Thesis.

William P. Jewett. Class, 1873.

Stone Arch Bridge.

I propose to design a Stone Arch Bridge to carry a carriage road over a tidal stream. The dimensions of arch to suit the heights of the banks and the width of the stream, must be as follows; Span 100 ft. Rise 12 ft. The material to be used must be of granite capable of sustaining a pressure of 10000 lbs. per sq. inch. The form of the arch ring is to be the segment of a circle. I choose a circular arch because the voussoirs can all be cut with the same dimensions, and therefore for a less price; dimension stone being required for the Catenary Transformed Catenary, or the Hydrostatic Curve The form of a segment of a circle of the above dimensions differs very slightly from the curves mentioned except near the springing point. It will be seen in the sequel that the curves of pressure approach the intrados near the springing point; thereby indicating that the circle is not the exact curve suited to the load.

I shall show however, that a circular arch for the present case is perfectly stable, from existing examples. The method I have used to construct the curves of pressure is that by Dr. Hermann Sheffler; translated into French by M. Victor Fournié, under the name "Traité de la Stabilité des Constructions", 1re partie "Theorie des voutes et des Murs de Soutenement". Prof. William Cain C. E. in, "a practical theorie of Voussoir Arches" has embodied the main principles of this work. I shall explain the method and then apply it to the present case. I do not propose to demonstrate the theorie; that is not the object of my thesis.

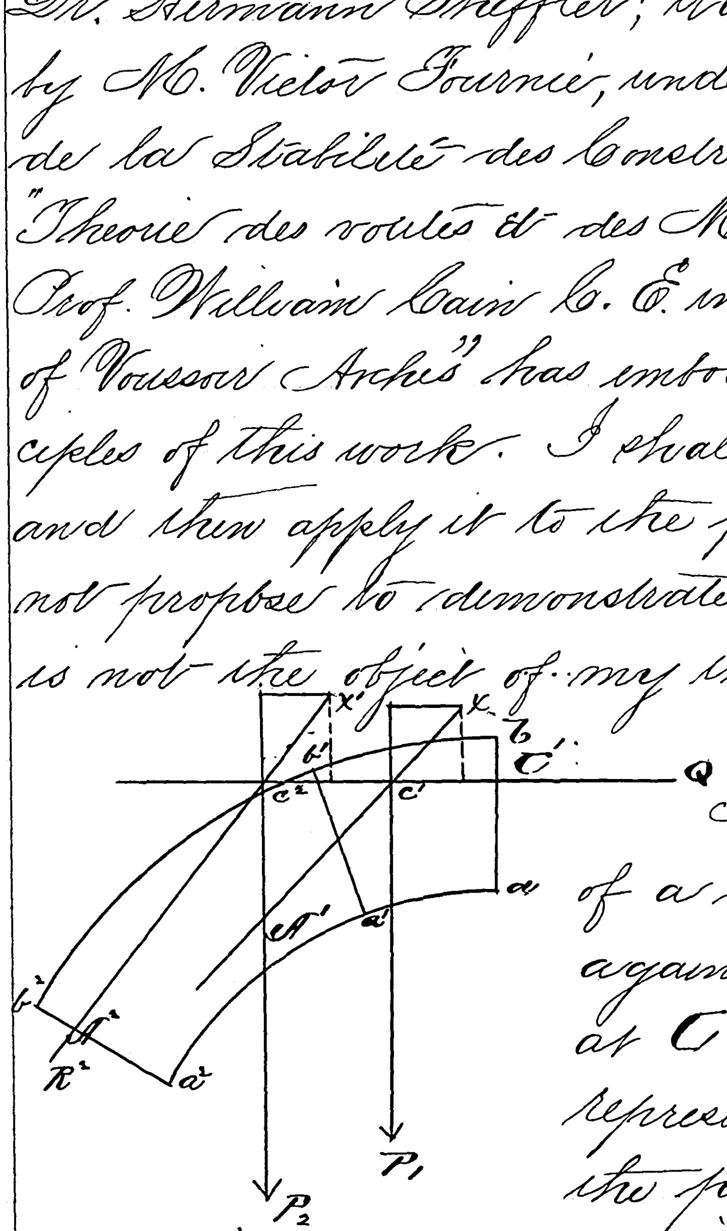

Fig 1.

Let: ab, $a'b'$, a^2b^2 Fig. 1. be the points of a half arch, leaned against a fixed surface at C'. Let P' and P² represent the weights of the parts $ab\,a'b'$; $a'b'\,b^2a^2$ including any load on these parts, both in amount and direction. P1 and

P_2 acting through the centres of gravity of these parts. A horizontal force Q combined with the reaction at R_2 is necessary to hold the part a b, $a^2 b^2$ in equilibrium; and the same with the other joints. At the points where the direction of Q cuts P_1, P_2 &c, combine those forces with Q. The resultant $c^1 4$ of Q and P_1 cuts the joint $a^1 b^1$ at A^1 and the resultant $c^2 4^1$ of Q and P_2 cuts the joint $a^2 b^2$ at A^2. These therefore are the centres of pressure on these joints. The line joining these points is the curve of pressure. In order that the arch may remain in equilibrium it is necessary in the first place, that the line of pressure shall cut each joint (according to Prof. Rankine) within the middle third of the arch ring. Dr. Sheffler says the arch is perfectly stable if the curve of pressure does not approach nearer the limits of the arch ring than one quarter the depth of the joint. 2nd. It is necessary that the directions $c^1 A^1$, $c^2 A^2$ of the pressures on the joints do not make angles with the normals to the joints, greater than the angle of friction. The friction of the materials is usually sufficient to assure the stability of the arch against sliding on its joints:

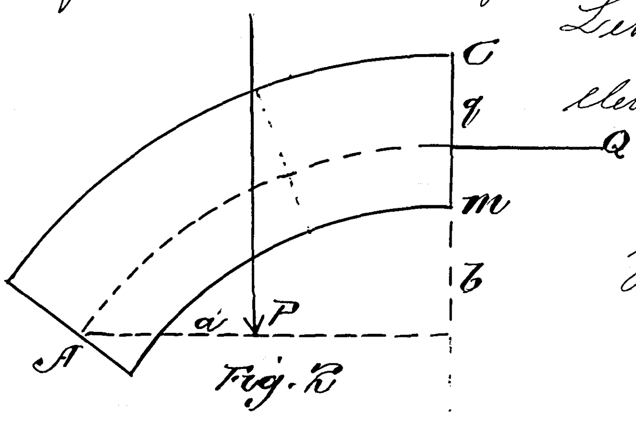

Fig. 2

Let Fig. 2 represent the elevation of a half arch 1 ft. thick. Let Q = horizontal thrust, q its vertical distance below the apex C, P = weight of the arch and load AC on the joint A, a = the horizontal distance from A, the centre of pressure on the joint, to the vertical passing through the centre of gravity of the arch. b = vertical distance between the point of application of Q at the crown and A. If we know the points m and A, we have for equilibrium ....

