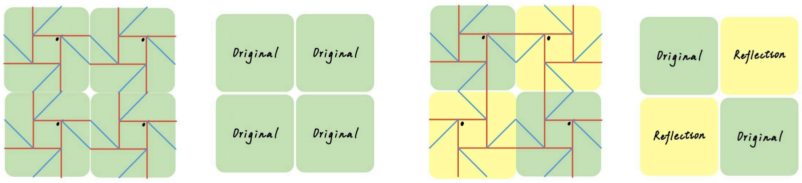


Step 4. **Iterative Reproduction**:

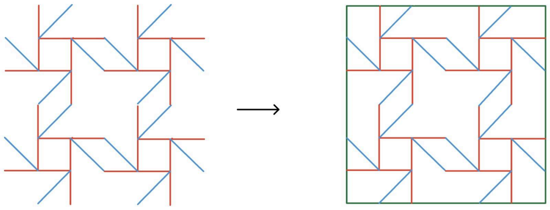
Rule 4A: Reflection Rule:



Rule 4B: Reproduction Rule:

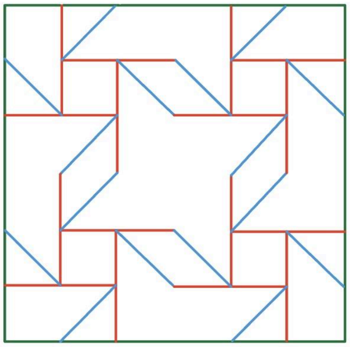


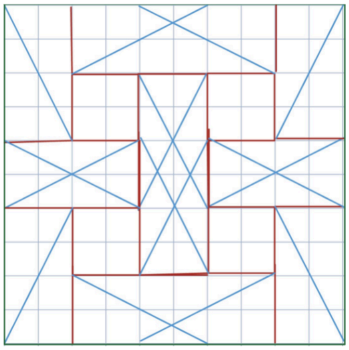
Rule 4C: Boundary Capturing Rule

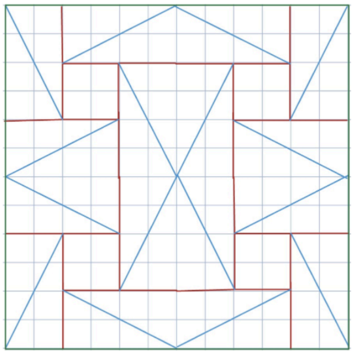


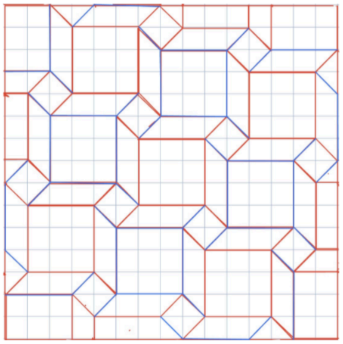
Appendix B:

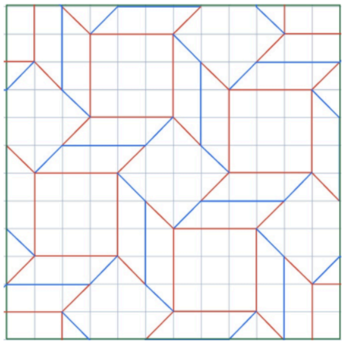
A Collection of Tessellation patterns generated using Shape Grammar