$aP = bQ \therefore Q = \frac{aP}{b}$. Now to find the centre of gravity of the weights $a\,b\,b_1\,a_1$, $a\,b\,b_2\,a_2$ Fig. 1 and the magnitude of those weights. If the arch is loaded, reduce this load to the same specific gravity as that of the masonry of the arch ring. Place those weights in their exact position on the arch, and reduce their vertical dimensions to conform

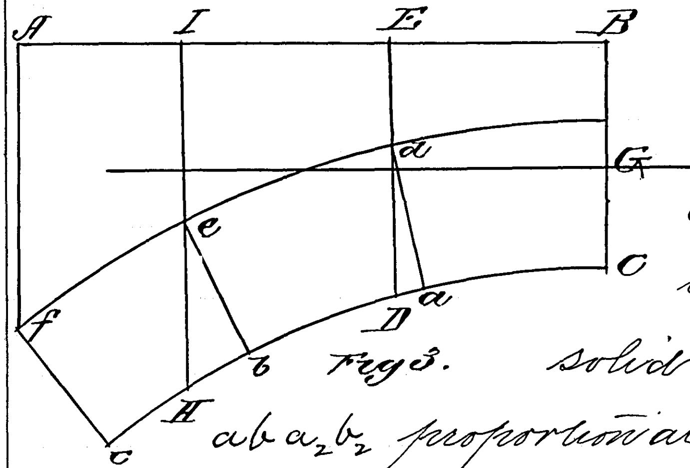

Fig 3.

to the specific gravity of the stone of the arch ring. In this way we have the solid contents of $a\,b\,a_1\,b_1$, $a\,b\,a_2\,b_2$ proportionate to the weights $P_1\,P_2$ &c. and so can be taken for them. Now draw vertical lines Fig. 3 through d, e. f up to A. B which limits the load. Find the area of each one of these trapezoids as C. B. D. E,. Each being 1 ft. thick this product expresses the weight of each one of these trapezoids in cubic feet of stone. The part d a D is in excess of the weight on the joint a d; but this error is not repeated and is very small in a flat arch. Now if we multiply this surface or weight by the horizontal distance of its centre of gravity from B C we get its moment; and the moments of the several trapezoids divided by the sum of the trapezoids, gives the distance of the centre of gravity of the whole mass from B C. So from this method we get the weights P_1 P_2 &c and the horizontal distances of their centres of gravity from the crown of the arch.

In applying this method to my arch the first thing to determine is the depth of the voussoirs From "Rankine's Civil Engineering," Section 290... The depth of the voussoirs for a single arch in feet $= \sqrt{.12 \times \text{rad. at the crown}} = \sqrt{.12 \times 110.167}$ $= 3.64$ ft. I have made them 3.5 ft. By referring to Thesis Drawing No. 1 it will be seen that I have divided each half arch ring into twelve equal divisions. This being sufficiently small to show the curves of pressure accurately. In Tables I. II. III. IV. V. VI which correspond to the same figures on the drawing... The first column gives the number of the joint from the crown. The 2nd (W) gives the widths of the different trapezoids. The 3rd (v) their middle heights. These are all taken by scale measurements, which is sufficiently correct as the curves of pressure must be contructed graphically. 4th (s) gives the products of the last two, or the weights of the trapezoids, considering a slice of the arch 1 ft thick. The next column (c) gives the distances of the centre of gravity of each trapezoid from the crown. Column (m) gives the product of (s) + (c) or the moment of each trapezoid with reference to a plane passing through the crown perpendicular to the axis of the arch. Now by uniting these trapezoids or weights; the weight on joint 2 consists of two trapezoids, that on joint 3 of three trapezoids &c. Column (S) gives the additions (s+c) each (S) being the given trapezoid added to the sum of all those which come before it; the last S of course should be equal to Σs. In the same way (M) is the sum of column (m) and the last M should be equal to Σm. Dividing the quantities in column (M) by those in column (S) each by each, we get column (C) which are the distances of the centres of gravity of the weights on each joint from the crown. The last C is the distance of the centre of gravity of the whole mass from the crown. The sum of column (s) = P the weight of the whole mass. Now from page 4 $Q = \frac{aP}{b}$

In each case I have considered the centre of pressure at the crown to be one third the distance down the joint; and at the abutment one third the distance up the joint. Therefore $a = 50.5$ ft. in each case, $b = 12\text{ ft} + \frac{2}{3} \text{ of } 3.5\text{ ft.} = 14.3$ ft. In Table II and Fig. II I have constructed the curve of pressures for the arch ring with its spandrel wall; and its surcharge owing to the stone railing, the railing being equivalent to a column of masonry 3½ ft. high and 1 ft. thick over a slice of arch 1 ft. wide over the whole length of the arch. This is shown on the drawing by the blue line A.B. Draw the horizontal line J.D through the point J 1/3 of the distance down the crown joint; on this line lay off the distances Jo, Jm, Jp in column C. At each one of these points o, m and p erect a vertical line and make it equal to quantity in column S expressing the force acting through each of these points, or distances of the centres of gravity from the crown. From the upper end of these lines draw horizontal lines, and make each equal to Q in Fig. II I have marked the forces corresponding to the lines. Diagonals joining the centres of gravity with the extremities of the Qs give the resultants of the forces acting on the joints in magnitude and direction. Where the prolongation of each resultant cuts the corresponding joint is the centre of pressure on this joint. It will be seen that the centre of pressure on the abutment joint approaches the intrados within .35 of a foot or 4.2 inches. The thrust on this joint is expressed by the line I, D in value and direction = 977 cu. ft. of stone or 164136 lbs., a cubic foot of granite weighing 168 lbs. Therefore a surface 4.2 X 12 inches has to sustain 164136 lbs. Granite will bear 10000 lbs. pressure per sq. inch. 4.2 X 12 X 10000 = 504000. $\frac{504000}{164136} = 3$ for a factor of safety. There is therefore little danger of crushing the material. The tendency of the arch is to sink at the crown and rise at the haunches; but the curve of pressure shows the arch to be perfectly stable at the crown. To prevent rising at the haunches, spandrel backing can easily be put in, and at a less expense than cutting the voussoirs of different dimensions to suit the curve of pressure. In Table III and Fig. III I have constructed the curve of pressure with earth backing two feet deep at the crown, and level over the whole length of the arch; substituting masonry backing for earth as indicated in the figure at a b c d e f to get greater weight on this part of the arch. This will prevent the joints opening on the side of the extrados as they have a tendency to do. Table IIII, Fig IIII gives the curves of pressure for the arch with earth backing alone 2 ft. deep at the crown and level over the whole arch. The blue line in this Fig. and in Fig. 3 shows the level of the earth. The dotted blue line shows the top of the earth with its vertical heights reduced to correspond to the specific gravity of stone. Considering that earth weighs two thirds as much as stone. Comparing Fig III and Fig. IV, it will be observed that the spandrel backing of stone in Fig III causes the curve of pressure to rise slightly on joints 9, 10, 11 and 12 which is the effect we wish to produce. In neither of these cases does the curve pass nearer the edge than one quarter the depth of the joint except at the abutment joint. Fig V Table V shows the curve of pressure of the arch with earth backing as in Fig. IV together with a weight of twenty tons over joints 3. 4. 5 & 6. I have supposed it to be placed over 13.4 feet in length and 5 ft. in breadth. Since the weight of a cubic ft. of stone is about .08 of a ton, the height of a mass of stone of these dimensions to weigh ~~20 tons~~ would be... $5 \times 13.4 \times h \times .08 = 20$, $h = \frac{20}{5 \times 13.4 \times .08}$ $= \frac{20}{5.36} = 3.7$. Therefore increase in Table IV v_4, v_5, v_6 by 3.7 ft. and in Table V v_4, v_5, v_6 $=$ 9.1, 9.7, 10.3. Table VI. Fig VI represents the same case as the preceeding with the load moved to the crown. This increases v_1, v_2, v_3 in Table IV. Each by 3.7 ft. It will be observed ~~that~~

in neither of these cases does the curve pass outside of the middle third except near the springing point. Table 1 Fig. 1 gives the curve of pressure for the arch ring without any other load. It will be seen in Table 1 that I have calculated each s, by multiplying together the depth of the arch ring and the $\frac{1/2 \text{ span}}{12}$. This curve is much the same as the rest except that it approaches nearer the intrados at the haunches. It will be observed that the different manner of loading changes the curve of pressure very little. In Fig V the curve approaches a little nearer the extrados at points 3, 4 and 5 but not outside the middle third. It may be inferred from the fact that the curves of pressure in the arch under all the loads, approach so near the intrados near the springing point that the arch is unstable. I rely upon the spandrel to prevent overturning. With such a height of stone on them bound together with the best cement which renders the mass almost solid rock, the voussoirs cannot move. As examples of flat circular arches which have been

Table I.

½ length of intrados = 52 feet

$\frac{52}{12} = 4\frac{1}{3}$ Depth of arch stones = $3\frac{1}{2}$ ft.

$4\frac{1}{3} \times 3\frac{1}{2} = 15$ ft. $= W =$

N.	W.	v.	s.	c.	m.	S.	M.	C.
1	4.3	3.5	15	2.2	33.0	15	33.0	2.2
2	"	"	15	6.5	97.5	30	130.5	4.3
3	"	"	15	11.0	165.0	45	295.5	6.6
4	"	"	15	15.4	231.0	60	526.5	8.8
5	"	"	15	19.9	298.5	75	825.0	11.0
6	"	"	15	24.0	360.0	90	1185.0	13.2
7	"	"	15	28.2	423.0	105	1608.0	15.3
8	"	"	15	32.4	486.0	120	2094.0	17.5
9	"	"	15	36.5	547.5	135	2641.5	19.6
10	"	"	15	40.7	610.5	150	3252.0	21.7
11	"	"	15	44.8	672.0	165	3924.0	23.8
12	"	"	15	48.7	730.5	180	4654.5	25.9
			180		4654.5			

$a = 50.5 - 25.9 = 24.6$ $b = 14.3$

$$Q = \frac{aP}{b} = \frac{24.6 \times 180}{14.3} = \frac{4428}{143} = 309.6$$

Q = 309.6 ~~cubic~~ feet of stone

N.	W	v	s	c	m	S	M	C
1	4.5	7.0	31.5	2.2	69.3	31.5	69.3	2.2
2	4.5	7.1	31.9	6.7	213.7	63.4	283.0	4.5
3	4.4	7.5	33.0	11.2	369.6	96.4	652.6	6.8
4	4.4	8.0	35.2	15.6	549.1	131.6	1201.7	9.1
5	4.4	8.7	38.7	20.0	774.0	170.3	1975.7	11.6
6	4.4	9.6	42.2	24.4	1029.7	212.5	3005.4	14.1
7	4.3	10.8	46.4	28.7	1331.7	258.9	4337.1	16.7
8	4.3	12.0	51.6	33.0	1702.8	310.5	6039.9	19.5
9	4.2	13.4	56.3	37.3	2100.0	366.8	8139.9	22.2
10	4.1	15.0	61.5	41.4	2546.1	428.3	10686.0	24.9
11	4.1	16.8	68.9	45.5	3134.9	497.2	13820.9	27.8
12	4.0	17.8	71.2	49.6	3531.5	568.4	17352.4	30.5
		P =	568.4		17352.4			

$a = 50.5 - 30.5 = 20$ $b = 14.3$

$$Q = \frac{aP}{b} = \frac{20 \times 568.4}{14.3} = \frac{11368}{14.3} = 794.9$$

Q = 794.9 cubic feet of Stone.