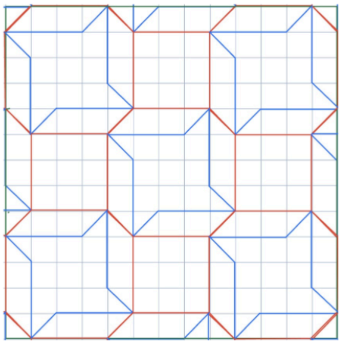


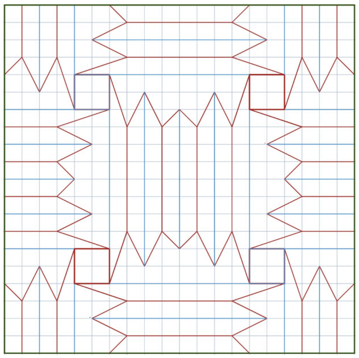


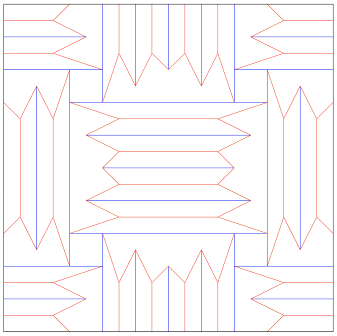


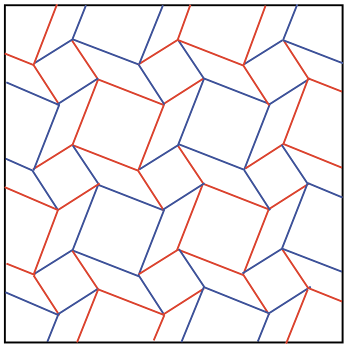


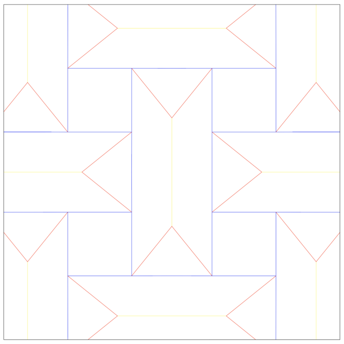


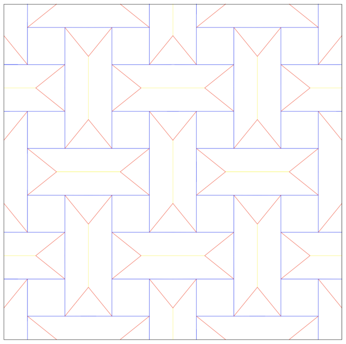


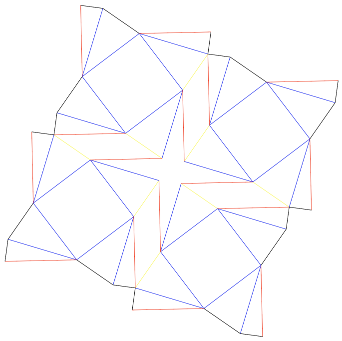




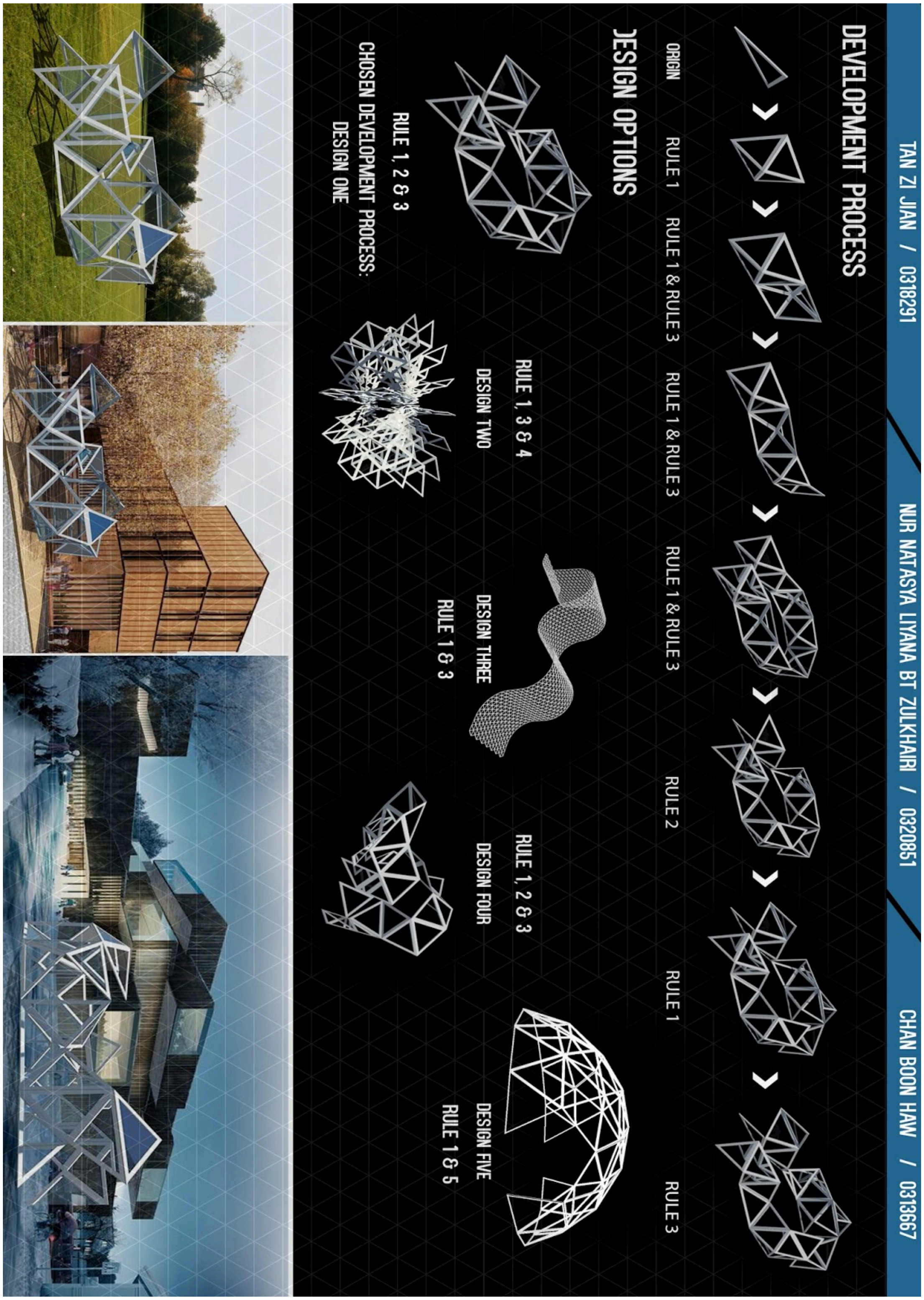








Appendix C: Figure 3a



Appendix D:

List of figures

Appendix E:

References

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