N.	W.	V.	s	c	m	S	M.	C.
1	4.5	4.8	21.6	2.2	47.5	21.6	47.5	2.2
2	4.5	4.9	22.0	6.7	147.4	43.6	194.9	4.5
3	4.4	5.1	22.4	11.2	250.9	66.0	445.8	6.8
4	4.4	5.4	23.8	15.6	371.3	89.8	817.1	9.1
5	4.4	6.0	26.4	20.0	528.0	116.2	1345.1	11.6
6	4.4	6.6	29.0	24.4	707.6	145.2	2052.7	14.1
7	4.3	7.2	31.0	28.7	889.7	176.2	2942.4	16.7
8	4.3	8.5	36.5	33.0	1204.5	212.7	4146.9	19.5
9	4.2	9.9	41.6	37.3	1551.7	254.3	5698.6	22.4
10	4.1	11.5	47.1	41.4	1949.9	301.4	7648.5	25.4
11	4.1	15.3	62.7	45.5	2852.8	364.1	10501.3	28.8
12	4.0	16.3	65.2	49.6	3233.9	429.3	13735.2	32.0
		P =	429.3		13735.2			

$a = 50.5 - 32. = 18.5 \quad b = 14.3$ ft.

$$Q = \frac{aP}{b} = \frac{18.5 \times 429.3}{14.3} = \frac{7942.05}{14.3} = 555.4$$

$Q = 555.4$ cubic feet of Stone –

N	W	v	s	c	m	S.	M.	C
1	4.5	4.8	21.6	2.2	47.5	21.6	47.5	2.2
2	4.5	4.9	22.0	6.7	147.4	43.6	194.9	4.5
3	4.4	5.1	22.4	11.2	250.9	66.0	445.8	6.8
4	4.4	5.4	23.8	15.6	371.3	89.8	817.1	9.1
5	4.4	6.0	26.4	20.0	528.0	116.2	1345.1	11.6
6	4.4	6.6	29.0	24.4	707.6	145.2	2052.7	14.1
7	4.3	7.2	31.0	28.7	889.7	176.2	2942.4	16.7
8	4.3	8.1	34.8	33.0	1148.4	211.0	4090.8	19.4
9	4.2	9.0	37.8	37.3	1409.9	248.8	5500.7	22.1
10	4.1	10.1	41.4	41.4	1714.0	290.2	7214.7	24.5
11	4.1	11.4	46.7	45.5	2124.8	336.9	9339.5	27.7
12	4.0	12.8	51.2	49.6	2539.5	388.1	11879.0	30.6
		P =	388.1		11879.0			

$a = 50.5 - 30.6 = 19.9$ $\qquad b = 14.3$ feet

$$Q = \frac{aP}{b} = \frac{19.9 \times 388.1}{14.3} = \frac{7723.19}{14.3} = 540.08$$

Q = 540.1 cubic feet of Stone —

N	W	v	s	c	m	S	M.	C
1	4.5	4.8	21.6	2.2	47.5	21.6	47.5	2.2
2	4.5	4.9	22.0	6.7	147.4	43.6	194.9	4.5
3	4.4	5.1	22.4	11.2	250.9	66.0	445.8	6.8
4	4.4	9.1	40.0	15.6	624.0	106.0	1069.8	10.1
5	4.4	9.7	42.7	20.0	854.0	148.7	1923.8	12.9
6	4.4	10.3	45.3	24.4	1105.3	194.0	3029.1	15.6
7	4.3	7.2	31.0	28.7	889.7	225.0	3918.8	17.4
8	4.3	8.1	34.8	33.0	1148.4	259.8	5067.2	19.5
9	4.2	9.0	37.8	37.3	1409.9	297.6	6477.1	21.8
10	4.1	10.1	41.4	41.4	1714.0	339.0	8191.1	24.2
11	4.1	11.4	46.7	45.5	2124.8	385.7	10315.9	26.7
12	4.0	12.8	51.2	49.6	2539.5	436.9	12855.4	29.4
		P	436.9		12855.4			

$a = 50.5 - 29.4 = 21.1 \qquad b = 14.3$

$$Q = \frac{aP}{b} = \frac{21.1 \times 436.9}{14.3} = \frac{9218.59}{14.3} = 644.6$$

Q = 644.6 cubic feet of Stone

N.	W.	V.	S.	c.	m	S'	M.	C.
1	4.5	8.5	38.2	2.2	84.0	38.2	84.0	2.2
2	4.5	8.6	38.7	6.7	259.3	76.9	343.3	4.5
3	4.4	8.8	38.7	11.2	433.4	115.6	776.7	6.7
4	4.4	5.4	23.8	15.6	371.3	139.4	1148.0	8.2
5	4.4	6.0	26.4	20.0	528.0	165.8	1676.0	10.1
6	4.4	6.6	29.0	24.4	707.6	194.8	2383.6	12.2
7	4.3	7.2	31.0	28.7	889.7	225.8	3273.3	14.4
8	4.3	8.1	34.8	33.0	1148.4	260.6	4421.7	17.0
9	4.2	9.0	37.8	37.3	1409.9	298.4	5831.6	19.5
10	4.1	10.1	41.4	41.4	1714.0	339.8	7545.6	22.2
11	4.1	11.4	46.7	45.5	2124.8	386.5	9670.4	25.0
12	4.0	12.8	51.2	49.6	2539.5	437.7	12209.9	27.9
		P =	437.7		12209.9			

$$a = 50.5 - 27.9 = 22.6$$

$$b = 12 + \tfrac{2}{3} \text{ of } 3.5 = 14.3$$

$$Q = \frac{aP}{b} = \frac{22.6 \times 437.7}{14.3} = \frac{9892.02}{14.3} = 691.7$$

Q = 691.7 cubic feet of Stone.

built I would mention the Darlaston Bridge in Staffordshire, 86 ft. span, 13½ ft. rise, voussoirs 3½ ft. deep. Jena Bridge over the Seine, at Paris 91.8 ft. span, 10.8 ft. rise, voussoirs 5 ft deep. The best example however, is an experimental arch built in 1864 at Souppes, sixty miles from Paris. I have not been able to ascertain whether the arch for which this experiment was tried has ever been built. The experimental arch was 13.78 ft. wide in the direction of its axis, had a span of 124.3 ft. and had a rise of 6.97 ft. The intrados was the segment of a circle. The depth of the voussoirs for a distance of 3.28 ft. back from the head of the arch was 3.61 ft. and under the roadway 2.62 ft. The increase in the depth of the voussoirs was on account of the spandrel wall, the foot way, and the cornice. The arch was built of the hardest stone the locality would afford. The cement was of the best quality and the greatest care was taken with the workmanship. One abutment was a solid ledge, the other was built 26.87 high, 49.54 wide at the bottom, 48.75 ft. wide at the top in a direction perpendicular to the axis of the arch. The arch was loaded with a uniform load of 135 lbs. per sq. foot. A moving load of 12122 lbs. was passed slowly over it. This produced a maximum lowering of .001 of one foot. A weight of 10945 lbs. was dropped from a height of one foot, directly upon the crown. This produced vibrations, the maximum amplitude of which were .009 of a foot. A surcharge of 206 lbs. per. sq. foot was then added to the 135 lbs. This produced a very slight settling at the crown. The surcharge of 206 lbs per sq. foot was then removed, and the abutment was cut away on the back until it was only 23.29 ft. wide. No joint opened and the masonry remained perfectly intact. The arch was then entirely unloaded and the voussoirs next to the crown cut away until there was a bearing surface of only 1.8 ft. The arch then slowly settled on the centre. This was the flattest arch ever built in the world and demonstrates very clearly the strength of flat circular arches. The account from which I take this in a "Traité de le Equilibre des Voutes et de la Construction des Ponts en Maçonerie". The curve of pressure for this arch loaded with a depth of earth of two feet at the crown and level over the whole arch, shows the arch to be unstable from the springing joint to the eighth voussoir, (counting from the skew-back) there being thirty eight in the half arch. The curve approaches nearer the intrados at these points than in my arch, thereby proving my arch to be more stable than this one. As I have mentioned before the voussoirs should be of the best of granite and they must be cut smooth enough on the bed joints so that no joint shall be wider than one third of an inch. The courses must be continuous through the whole arch. The joints perpendicular to these bed joints, must not be more than one half an inch wide, and the voussoirs in this direction must break joints. No voussoir being less than two feet in the direction of the axis of the arch. They must be laid in cement which will stand a pull of one thousand pounds per sq. inch. The lower side of the voussoirs is to be hammer dressed so that the intrados shall present an even surface. The heads of the arch ring are to built of arrised edged stone as shown in Fig. 1 Thesis Drawing No. 3 The rest of the stone being rock faced. As shown in the drawing the arch ring requires sixty six courses of voussoirs beside the keystone course which must be as thick as two voussoir courses, and project nine inches above the rest of the arch ring. The length of the intrados is 103.7 feet = 1244+ inches. 36 inches will be taken up by the keystone, leaving 1208 inches. $\frac{1208}{66}$ = 18 in and 20 inches to spare for the joints; hence we have sixty-six courses eighteen inches wide. The spandrel walls and the faces of the abutments are to be built of line dressed ashlar masonry of granite in regular courses laid in cement with alternate headers and stretchers; no stone to have less dimensions than two and one half feet by two feet by fifteen inches. The thickness of these walls is shown on the drawing Figs. II, III, IV. The skew-backs must be of larger stone. the dimensions of these are shown on the drawing The spandrel backing and the backing of the abutments may be of irregular granite blocks laid in best cement. The railing and ornamental posts over the abutments and crown are to be of hammered stone of the dimensions shown in Figs. V, VI. The joints to fit within one third of an inch. The back of the arch must be grouted and covered with a coat of cement two or three inches thick to keep out the water. The rest of the backing up to the level of the roadway to be of good gravel as shown on the drawing. I have marked on the drawing the form and dimensions of the roadways and footways. The next step is to determine the pressure on the abutments in amount and direction and from this to determine the width of the abutments to sustain this pressure. In Fig. II Drawing No 1 the line L, D represents this pressure in amount and direction. This can be obtained however more accurately by computation. From Table II. Page 14 $P' = 568.4$ cu. ft. of stone, $Q = 794.9$ cu. ft. of stone, P the pressure on the abutment $= \sqrt{P'^2 + Q^2}$ $= \sqrt{568.4^2 + 794.9^2} = 977$ cu. ft. of stone or $977 \times 168 =$ 164136 lbs. The tang. of θ, its angle of inclination to the horizon $= \frac{568.4}{794.9} = .71506$ $\therefore \theta = 35°34'$ cos. $\theta = .81344$, sin $\theta = .58165$. From Rankine's Civil Engineering, Sect. 264. I. 6, the least thickness of the abutment, at any joint consistent with the stability of position $= \sqrt{(A + B^2)} - B$ $A = \frac{P k \cos\theta}{n(q+q')whb}$, $B = \frac{(q+k) P \sin\theta}{2n(q+q')whb}$. In these formulae P = the pressure θ its inclination. q is a factor expressing the greatest deviation of the centre of resistance at the joint from its centre of gravity consistent with stability. q' = the distance between the centre of the joint and a vertical line passing through the centre of gravity of the mass of masonry above this joint. n is a factor depending on the form of the abutment. I propose to make the abutments rectangular for calculation adding enough masonry afterwards to make the walls batter. The height of the abutment h = 29 ft. above the point C. D the width of which I wish to ascertain. I shall consider a slice of the abutment 1 ft. thick ∴ $q = 1/6$ for a rectangle (Rankine pp. 294) $n=1$, $q'=0$, x'; the height of the abutment from point C. D to the point of application P = 12.2 ft. $b = 1$ ft. w = weight of a cubic foot of the material = 125 lbs therefore $A = \frac{164136 \times 12.2 \times .81344}{1(1/6+0)(125 \times 1 \times 29)} = \frac{162880.411648}{604.167} = 2696.0876$ $B = \frac{(1/6+1/2)\,164136 \times .58165}{2(1/6+0)\,125 \times 29 \times 1} = \frac{63646.4696}{1208.33} = 52.673$, $t = \sqrt{A+B^2} - B =$ $\sqrt{2696.0876+2774.4449} - 52.673 = \sqrt{5470.5227} - 52.673 = 73.96 - 52.673 = 21.29$ ft. According to Trautwine's formula, if the height of the springing point above the point under consideration does not exceed one and one half times its width, the width $= \frac{\text{rise in feet}}{10} + \frac{\text{radius}}{5} + 2 \text{ ft} = \frac{12}{10} + \frac{110}{5} + 2 = 25.2$ ft. According to a formula in Spons Dictionary of Engineering, T the thickness of the abutment $= \sqrt{\frac{2P}{C} + \left(\frac{W}{CH}\right)^2} - \frac{W}{CH}$. H = height of the abutment = 29 feet. C = weight of a cubic foot of the abutment and arch stones = 150 lbs. P = horizontal thrust of the half arch = $164136 \times \cos\theta = 133574.78784$. W = weight of the half arch = $568.4 \times 168 = 95491.2$

$$T = \sqrt{\frac{2P}{C}} = \frac{2 \times 133574.78784}{150} + \left(\frac{W}{C.H}\right)^2 = \left(\frac{95491.2}{29 \times 150}\right)^2 \quad \frac{W}{C.H} = \frac{568.4 \times 168}{150 \times 29}.$$

$$T = \sqrt{1780.19717 + 481.8025} - 21.95 = 25.61.$$

To be sure to have the abutment strong enough I have made it twenty six feet wide at the joint E. F. and twenty-eight ft. at the joint C. D. In the experimental arch before referred to, the abutment was 26.87 ft. high and was cut away to 23.29 ft. in width, and it was perfectly stable. The thrust of the experimental arch was as much greater than in the present case, owing to the greater span and flatness, and acted with a lever arm of more than twice the length of that in the arch under consideration. The next step is to find out the width of the abutment immediately above the springing point or point of application of the resultant thrust on the abutment joint consistent with the stability of friction. T_0 = the required thickness. $= \frac{\frac{P \cos \theta}{\tan \varphi} - \sin \theta}{n w h_0 b_0}$, $\varphi = 36°$ for masonry and brick work with slightly damp mortar. $\theta = 35°34'$; $h_0 = 17$ ft = height of the abutment above the joint immediately above the springing point, $b_0 = 1$, $n = 1$, $w = 125$ lbs.

$$\therefore T_0 = \frac{\frac{164136 \times .81344}{.72654} - .58165}{125 \times 17} = \frac{88298.60256}{2125} =$$

41.55 ft. This being so much wider than is required to prevent overturning, to save material I have made the bed joints of the abutments nearly perpendicular to the thrust as shown on the drawing. This method is recommended by Trautwine; he also recommends dowels and cramps. Instead of these last it will be seen that in some places I have broken the regular courses by stones reaching through two courses. These stones must all be laid in cement making a solid mass of masonry no more likely to slide at this joint than at any joint below it, although the lower joint has more weight on it. The back part of the abutment will be filled in with earth from the bottom up to the level of the roadway. This I do not take into consideration at all except as a factor of safety

Thesis drawing No. 3 shows the elevation of the whole arch and a longitudinal section through the arch and abutment, showing earth backing and giving dimensions; also a cross section at the crown and at the springing point. It will be seen that ~~the~~ the faces of the abutments project one foot beyond the arch ring, and spandrel walls. The general method of building I have indicated in the drawing. The abutment backing must be firmly bound throughout its whole height to the abutment walls by stretchers extending well into each. I have shown weeping holes to let off the water that soaks through the roadway. I should put in four of these at each end. These should be of glazed stoneware pipe, four inches in diameter. A quantity of small stones should be put in at the end of these pipes to prevent the earth from clogging them.

The next step is to design a timber centre upon which to construct the arch I propose to use spruce timber. In Van Nostrand's Engineering Magazine of last year is a method of distributing the timbers for arch centres, which I have partially followed. The form of the centre and size of the timbers I have shown in Thesis Drawing No 2. I have designed to use nine trusses. The width of the arch being 35 ft. 2 in. each half truss will have to sustain a weight of 133770 lbs. for 52 feet in length × 3.5 ft. in depth × $\frac{35'2''}{8}$* = 133770 or since since I have six points of support, each strut or pair of struts must bear $\frac{133770}{6}$ = 22295 lbs. pressure These forces all act towards the geometrical centre of the arch. I have represented each on the drawing by a full red line. I have resolved each one of these forces where a pair of struts act into two components acting in the direction of each strut, as shown by the dotted red lines; each line being marked with the force it represents. I have also resolved the force acting along each strut into horizontal and vertical components.

The strut a 127 in. long. has to bear a thrust of 22295 lbs.
" " b 147 " " " " " " " 12700 "
" " c 134 " " " " 13850 "
" " d 112 " " " " 22295 "
" " e 127 " " " " 16950 "

* This should be multiplied by 168 lbs. per cubic foot.

The strut f 108 in long has to bear a thrust of 12200 lbs.
" " g 66 in " " " " 22295 "
" " h 97 " " " " 16658 "
" " i 26 " " " " 17300 "
" " v 121 " " " " 17300 "

From Rankine Section 158 when the length of a pillar is ~~length~~ less than thirty times the diameter, the following formula, deduced by Mr. Lewis Gordon from Hodgkinson's experiments, answers the present case. The crushing load $P = \frac{fs}{1+a\frac{l^2}{h^2}}$ $f = 7200$ for timber, $a = \frac{1}{250}$ for rectangular pillars h = least diameter, l = length, s = sectional area of the timber, is the quantity I wish to find, since I have P and assume the least diameter $s = \frac{P \times 1 + a \cdot \frac{l^2}{h^2}}{f}$ for the strut g assuming $h =$ 6 in. $s = \frac{22295 \times 1 + \frac{1}{250} \times \frac{16129}{36} \times 7}{7200} = \frac{435733.480}{7200} =$ 60.5, $\frac{60.5}{6} = 10$ ∴ 6×10 is the required size of this timber with a factor of safety of 7. Rankine says from 4 to 5 for a dead load with good ordinary workmanship and materials. This centre must bear more than a dead load from the fact that it is subject to sudden shocks from falling stones. On the other hand it is a temporary structure and not subject to rotting &c. The factors of safety I have used range from 5 to 10. For the strut b assuming $h = 5$, $s = \frac{12700 \times 1 + \frac{1}{250} \times \frac{21609}{25} \times 6.5}{7200} = 57.1$, $\frac{57.1}{5} = 10$ ∴ This strut should be 5×10 with a factor of safety of 6.5 For the strut c assuming $h = 5$ $s = \frac{13850 \times 1 + \frac{1}{250} \times \frac{17956}{25} \times 7}{7200} = 52.1$, $\frac{52.1}{5} = 10$ ∴ This strut should be 5×10 with a factor of safety of 7. For the strut d assuming $h = 6$ in $s = \frac{22295 \times 1 + \frac{1}{250} \times \frac{12544}{36} \times 8}{7200} = 59.3$, $\frac{59.3}{6} = 10$, ∴ This strut should be 6×10 with a factor of safety of 8. For the strut e assuming $h = 5$ in, $s = \frac{16950 \times 1 + \frac{1}{250} \times \frac{16129}{25} \times 6}{7200} = 50.5$, $\frac{50.5}{5} = 10$ ∴ this strut must be 5×10 with a factor of safety of 6. For the strut f, assuming $h = 4$, $s = \frac{12200 \times 1 + \frac{1}{250} \times \frac{11664}{16} \times 6}{7200} = 39.8$; $\frac{39.8}{4} = 10$ ∴ This strut must be 4×10 with a factor of safety of 6. For the strut g assuming assuming $h = 4$, $s = \frac{22295 \times 1 + \frac{1}{250} \times \frac{4356}{16} \times 6}{7200} = 38$ $\frac{38}{4} = 10$ ∴ This strut must be 4×10 with a factor of safety of 6. For the strut h assuming $h = 4$, $s = \dfrac{16650 \times 1 + \frac{1}{25} \times \frac{9404}{16} \times 5}{7200} = 38.7$. $\frac{38.7}{4} = 10$. ∴ This strut should be 4×10 with a factor of safety of 5. For the strut i assuming $h = 3$, $s = \dfrac{17300 \times 1 + \frac{1}{250} \times \frac{676}{9} \times 10}{7200} = 31.2$, $\frac{31.2}{3} = 10$ ∴ this strut should be 3×10 with a factor of safety of 10. For the strut v, assuming $h = 5$, $s = \dfrac{17300 \times 1 + \frac{1}{250} \times \frac{14641}{25} \times 6.5}{7200} = 52.2$, $\frac{52.2}{5} = 10$ ∴ this strut should be 5×10 with a factor of safety of 6.50. To find the section of the pillar l whose length is 71 in. This pillar must bear $22295 + 9850 \times 2 = 41995$ lbs. Assuming $h = 7$ $s = \dfrac{41995 \times 1 + \frac{1}{250} \times \frac{5041}{49} \times 8}{7200} = 65.8$, $\frac{65.8}{7} = 10$ ∴ This strut must be 7×10 with a factor of safety of 8. The next step is to find the resultant of the vertical thrusts at (2) and from that the thrust on m, and from that the cross section of m. This vertical thrust mentioned above is equal to the sum of the vertical components of the thrusts on c, d, and e = 12350 + 22000 + 11650 = 46000 lbs. I have represented this force by the full red line o, q. Complete the triangle of forces and the dotted red line o, r, represents the thrust in the direction of m = 104650 lbs. In the same way the vertical thrust at (3) = the sum of the vertical components of f, g, h, 9950 + 21150 + 4950 = 36050 represented by the line o p. Complete the triangle of forces, the line o s, represents the thrust on the strut n = 70500. For the strut m, l = 160 inches, P 104650. Assuming h = 12, $s = \frac{104650 \times 1 + \frac{1}{250} \times \frac{25600}{144} \times 6}{7200} = 149$, $\frac{149}{12} = 12.4$ ∴ this strut must be 12×12 with a factor of safety of a little less than six. For the strut n, l = 210 in. P = 70500. Assuming h = 12 $s = \frac{70500 \times 1 + \frac{1}{250} \times \frac{44100}{144} \times 6.5}{7200} = 141.5$ $\frac{141.5}{12} = 12$ ∴ this strut should be 12×12 with a factor of safety of 6.5. The next step is to find the dimensions of the tie beam at its different divisions j, j_1, j_2 & j_3. The thrust on j = the resultant of the horizontal thrusts of b, c, d and e, and f, g, h & i. the horizontal thrust of b is counteracted by b, on the other side of the centre. The thrusts towards the right consist of d, e, g and h. 3550 + 12250 + 7000 + 15900 = 38700. The thrusts towards the left consist of the horizontal components of the thrusts on c, f and i, 6200+7000+7300=20500. 38700 − 20500 = 18200 lbs. To find the cross section of j. assuming $h=6$ $s = \frac{18200 \times 1 + \frac{1}{250} \times \frac{32400}{36} \times 6}{7200} = 69.7$, $\frac{69.7}{6} = 12$. ∴ this tiebeam must be 6 × 12 with a factor of safety of a little more than 6. The tie-beam at j. must bear the resultant of the horizontal thrusts on c, d, e, f, g, and h; that on f balances that on g ∴ we have a pull on j. to the righ of 12250 + 3550 − 6200 = 9600 lbs. There is a thrust to the right on j. of 15900 lbs.– The tie-beam j, j_1, j_2 must be made of the same dimensions throughout its whole length to preserve the symetry. If it sustains a thrust of 18200 lbs. at j. it is strong enough to sustain 15900 lbs. thrust at j. At j_2 it has to sustain a pull in opposite directions of 15900 lbs. and 7300 lbs = 23200 lbs. For spruce the resistance to stretching = 160000 per. sq. inch. $\frac{160000}{23200} \times 72 =$ the factor of safety. j_3 has to resist a thrust of 7300 and of course is amply strong enough. This tie-beam should be spliced at, or very near 5, 7 and 9. The beam 1, 2 and 3 immediately below the last, has no thrust to bear but it must be wide enough to carry the wedges used in striking the centre and to receive the forces from above, and to transmit them to x, m and n. I have constructed it of two timbers 4×12 with a middle piece at 1, 2, 3 &c as indicated by dotted lines. At these points the three pieces should be fastened together with wooden pins as indicated in details at B. These timbers should be spliced midway between 1, 2 and 3 so that no two splices shall come directly opposite. t must be a 12×12 timber bolted to the timbers above as indicated in details. The next step is to design the circular rib, the whole length of which is 104 ft. Dividing this into twelve divisions each one of these parts will be $\frac{104 \text{ ft}}{12} = 8'.8\text{ in} = 104\text{ in}$. This length l, u, on the drawing must hold up a mass of masonry weighing 22295 uniformily distributed over its whole length. To get the cross section of this rib I use Rankine's Formula, Sec. 162 (6) $M^0 = mWl = nfbh^2$. $m = 1/8$, $W = 22295$ $l = 104$, $n = \frac{1}{6}$. f = a mean between 9900 and 12300 $= 11100$, $b = \frac{mWl}{nfh^2} = \frac{1/8 \times 22295 \times 104}{1/6 \times 11100 \times 144} \times 6 = 6.5$ with a factor of safety of 6. Hence two 3 in planks placed edgewise will be sufficiently strong. I have, however, constructed the rib with a 4 in. plank in the centre, with a 3 in plank on each side of it,—each being 12 in in depth and firmly bolted together as shown in details at E; The timbers must be cut to fit the curve on the upper side. The joints of the middle piece must come directly over the ends of the struts, and the joints of the outside pieces must come midway between the struts, no two joints coming directly opposite each other. Where there is but a single strut as at C it is to be morticed into the rib 2 in. Where there are two struts as at A they must abut against a piece of oak set into the rib two inches, as shown in details at A. To test the divisions of this rib, as to bending, I use Rankine's Formula Sec 169 (12). Deflection under a given load $W = 22295$ lbs. $v_1 = \frac{n''' W c^3}{E n' b h^3}$, $n''' = \frac{5}{48}$, $c = \frac{l}{2}$ for beam supported at both ends, $E = 1600000$, $n' = \frac{1}{12}$, $b = 10$, $h = 12$, $v_1 = \frac{5/48 \times 22295 \times 140608}{1600000 \times \frac{1}{12} \times 10 \times 1728}$ $= .142$ inches so that the rib is stiff enough.

Next to find the dimensions of the planking w, w, w, to cover the ribs. These will be 48 in. long and must bear a mass of stone 1.5 × 4 ft. × 3.5 ft. × 168 lbs = 3528 lbs. To find the least width of plank to be used, Assuming $h = 3$ $b = \frac{1/8 \times 3538 \times 48 \times 6}{6 \times 11100 \times 9} = 7$ in. ∴ no plank less than 7 in wide should be used; this allows a factor of safety of 6. To find the deflection of such a plank under this load. $v_1 = \frac{5/48 \times 3528 \times 188\ 24}{1600000 \times 1/12 \times 7 \times 27}$ = .20 inches. This deflection is to great; the plank should be wider. In no case however, will any one course of voussoirs come entirely on one plank of this width. It is not intended that more than one half a course shall bear on one plank, since plank even 12 in. wide can be used. On these laggings are to be placed strips of two inch plank 3 in. wide running through the whole width of the arch, at the bed joints of the voussoirs. These I have show in the details at A 1, 2 & 3. I propose to strike the centre by means of wooden wedges shown in the details The Drawing of Details at B shows the method of framing the struts. They are to be let into

* I have not taken into account the weight of the timbering itself; but in all cases I have used a larger factor of safety than is required.

the tie-beam not over 2 inches and 2 in. planks are to be spiked to each side of the tie-beam and strut for a distance of two feet up the strut. Planks are also to be spiked to the rib and down the struts a distance of two or three feet. On the Drawing of details from D to E may be seen a method of cross bracing; these braces should also be placed at a, d & g. I would als use diagonal braces spiked to the under sides of the circular ribs. The centre supports I have designed to rest each on eight piles of not not less than 10 in. in diameter capped with 12×12 timber in the direction of the axis of the arch; and the block E to rest on another 12×12 timber placed at right angles to the first. *The piles should be driven until a 20 ft. pile sinks not more than 2 in at the last blow of the ram. From Rankine, Sec. 402

P = greatest load a pile will bear without sinking further $= \sqrt{\frac{4ESWh}{l} + \frac{4E^2S^2x^2}{l^2}} - \frac{2ESx}{l}$

E = Modulus of elasticity = 1600000

S = Sectional area of the pile = 78 in.

W = Weight of ram = 1500 lbs.

h = Height from which it falls = 240 in.
x = Distance the pile is driven by last blow = 2 in
l = Length of pile = 240 in
P the load to be held up = 46000 × 2 + 22295 + 9850 × 2 = 133995. This should equal

$$\frac{4 \times 1600000 \times 78 \times 1500 \times 240}{240} = 788,800,000,000 +$$

$$\frac{4 \times 2560000000000 \times 6084 \times 4}{57600} = 4326400000000 -$$

$$\frac{2 \times 1600000 \times 78 \times 2}{240} = 2080000 = 172828 \text{ lbs.}$$

This is what one pile will bear. 8 piles will bear 1382592 ∴ 133995 lbs. will be sustained by these eight piles, allowing a factor of safety of 10. It is not necessary to use a pile foundation under the abutments as there is good gravel bottom; under two or three ft. of mud, which must be removed. The quantities of masonry to be used are as follows: 100 cu. yds. of Ashlar masonry for spandrel walls, 475 cu. yds. of voussoirs for arch ring, 1496 cu. yds. of abutment and spandrel backing, 934 cu. yds of Ashlar masonry for abutment facings, 256 running ft of stone railing shown in section at Fig 5 Drawing No. 3 and 4 large stone posts as shown in Section at Fig. 6.

475 yds. voussoirs
1034 " Ashlar masonry
1496 " backing

The quantity of timber required for the arch centre, expressed in board measure = 65767 making no allowance for waste and cutting

Respectfully Submitted
with the accompanying drawings.

William P. Jewett Class '73.
Savin Hill Ave. Boston
April 17th 1876